Optimal Placement of Training for Time Varying ISI Channels

Srihari Adireddy and Lang Tong
Dept. of ECE
Cornell University
Ithaca, NY 14853
{srihari,ltong}@ee.cornell.edu

Abstract — **We consider the problem of optimal placement of training for time varying channels with inter symbol interference (ISI). We assume that channel has a finite impulse response of length $(L+1)$ and that each tap of the channel fades independently and identically. The time variation of the fading is modeled by a Gauss-Markov process. We constrain the training symbols to be placed in clusters of length $(2L+1)$ and each cluster contains only one non-zero training symbol. We show that at high SNR, the cumulative channel MSE is minimized by placing the training clusters periodically in the packet. A special case ($L = 0$) of this result is the optimal placement of pilot symbols for Rayleigh fading channels with Gauss-Markov variation.**

I. Summary

One of the main obstacles in achieving high data rates in wireless communication is the time varying nature of the propagation channel. Inserting training symbols in the data stream is a popular technique for tracking the channel in such an environment. The optimization of the placement of training in a data stream is thus an important issue both from theoretical and practical points of view.

We assume that the channel has a finite impulse response of length $(L+1)$. The channel at time instant k is denoted as $\mathbf{h}_k = (h_k^{(0)}, h_k^{(1)}, \cdots, h_k^{(L)})^t$. We assume that $\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I})$. The time variation of the fading is modeled as a Gauss-Markov process with correlation α. That is,

$$\mathbf{h}_k = \alpha \mathbf{h}_{(k-1)} + \mathbf{z}_k, \tag{1}$$

where $\mathbf{z}_k$ is an i.i.d process distributed as $\mathcal{CN}(0, (1-\alpha^2)\mathbf{I})$. The channel output is corrupted by the additive, zero mean, white Gaussian noise n_k with variance σ^2.

On this channel, data is transmitted through packets of length $(N+P)$ where N is the number of unknown symbols and P is the number of known symbols. Known symbols are inserted into the packet in order to estimate the time-varying channel. The placement of known symbols can be specified by a set $\mathcal{P}$ that contains the positions at which they are present or, as shown in Fig. 1, the placement can be specified by $\mathbf{r} = (\mathbf{m}, \mathbf{n})$, that is, two tuples $\mathbf{m} = (m_1, \cdots, m_J)$ and $\mathbf{n} = (n_1, \cdots, n_{J+1})$ where $\mathbf{m}$ gives the lengths of unknown symbol blocks and $\mathbf{n}$ the lengths of known symbol clusters. For placements that end with unknown symbols we have $n_{J+1} = 0$. The number of elements in each of these tuples is a variable and depends on the placement scheme.

*This work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564.

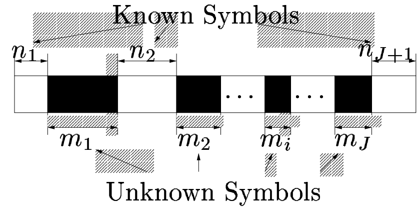

Fig. 1: Representation of Placement Schemes

We limit ourselves to the case when P is of the form $i(2L+1)$, where i is a positive integer. We also assume that every training cluster is of length $(2L+1)$. Further, we assume that each training cluster starts and ends with L zeros and has only one non-zero training symbols that is placed at the center of the cluster. Such training clusters are shown to be optimal in a certain sense in [1, 2]. We are interested in the optimal placement of these training clusters. We also assume that every packets starts and ends with at least one training cluster implying $n_1 \geq (2L+1)$ and $n_{(J+1)} \geq (2L+1)$. This constraint also implies that $i \geq 2$.

The receiver forms an MMSE estimate of the channel taps using training only. The objective is to optimize $\mathbf{r}$ so as to minimize the cumulative channel MSE over all those channel taps that affect the output due to data symbols. In other words,

$$\mathcal{P}^* = \arg\min_{\mathcal{P}} \sum_{(k-i)\notin\mathcal{P}} \left(\mathrm{E}\left\{ \left| \tilde{h}_k^{(i)} \right|^2 \right\} \right), \tag{2}$$

where $\tilde{h}_k^{(i)}$ is the MMSE estimate of $h_k^{(i)}$.

The following theorem gives the optimal placement at high SNR.

Theorem 1 *Under the assumption that $P = m(2L+1)$, where $m \geq 2$, the placement $\mathbf{r}^*$, that is optimal with respect to (2) at high SNR is given by*

$$n_i = 2L+1 \quad i = 1, \cdots, m \tag{3}$$

$$m_i \in \left\{ \lfloor \frac{N}{m-1} \rfloor, \lfloor \frac{N}{m-1} \rfloor + 1 \right\}. \tag{4}$$

This placement scheme is the QPP-$(2L+1)$ placement scheme that was introduced in [2].

References

[1] M.Dong and L.Tong. Optimal Placement of Training for Channel Estimation and Tracking. In *Proceedings MILCOM'02*, Washington, DC., October 2001.

[2] S.Adireddy, L.Tong, and H.Viswanathan. Optimal placement of known symbols for unknown channels . *Submitted to IEEE Trans. Info. Theory, See also http://www.acsp.ece.cornell.edu*, March 2001.