Information Theory at the Extremes

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Information Theory in Wireless

Wireless communication is an old subject.

Only in the past decade has information theory been applied to study wireless channels in a serious way.

Fundamental new ways of looking at wireless fading channels have emerged.

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Opportunistic Communication

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- key concept: multiuser diversity

- implementation in IS-856 (HDR), a 3G wireless standard for data. (Tse 99)

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Fading is exploited rather than mitigated.

Example #2:
Space Time Communication

- capacity of MIMO channels (Foschini 96, Telatar 99)

- invention of space-time codes (Alamouti 98, Tarokh et al 98)

- Implementation of Alamouti scheme in 3G standards

- prototypes and startups on MIMO systems

Role of Information Theory in Wireless

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Provides a new and fundamental way of looking at the problem.

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We need qualitative insights from information theoretic results.

Current State of the Field

Communication over point-to-point, many-to-one and one-to-many wireless links are reasonably well-understood.

In more complex scenarios (ad-hoc networks, spectrum sharing by multiple systems), new and important issues come to the forefront:

- how to deal with interference?

- how to optimally use relays?

- how to use multiple antennas in networks?

Unfortunately, network information theory problems are very difficult.

How to Cop Out

Problems in network information theory are open for ∞ years.

Unclear even if they can be "solved" to the extent that point-to-point communication was solved by Shannon.

In designing communication systems, one usually thinks in terms of an asymptotic regime to describe its operation.

By looking at various asymptotic regimes, more progress is possible.

Three Extremes

- very quiet (high SNR)

- very noisy (low SNR)

- very large (many nodes)

We will review recent results in these 3 regimes and assess how they may be able to answer the questions posed.

High SNR Regime

Three main sources of randomness in a wireless link:

- fading

- interference

- noise

The high SNR regime is relevant for systems in which the first two sources are dominating.

Particularly appropriate for studying multiple antenna systems.

High SNR Capacity

Spectral efficiency of the AWGN channel:

$$C = \log(1 + \mathsf{SNR}) \quad \text{bits/s/Hz.}$$

At high SNR,

$$C \sim 1 \log \mathsf{SNR} \quad \text{bits/s/Hz.}$$

System is degree of freedom limited.

Increasing the available degrees of freedom gives the biggest bang for the buck.

Multiple Antennas

In a wireless link with m transmit and n receive antennas and rich scattering (Foschini 96):

$$C \sim \min\{m, n\} \log \mathsf{SNR} \quad \text{bits/s/Hz.}$$

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If channel is not *a priori* known at the receiver but remains constant for coherence time of t_c symbols (Zheng,Tse 02):

$$C \sim m^*(1 - m^*/t_c) \log \mathsf{SNR} \quad \text{bits/s/Hz,}$$

where $m^* = \min\{m, n, t_c/2\}$.

Diversity

Capacity assumes infinite-depth interleaving over the fluctuations of the channel.

In a slow fading environment this is impossible.

Unreliability of the channel due to fading becomes a first-order issue.

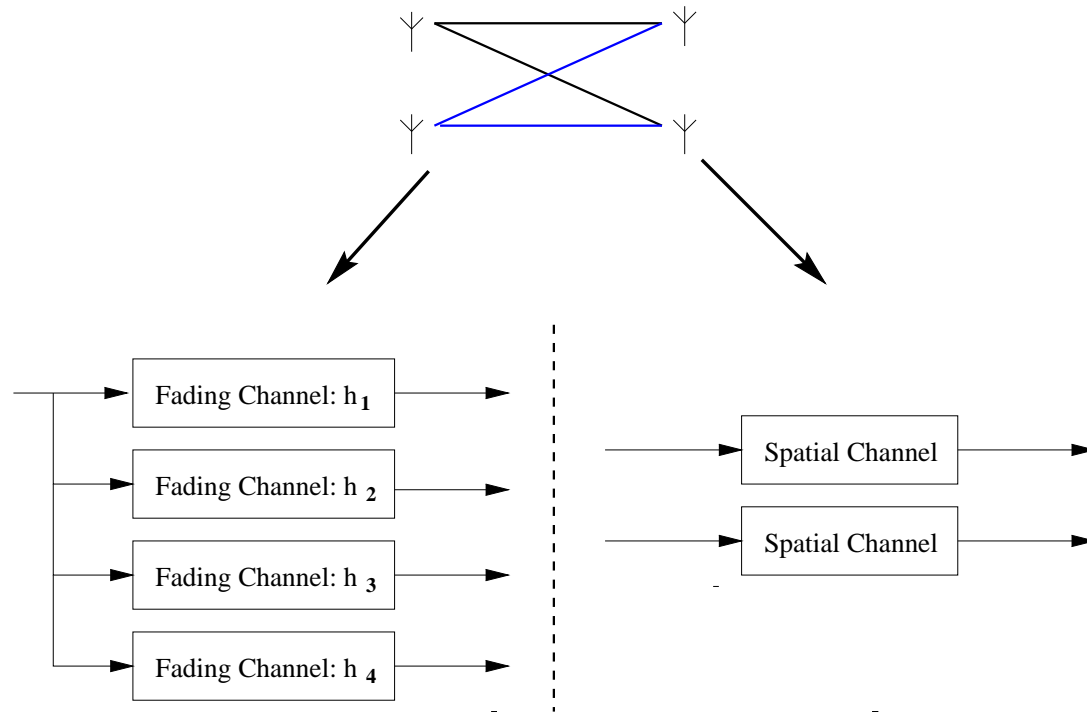In a 1 by 1 Rayleigh fading channel at high SNR, very poor error probability:

$$P_e \sim \mathsf{SNR}^{-1}$$

In a m by n channel, however,

$$P_e \sim \mathsf{SNR}^{-mn}$$

Multiple antennas provide a maximum of mn diversity gain.
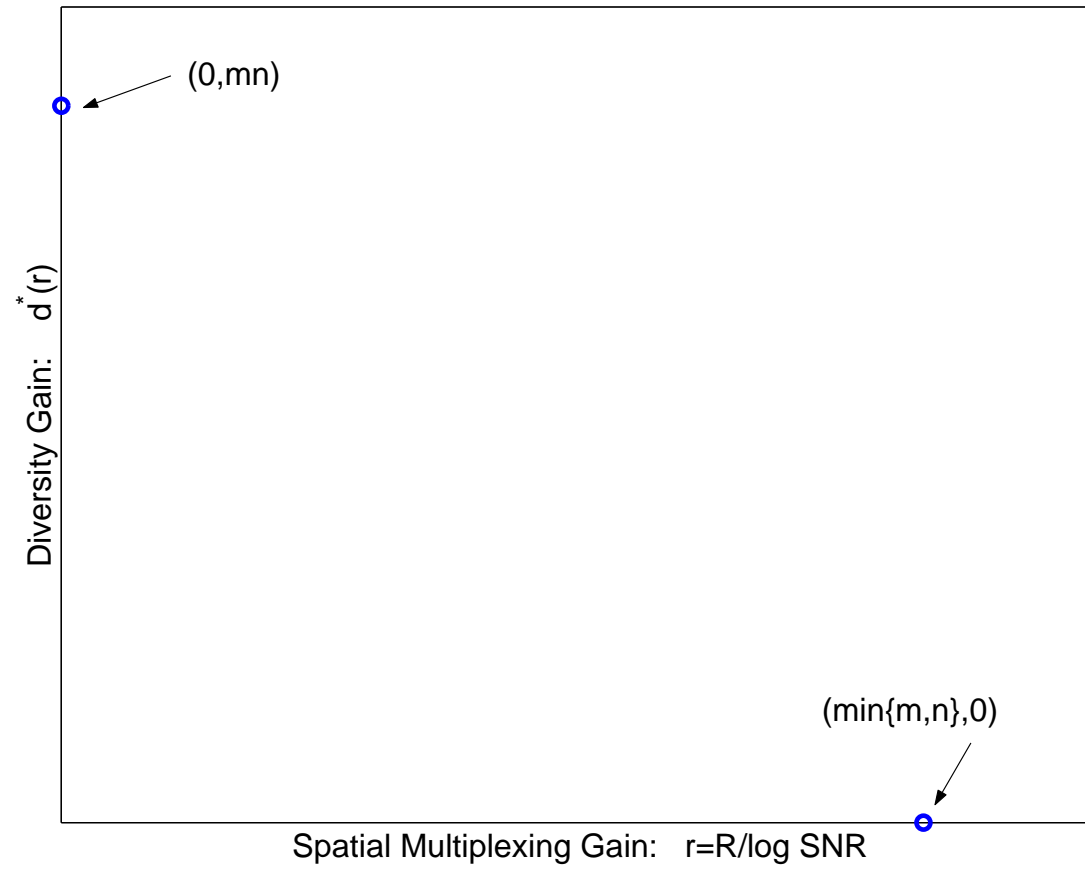
Diversity and Multiplexing

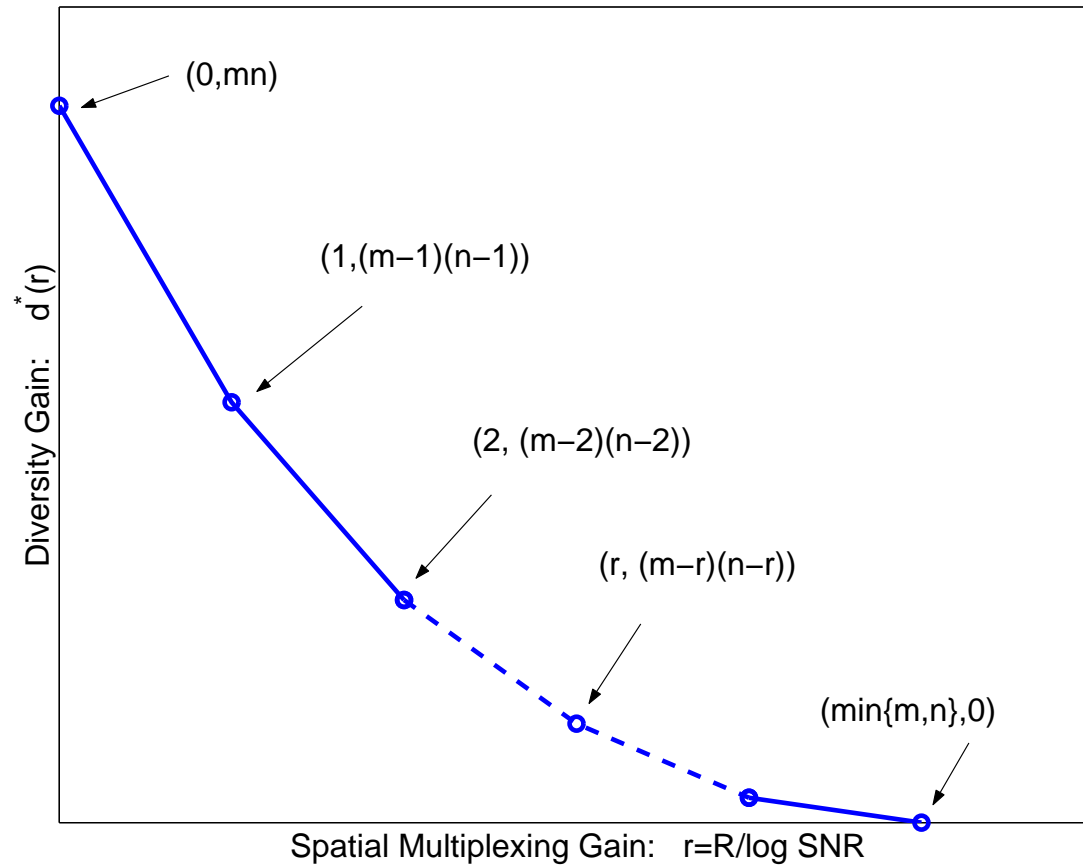But each is only a single-dimensional view of the situation.

The right way to formulate the problem is a tradeoff between the two types of gains.

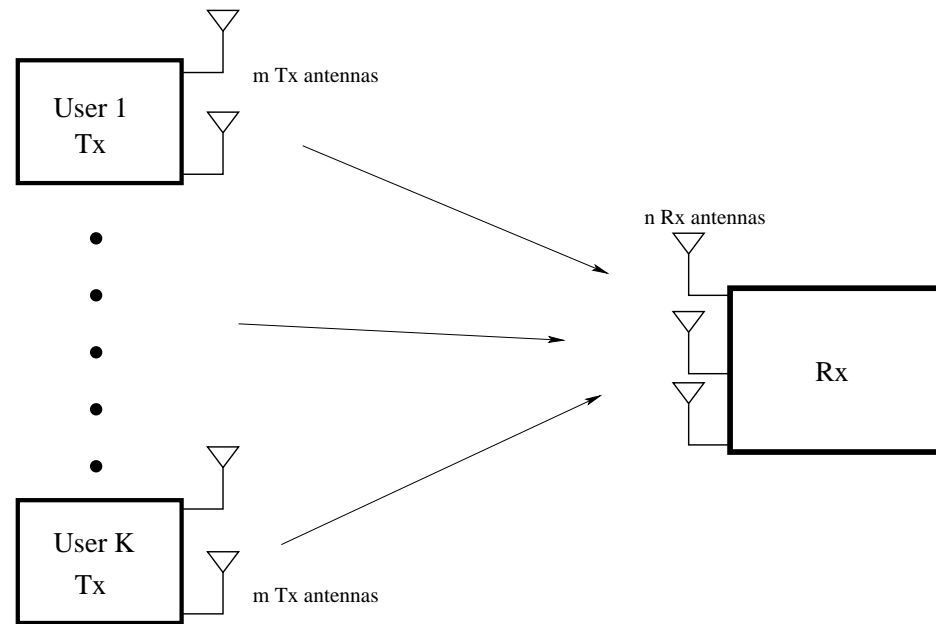But what is the fundamental tradeoff achievable by any space-time code?

Optimal Tradeoff

(0,mn)

Diversity Gain: d*(r)

(min{m,n},0)

Spatial Multiplexing Gain: r=R/log SNR

Optimal Tradeoff

To guarantee a multiplexing gain of r (r integer), the best diversity gain achievable for any space-time code is $(m-r)(n-r)$.
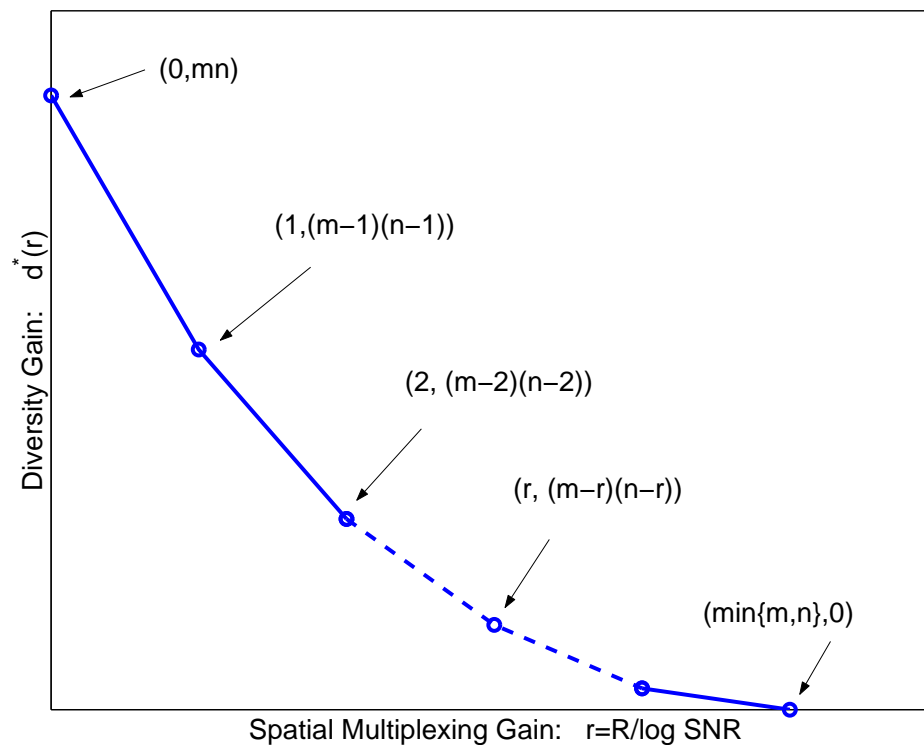
(Zheng,Tse 02)

Multiple Access

In a point-to-point link, multiple antennas provide diversity and multiplexing gain.

In a system with K users, multiple antennas can discriminate signals from different users too.
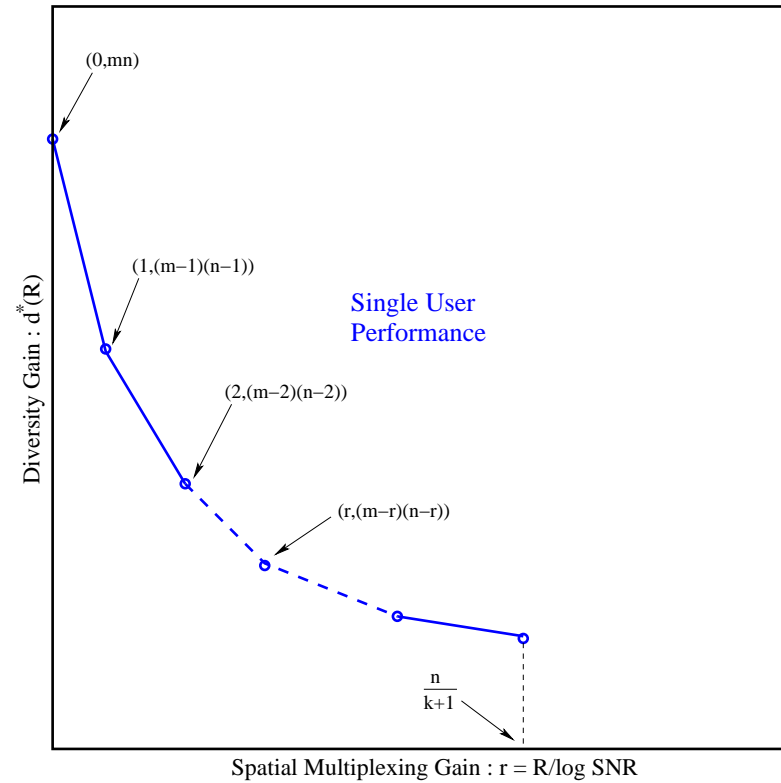
Multiuser Diversity-Multiplexing Tradeoff

If $m < \frac{n}{K+1}$, diversity-multiplexing tradeoff of each user is as though it is the only user in the system.
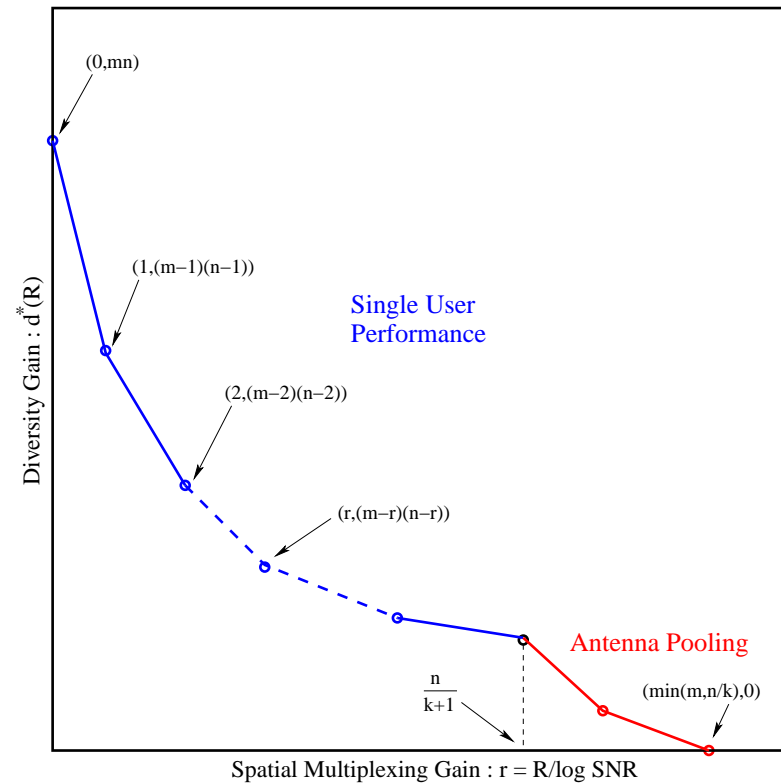
(Tse,Viswanath,Zheng 02)

Multiuser Tradeoff: $m > \frac{n}{K+1}$

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- Single-user diversity-multiplexing tradeoff up to $r^* = \frac{n}{K+1}$.

- For r from $\frac{n}{K+1}$ to $\frac{n}{K}$, tradeoff is as though the K users are pooled together into a single user with mK antennas.

Multiple Antennas in General Networks

Multiple antennas serve multiple functions:

- diversity

- spatial multiplexing

- multiple access

- broadcast

- interference suppression

- cooperative relaying

- etc ....

What is the fundamental performance tradeoff in general?

High SNR analysis may give a simple picture.

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Energy-Limited Communication

AWGN channel:

$$C = \log(1 + \mathsf{SNR}) \quad \text{bits/s/Hz.}$$

To minimize energy per bit, one would operate at as low SNR as possible, spreading energy over many degrees of freedom.

Essentially: Ultra-wideband communication (UWB)

Energy-limited Communication

Result is for the AWGN channel. What about for fading channels?

Same conclusion. (Kennedy 60's)

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Caveat:

Transmitted signals have to be very peaky in time and in frequency (need good estimate of the channel).

If signals are spread evenly over the degrees of freedom , then energy per bit goes to infinity as the bandwidth goes large. (cannot track the channel) (Medard, Gallager 02)

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Lesson:

In fast time-varying channels, one must be very careful how to spread.

Energy Efficient Communication in Networks

For multiaccess (many-to-one), broadcast (one-to-many) and interference channels, problem decomposes into separate point-to-point links.

Transmissions between different users pairs are kept orthogonal, and each user pair should do UWB.

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Does that mean the network problem is solved?

A Simple Relay Network

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UWB is then optimal for each link.

However, this is not necessary.

eg. Relay can amplify and forward the analog signals it receives to the final destination.

This creates a tension:

UWB transmission is energy efficient for the direct S-D link but may make it difficult for the relay to help out (too much noise amplification).

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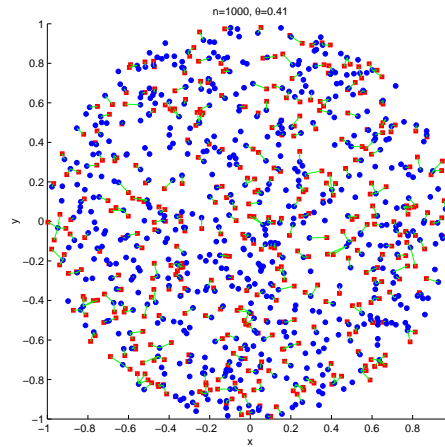
Very Large Networks

Traditionally, information theory looks at simple networks and try to analyze their capacity.

A different approach: look at large networks and ask how the capacity scales with number of nodes.

(Gupta and Kumar 00)

Motivated by trying to understand how ad hoc networks scales.

Basic Model

n=1000, θ=0.41

- n nodes in a disk of area A.

- There are n source-destination pairs.

- Each node can also serve as a relay for other nodes.

- Relaying is performed by decoding and re-transmitting packets.

- Packet reception is successful if the signal-to-interference ratio exceeds a threshold.
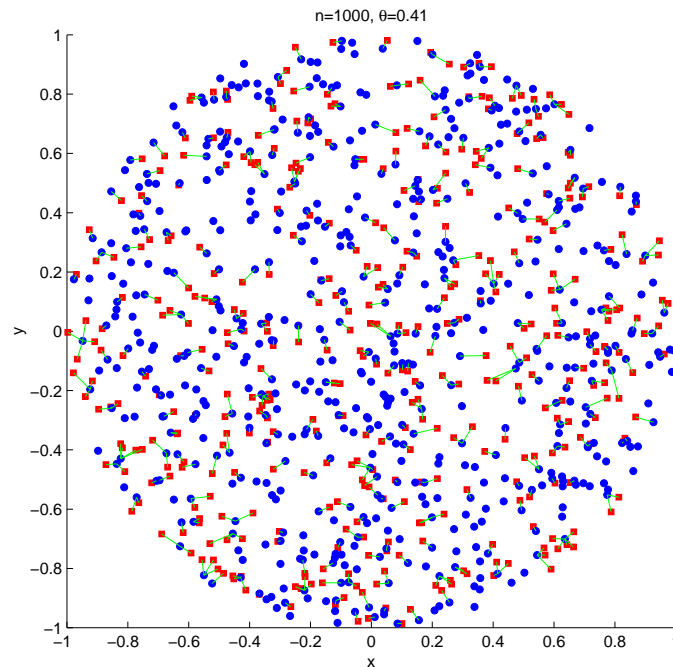
Throughput Scaling

Main results:

- Upper Bound: Total bit meter per second communicated in the network is $O(\sqrt{nA})$.

- Achievability:: If the n source-destination pairs are randomly chosen, then this upper bound can be achieved.

Note: since distance between each S-D pair is $O(\sqrt{A})$, this implies the throughput per pair scales like $1/\sqrt{n}$ bits per second.

What's the Problem?

- communication confined to nearest neighbors to allow dense spatial reuse.

- a typical route has order $\sqrt{n}$ relay nodes

- each packet is transmitted of the order $\sqrt{n}$ times.

System is interference-limited.

Large-scale Information Theory

The communication schemes are information theoretic sub-optimal.

- Why should interference be treated as interference?

- Why should we insist that relays have to fully decode the received signal?

Can one do better using information theoretically optimal techniques?

Large-scale Information Theory

(Xie and Kumar 02)

Large-scale Information Theory

No.

(Xie and Kumar 02)

But under some further assumptions:

- Each node is power-constrained.

- There is a minimum distance between the nodes as the network scales.

- Path loss: received power decays faster than $1/r^6$ with the distance r.

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- Path loss: received power decays faster than $1/r^6$ with the distance r.

On the other hand, if received power decays slower than $1/r$, then information theory helps!

For path loss between $1/r$ and $1/r^6$, problem is still open.

Comments and Questions

- In the information theoretic setting, capacity is power-limited rather than interference-limited.

- Area A of the network goes to infinity. What happens in a dense network?

- Growth rate depends on details of channel model; makes me feel a bit queasy.

- Traffic pattern is assumed to be homogeneous and point-to-point. Is "bit meter per second" the right invariant for other traffic patterns?

Conclusion

Three asymptotic regimes give different insights to fundamental limits of wireless networks.

Still many open questions remain.

We need to look more at applications on how networks are used to find the right questions.