Exploiting Decentralized Channel State Information for Random Access

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***Abstract*—We study the use of channel state information (CSI) for random access in fading channels. Traditionally, random access protocols have been designed by assuming simple models for the physical layer where all users are symmetric, and there is no notion of channel state. We introduce a reception model that takes into account the channel states of various users. Under the assumption that each user has access to its CSI, we propose a variant of Slotted ALOHA protocol for medium access control, where the transmission probability is allowed to be a function of the CSI. The function is called the *transmission control*. Assuming the finite user infinite buffer model we derive expressions for the maximum stable throughput of the system. We introduce the notion of asymptotic stable throughput (AST) that is the maximum stable throughput as the number of users goes to infinity. We consider two types of transmission control, namely, population-independent transmission control (PITC), where the transmission control is not a function of the size of the network and population-dependent transmission control (PDTC), where the transmission control is a function of the size of the network. We obtain expressions for the AST achievable with PITC. For PDTC, we introduce a particular transmission control that can potentially lead to significant gains in AST. For both PITC and PDTC, we show that the effect of transmission control is equivalent to changing the probability distribution of the channel state. The theory is then applied to code-division multiple-access (CDMA) networks with linear minimum mean-square error (LMMSE) receivers and matched filters (MF) to illustrate the effectiveness of using channel state. It is shown that through the use of channel state, with arbitrarily small power, it is possible to achieve an AST that is lower-bounded by the spreading gain of the network. This result has implications for the reachback problem in large sensor networks.**

***Index Terms*—Distributed channel state information (CSI), distributed transmission control, fading channels, maximum stable throughput, random access, Slotted ALOHA.**

I. Introduction

THE rapid increase in the demand for data rate over wireless channels has led to a rethinking of the traditional network architecture and design principles. Cross layer design, where information is exchanged between layers is being explored as an alternative to the traditional design paradigm [1]. In this context, allowing interaction between the medium access layer (MAC) and physical layer (PHY) layers seems natural, especially for mobile wireless communication where the channel quality is changing with time. As illustrated in Fig. 1, users might experience different channel conditions and this knowledge can be used to control the access to medium and improve the throughput of the network. The source of asymmetry between users might be due to various parameters such as propagation channel gain, distance from the base station, transmit power capabilities, etc.

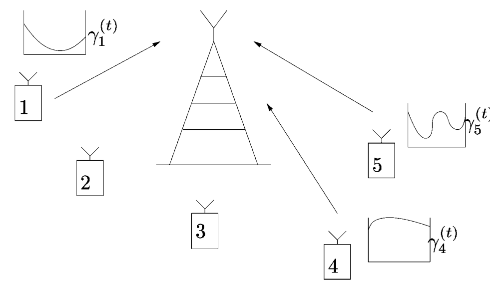

Fig. 1. Cellular uplink.

There is a recent line of work that studies the effect of channel state information (CSI) on resource allocation in multiple-access fading channels [2]–[7], [9], [10]. These papers however assume a centralized controller that has the knowledge of the channel states of all the users in the network. While this assumption might be reasonable for channel allocation on the downlink, a similar assumption on the uplink is not easy to justify. Resource allocation on the uplink, specifically power control, with each user having access to his channel state alone was considered by Telatar and Shamai in [11]. A simple threshold power control scheme is proposed in which each user transmits when his channel state is better than a certain threshold. The threshold is chosen so as to keep the number of active users small compared to the total number of users. It is demonstrated that this scheme achieves a sum capacity that is close to that obtained by the optimal centralized power control scheme. Decentralized schemes have also been considered for code-division multiple-access (CDMA) networks. Viswanath *et al.* [12] have shown the asymptotic optimality of a decentralized power control scheme for a multiple-access fading channel that uses CDMA with an optimal receiver. The effect of decentralized power control on the sum capacity of CDMA with linear receivers and single-user decoders was studied by Shamai and Verdú in [13].

In this paper, we complement the existing information-theoretic literature by considering the effect of decentralized CSI

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under random-access framework. Each user uses the CSI to determine the probability of transmission, whereas the power of transmission is kept constant. We are interested in determining transmission control schemes that maximize the stable throughput of the system. As has been noted in [14], [15], the field of random access is built upon simplistic models for the physical layer. Random-access protocols such as traditional ALOHA, splitting algorithms, and carrier sense multiple access (CSMA) have all been developed assuming that the physical layer behaves like a collision channel. To conduct a meaningful study of the use of CSI in random access, it is necessary to develop models that can first incorporate the channel states of the transmitting users and, second, abstract the increasing sophistication of the underlying signal processing algorithms. One such model is the multipacket reception (MPR) model introduced by Ghez *et al.* [16], [17]. It is possible to model the simultaneous reception of multiple packets using this model but the level of abstraction does not allow for the incorporation of the CSI of the transmitting users. As a result, the version of ALOHA proposed in [16], [17] is symmetric with respect to the users. Random-access protocols that are built upon the MPR model have been proposed by Zhao and Tong [18], [19]. Again, there is no concept of channel state in these protocols. Random access for general reception models without using channel state have also been considered in [20]–[22].

The contents and contributions of this paper can be broadly separated into two parts. In the first part, we focus on deriving a general theory of random access with CSI. Our main contributions in this part can be summarized as follows.

- We introduce a model for the physical layer where the reception is allowed to depend on the *channel states* of the transmitting users and it is also possible to model the simultaneous reception of multiple packets. Any parameter that influences the reception could be chosen as channel state. Examples include propagation channel gain, position of the mobile with respect to the base station, etc. This model can be considered as a generalization of the MPR model proposed by Ghez *et al.* in [16], [17]. Similar generalizations have also been considered in [20]–[22].
- A variant to the classical Slotted ALOHA protocol is proposed where the knowledge of channel state is utilized to vary the transmission probability. The function that maps the CSI to the probability of transmission is termed the *transmission control*.
- Maximum stable throughput [23] is used as a figure of merit to compare different transmission control schemes. We assume a network with finite number of users and infinite buffers and derive the expression of maximum stable throughput of the network as a function of the reception model, CSI distribution, and the transmission control used. The notion of asymptotic stable throughput (AST) defined as the maximum stable throughput of the network as the number of users go to infinity is introduced. The AST expression allows us to derive "good" transmission control algorithms.
- Two types of transmission control schemes are studied namely, population-independent transmission control (PITC) and population-dependent transmission control (PDTC). PITC does not use the size of the network. Such a strategy is attractive when nodes are added and eliminated from the network from time to time because it is not necessary to keep track of the size of the network. We derive expressions for AST with population-independent transmission and characterize what can be achieved by varying the transmission probability as a function of channel state but not the size of the network. In contrast, PDTC, as the name suggests, refers to transmission control schemes that are a function of the size of the network. We introduce a particular PDTC scheme, evaluate its AST, and show that it can be used to obtain significant gains. For either type of control, the effect of using a transmission control sequence is shown to be equivalent to changing the probability distribution of the channel state. Thus, the problem is one of identifying the good target distributions for various reception models.

In the second part, we apply the results of the general theory to CDMA networks and demonstrate the effectiveness of the proposed strategies. We focus on the application of results to CDMA networks that use either a linear minimum-mean square error (LMMSE) multiple-user receiver or a matched filter (MF). This context provides us with two particular reception models for which the theory can be applied. For this application, we assume that the propagation channel gain is used as the channel state and it is assumed that the channel undergoes Rayleigh fading. Our main contributions in this part are as follows.

- We characterize the gain in AST through PITC. It is shown that the gain possible through this technique is quite limited.
- For PDTC, we identify the class of distributions that are good target distributions and construct transmission control schemes that can achieve this target distribution.
- We show that if we use an MMSE multiple-user detector as the receiver, with arbitrarily small power, it is possible to obtain an AST that is lower-bounded by the spreading gain of the system.

The final comment above is important for the uplink of networks that have a large number of nodes but each is equipped with small power. The regime of large number of nodes and small power is relevant to sensor networks [46]. Thus, the theory that we have derived finds an important application in the reachback problem in sensor networks. For us, reachback refers to the data gathering phase of the operation of sensor networks. Typically, hundreds and thousands of sensors, each with limited transmission power capabilities, are deployed in order to collect some information and this information has to be relayed back through some collecting agent like the airplane that is shown in Fig. 2. Thus, our results for CDMA networks have an important implication on the design of the protocol stack for sensor networks.

Almost all other related work in collision resolution is in the analysis and design of the Slotted ALOHA protocol for the cap-

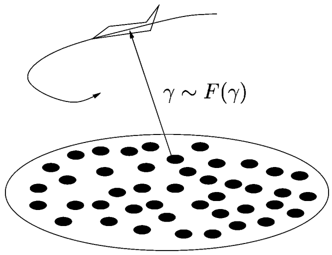

Fig. 2. Reachback in sensor networks.

ture model, a specific model that can be represented with the proposed general reception model. The performance of Slotted ALOHA for uplink in fading channels both with and without capture has been previously explored in [24]–[27] and the references therein. But these papers did not assume that the users have access to their CSI. Design of retransmission probability was considered in [28]–[30]. An important concern in these papers was to make the protocol fair to all the users. In [31], Liu and Polydoros study the design of retransmission probabilities to maximize the throughput, but it was assumed that the design was done by a central controller who has access to channel state of all the users. The Slotted ALOHA scheme where mobiles have knowledge of the uplink signal-to-noise ratio (SNR) was considered in [32], [33]. In [32], Qin and Berry used this knowledge to vary the power of transmission but the transmission probability was kept fixed. It was shown that with the choice considered, the throughput increases with the number of users. The reception model considered was a collision model. In [33], the design of transmission probability was chosen in a heuristic fashion and it was not optimized. In [34], Chockalingam *et al.* studied the design of Slotted ALOHA for correlated Rayleigh-fading channel. It was not assumed that the mobiles have access to the channel state but it was shown that the correlation in the fading channel can be exploited to improve the throughput of ALOHA. Stability analysis for capture model was considered in [35] by Sant and Sharma. It was not assumed that the nodes have access to their CSI. The retransmission probabilities of different users was therefore kept fixed. Characterization of stability region for Slotted ALOHA in networks with multiple antennas (without the use of CSI) has been considered in [21], [22].

The rest of the paper is organized as follows. In Section II, we describe the system model in detail. In Section III, we derive the expression for the maximum stable throughput of the system under consideration. In Section IV, we introduce the notion of AST and derive the expressions for AST for various types of transmission. In Section V, the theory is applied to CDMA networks. In Section VI, we list our concluding remarks and describe some interesting directions for related future research. The proofs of all the theorems and propositions have been included in the Appendix.

II. System Model

We consider a network where n users are communicating with a base station over a common channel. Each user has a buffer of infinite length for the incoming packets until they are sent successfully to the base station. Time is slotted into intervals equal to the time required to transmit a packet. We make the slot time equal to one time unit and slot t is assumed to occupy the time $[t, t+1)$. We denote by $X_m^{(t)}$ the number of incoming packets to user m during time slot t. The packet arrival process for different $X_m^{(t)}$ for $m = 1, \ldots, n$ and $t \in \mathbb{N}$ is assumed to be i.i.d. as well. The arrival process has a finite mean $\frac{\lambda}{n}$ (so that the cumulative input rate is λ) and finite variance. The above model for the arrival process is the same as that in [23] for a symmetric system.

The channel between the m^ith user and the base station during slot t is parametrized by $\gamma_m^{(t)}$. It is assumed that the quantities $\gamma_m^{(t)}$ for $m = 1, \ldots, n$ and $t \in \mathbb{N}$ are i.i.d. with probability distribution $F(\gamma)$. Further, we assume that the user m has access to the uplink CSI $\gamma_m^{(t)}$ at time t.

We define a general reception model that is given by a set of n functions. The kth function assigns probabilities to all the possible outcomes conditioned on the event that k users transmitted and that their channel states are given by $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)$. Assuming that k users transmitted, we let $\Theta_k = (\theta_k^{(1)}, \ldots, \theta_k^{(k)})$ be a binary k-tuple that represents the outcome of a slot. The bit $\theta_k^{(1)}$ equal to one represents the success of user 1 and so on. The kth function $\Phi^{(k)}(\gamma_1, \ldots, \gamma_k; \Theta_k)$ is the probability of outcome Θ_k when k users whose CSI is given by $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)$ transmit. That is,

$$\Phi^{(k)}(\gamma_1, \ldots, \gamma_k; \Theta_k) = \Pr\left\{\Theta_k | k \text{ users transmit, } \boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)\right\}. \quad (1)$$

Define $\Psi^{(k)}(\gamma_1, \ldots, \gamma_k)$ as the expected number of packets successfully demodulated when the CSI of the transmitting users is $(\gamma_1, \ldots, \gamma_k)$, that is,

$$\Psi^{(k)}(\gamma_1, \ldots, \gamma_k) \triangleq \sum_{i=1}^{k} \mathrm{E}\left\{\theta_k^{(i)} | k \text{ users transmit, } \boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)\right\}. \quad (2)$$

Given a distribution function $F(\cdot)$, define $C_k(F(\cdot))$ as the expected number of packets received conditioned on k users transmit and their CSI is distributed i.i.d. according to $F(\cdot)$. That is,

$$C_k(F(\cdot)) \triangleq \sum_{i=1}^{k} \mathrm{E}\left\{\theta_k^{(i)} | k \text{ users tx}\right\}. \quad (3)$$

Note that this model allows the reception of multiple packets simultaneously. Special cases of this reception model are the classical collision model, capture model and MPR model [16].

We impose some constraints on the reception model that hold for many practical scenarios. For each k, we assume that if we permute the CSI $(\gamma_1, \ldots, \gamma_k)$ and apply the same permutation to the bits of Θ_k, the value of $\Phi^{(k)}(\cdot)$ does not change. That is, we assume long-term symmetry among the users. This condition has been relaxed in the reception model considered in [20], [22]. Further, we assume that for any given $(\gamma_1, \ldots, \gamma_k)$, adding an extra user decreases the probability of packets success for each of the k users. That is, for all $\gamma_1 \geq 0, \ldots, \gamma_{k+1} \geq 0$, for all $1 \leq i \leq k$

$$\sum_{\Theta_k} \theta_k^{(i)} \Phi^{(k)}(\gamma_1, \ldots, \gamma_k; \Theta_k) \geq \sum_{\Theta_{k+1}} \theta_{k+1}^{(i)} \Phi^{(k+1)}(\gamma_1, \ldots, \gamma_{k+1}; \Theta_{k+1}). \quad (4)$$

The parameter $\gamma_m^{(t)}$ can be used to model various parameters that influence the reception. Examples include physical channel gain, position of the mobile, etc. In some cases, it is possible to abstract the reception of an uplink using multiple antennas into the above model.

In the ALOHA protocol analyzed in [23], if the user m has a packet to transmit, he transmits it with a probability p_m. We consider a more general random-access scheme where the probability of transmission for each user is allowed to be a function of his CSI $\gamma_m^{(t)}$. The function is called the *transmission control* scheme and is denoted by $s(\cdot)$. Thus, we assume that in slot t, if user m has a packet then it is transmitted with probability equal to $s(\gamma_m^{(t)})$. At the end of slot t, the base station broadcasts the indexes of those users whose packets it was able to demodulate successfully. The type of ALOHA protocol considered in this paper, where the new arrivals are not transmitted immediately, is known as ALOHA with delayed first transmission. This is in contrast to ALOHA with immediate first transmission where new arrivals are transmitted in the slot immediately following their arrival.

III. Maximum Stable Throughput

In this section, we derive the expression for maximum stable throughput as a function of the CSI distribution, reception model, and the transmission control. The system is defined to be stable if for each node the queue size does not go to infinity. In other words, given a positive number $0 < \epsilon \leq 1$, there exists a buffer size such that the probability of buffer overflow is less than ϵ. It should be obvious that stability is one of the important requirements for a network. The requirement of stability can be said to impose a mild requirement on delay.

We now define the notion of maximum stable throughput in a formal manner. Let the n-tuple $\boldsymbol{N}^{(t)} = (N_1^{(t)}, N_2^{(t)}, \ldots, N_n^{(t)})$ be the length of the buffers at each node at the beginning of slot t. We say that the system is stable for a particular arrival process, if for $\boldsymbol{x} \in \mathbb{N}_+^n$, there exists an $H(\boldsymbol{x})$ such that

$$\lim_{t\to\infty} \Pr\{\boldsymbol{N}^{(t)} < \boldsymbol{x}\} = H(\boldsymbol{x}) \quad \lim_{\boldsymbol{x}\to\infty} H(\boldsymbol{x}) = 1 \quad (5)$$

where $\mathbb{N}_+$ is the set of nonnegative integers. This notion of stability is also used in [23]. We will see that the stability of the system can be characterized by λ, the cumulative mean of the arrival process alone. This will allow us to define maximum stable throughput as the supremum of all cumulative input rates λ for which the system is stable. The following theorem gives the expression for the maximum stable throughput of the system in terms of the transmission control, reception model, and the underlying CSI distribution.

Theorem 1: Given $F(\gamma)$ the distribution function of the CSI, the transmission control $s(\gamma)$ and the reception functions $\{\Phi^{(k)}(\cdot)\}_{k=1}^n$, the maximum stable throughput is given by

$$\lambda_n(s(\cdot)) = \sum_{k=1}^n \binom{n}{k} \left(1 - \int_0^\infty s(\gamma) dF(\gamma)\right)^{n-k} \times \left(\int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_k) \times \Psi^{(k)}(\gamma_1, \ldots, \gamma_k) dF(\gamma_1) \cdots dF(\gamma_k)\right). \quad (6)$$

If $p_s \triangleq \int s dF \neq 0$, then

$$\lambda_n(s(\cdot)) = \sum_{k=1}^n \binom{n}{k} (1 - p_s)^{n-k} p_s^k C_k(G_s(\cdot)) \quad (7)$$

where the distribution function $G_s(\cdot)$ is

$$G_s(\cdot) = \frac{\int_0^\gamma s dF(\gamma)}{p_s}. \quad (8)$$

Proof: Refer to Appendix I.

It should first be noted that p_s defined above is the unconditional probability of transmission, and the distribution $G_s(\cdot)$ is the distribution of CSI conditioned on the event that a user transmits, that is, it is the *a posteriori* distribution of the channel state. It is intuitively reasonable that the maximum stable throughput should depend on $F(\gamma)$ only through $G_s(\cdot)$ because this is the distribution of channel state that the base station "sees"; the underlying distribution of CSI is not relevant. The power of using a transmission control is that it allows us to manipulate the *a posteriori* CSI distribution $G_s(\cdot)$. Thus, we would like to steer the underlying distribution to "good" *a posteriori* distributions by the use of the transmission control. The problem however is more complicated because the transmission control also affects the probability of transmission p_s. Thus, it is possible that transmission controls that lead to good *a posteriori* distributions might lead to an extremely low probability of transmission. It is this coupling that makes it difficult to find optimal transmission controls for various reception models. In the following section, we will consider the SNR threshold model as an example for which it is possible to obtain the optimal transmission control. The optimal transmission control for a simplified capture model was considered in [36]. Obtaining the optimal transmission control for the general capture model and other reception models is interesting and useful but is also hard.

A. An Example

In this section, we apply the results derived in the previous section for the SNR threshold model and obtain the maximum stable throughput. We then optimize the transmission control by maximizing this stable throughput. These results also shed light on how a transmission control can be used to increase the stable throughput of the system.

The SNR of the uplink is taken as the channel state parameter and the reception model is defined as follows. We assume that a user is successfully demodulated if no other user transmits, and

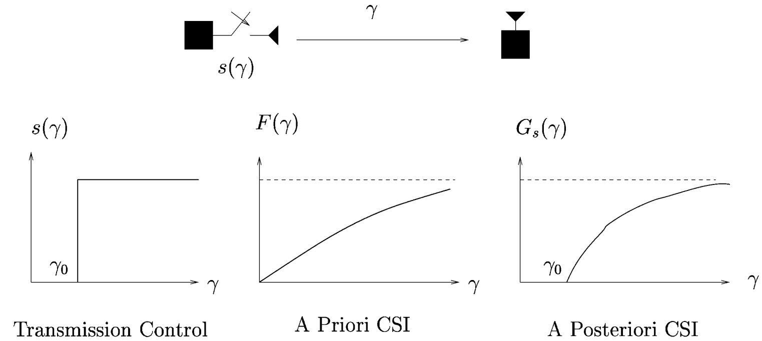

Fig. 3. Shaping of the *a priori* distribution.

if his SNR is larger than a given threshold γ_0. The reception model for this is given by

$$\Phi^{(1)}(\gamma;1) = \begin{cases} 1, & \gamma \geq \gamma_0 \\ 0, & \gamma < \gamma_0. \end{cases} \tag{9}$$

The function $\Phi^{(1)}(\gamma;0)$ is of course equal to $1-\Phi^1(\gamma;1)$. For $k \geq 2$, $\Phi^{(k)}(\cdots)$ is identically equal to zero. This model is similar to the collision channel except that it also takes into account the channel state of the transmitting user.

Given a transmission control $s(\gamma)$, the maximum stable throughput is given by

$$\lambda_n(s(\cdot)) = n\left(1-\int_0^\infty s(\gamma)dF(\gamma)\right)^{n-1}\left(\int_{\gamma_0}^\infty s(\gamma)dF(\gamma)\right). \tag{10}$$

The optimal transmission control is then obtained as

$$s^*(\cdot) = \arg\max_{s(\cdot)} \lambda_n(s(\cdot)). \tag{11}$$

We then have the following theorem.

Theorem 2: Denote $p_{\gamma_0} = P\{\gamma \geq \gamma_0\}$. The optimal transmission control is

$$s^*(\gamma) = \begin{cases} 0, & \gamma < \gamma_0 \\ \min\left(\frac{1}{np_{\gamma_0}}, 1\right), & \gamma \geq \gamma_0 \end{cases} \tag{12}$$

and the corresponding maximum stable throughput is

$$\lambda_n(s^*(\cdot)) = n\min\left(\frac{1}{n}, p_{\gamma_0}\right)\left(1-\min\left(\frac{1}{n}, p_{\gamma_0}\right)\right)^{n-1}. \tag{13}$$

Proof: Refer to Appendix III.

The transmission control is a step function and we find that, as expected, if $\gamma < \gamma_0$, the mobiles do not transmit. If $\gamma \geq \gamma_0$, the probability of transmission is chosen such that the average number of transmitting users in each slot is equal to one. In order to understand the role of transmission control, we illustrate the *a priori* and *a posteriori* distributions of CSI in Fig. 3. We see that the *a posteriori* distribution starts from $\gamma = \gamma_0$ which means that the base station believes that the channel states below γ_0 do not occur. We would like to finally note that the optimal transmission control is not unique.

Fig. 4 illustrates the variation of average delay with total input rate under the SNR threshold model. The threshold γ_0 was set at -5 dB. For the conventional transmission control, when a station has a packet to transmit it transmits the packet with a probability $\frac{1}{n}$. It can be seen that the optimal transmission control has a higher maximum stable throughput and also a lower delay at every load.

An interesting problem that we have not considered is the selection of γ_0 and how the various physical layer parameters and the signal-processing algorithms influence its choice.

IV. Asymptotic Stable Throughput

In this section, we define the notion of AST of a network and consider the problem of designing transmission controls that are optimal with respect to the AST. We consider two types of transmission control: the PDTC where the transmission control depends on the total number of users in the network, and the simpler PITC where the transmission control is not allowed to be a function of total number of users in the network.

We know that given the number of users in the network n, reception model $\Phi^{(k)}(\cdot)$, the CSI distribution $F(\gamma)$, and the transmission control $s(\cdot)$ such that $p_s > 0$, the maximum stable throughput for the network is given by

$$\lambda_n^*(s(\cdot)) = \sum_{k=1}^{n}\binom{n}{k}(1-p_s)^{n-k}p_s^k C_k(G_s(\cdot)). \tag{14}$$

The AST is defined as the maximum stable throughput as the number of users in the network goes to infinity. Such a metric is of value for "large" networks, and it is possible to obtain transmission controls that are asymptotically good based on this metric. This enables us to design transmission controls for those reception models for which it is difficult to find the transmission controls that are optimal with respect to the maximum stable throughput. The formal definition of AST is as follows.

Definition 1: Given the distribution function of CSI $F(\gamma)$, the transmission control sequence $s_n(\gamma)$ and the reception functions $\{\Phi^{(k)}(\cdot)\}_{k=1}^\infty$, the AST is defined as

$$\lambda_\infty(\{s_n(\cdot)\}) \triangleq \liminf_{n\to\infty}\sum_{k=1}^{n}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k C_k(G_{s_n}(\cdot)). \tag{15}$$

For what follows, we impose the following technical restrictions on the kind of reception models $\{\Phi^{(k)}(\cdot)\}_{k=1}^\infty$ considered.

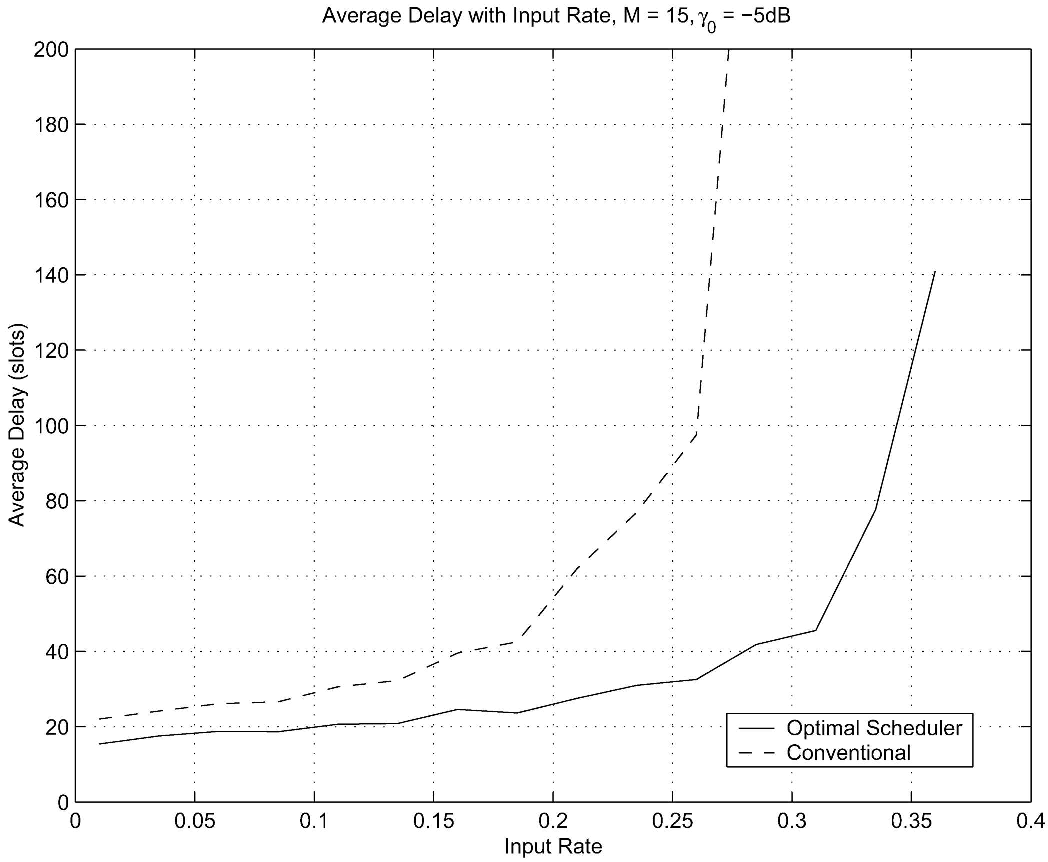

Fig. 4. Delay versus input rate.

A1: For any distribution function $F(\gamma)$

$$\lim_{k\to\infty} C_k(F(\cdot)) \triangleq C_\infty(F(\cdot))$$

exists.

This restriction is quite mild and in fact holds for most reception models considered.

A. Population-Independent Transmission Control (PITC)

We first consider the scenario where the transmission control sequence is such that it does not depend on n. That is, $s_n(\gamma) = s(\gamma),\ \forall\, n$. This kind of transmission control is termed population-independent transmission control (PITC). Such transmission controls are interesting because they are simpler to implement and they can be expected to be robust to the size of the network. In cases where nodes may enter and leave the network, it is easier to use a PITC because it is not necessary to keep track of the size of the network.

The AST with PITC becomes

$$\lambda_\infty(s(\cdot)) \triangleq \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} (1-p_s)^{n-k} p_s^k C_k(G_s(\cdot)). \quad (16)$$

The AST can be given a simpler characterization as follows.

Proposition 1: Given the transmission control, $s(\cdot)$, the AST is given by

$$\lim_{k\to\infty} C_k(G_s(\cdot)). \quad (17)$$

Proof: The proof follows from [17].

In contrast, the AST for PITC that does *not* depend on the channel state is given by

$$\lim_{k\to\infty} C_k(F(\cdot)). \quad (18)$$

Thus, the effect of the transmission control for PITC is equivalent to changing the underlying CSI distribution. It is therefore important to determine the set of probability distributions that can be reached through PITC from $F(\gamma)$. Given $F(\gamma)$, it is easy to see that the set of distributions that can be reached through PITC is given by

$$\Lambda_F \triangleq \left\{ G(\gamma) : \exists s(\cdot) \in \mathcal{S},\ s.t \int_0^\infty s(\gamma) dF(\gamma) > 0, \right.$$
$$\left. G(\gamma) = \frac{\int_0^\gamma s(\gamma) dF(\gamma)}{\int_0^\infty s(\gamma) dF(\gamma)} \right\} \quad (19)$$

where

$$\mathcal{S} \triangleq \{ s(\gamma) : 0 \le s(\gamma) \le 1,\ \forall\, \gamma \}. \quad (20)$$

Thus, we have the following proposition.

Proposition 2: The supremum of all possible stable throughput by optimizing the transmission control function is given by

$$\sup_{s(\cdot)} \lambda_\infty(s(\cdot)) = \sup_{G(\cdot)\in\Lambda_F} \lim_{n\to\infty} C_n(G(\cdot)). \quad (21)$$

In what follows, we derive the properties of the distributions in Λ_F and try to ascertain how large this set is. We first list some simple properties of the functions $G(\gamma) \in \Lambda_F$.

P1 $G(\gamma)$ is a distribution function.
P2 $\mu_F(A) = 0$ implies $\mu_G(A) = 0$. (Notation: $G \ll F$.)
P3 There exist a positive constant $C < \infty$ such that the Radon–Nikodym derivation $\frac{dG}{dF} < C$ for all γ.

We now show that in fact the above three properties characterize the set Λ_F, namely, if there exists a function $G(\cdot)$ satisfying the properties above then it belongs to Λ_F. Given $G(\cdot)$ and $F(\cdot)$ satisfying the properties given above, define the transmission control as

$$s(\gamma) = \frac{1}{C}\frac{dG}{dF}. \tag{22}$$

It is easy to see that the *a posteriori* CSI distribution with this transmission control is equal to $G(\gamma)$ and therefore $G(\cdot) \in \Lambda_F$.

Thus, if the underlying channel state distribution is $F(\gamma)$, it is possible to steer the conditional distribution of the channel state to any $G(\gamma)$ that satisfies the properties listed above by choosing the transmission control as

$$s(\gamma) = \frac{dG}{dF}\frac{1}{\sup \frac{dG}{dF}}. \tag{23}$$

It is important to determine how limiting the restriction to the set Λ_F is. As we shall see later, this restriction has an important bearing on the maximum achievable AST with PITC for many reception models and state distributions $F(\gamma)$.

B. Population-Dependent Transmission Control (PDTC)

We now consider the more general case, when the transmission control is allowed to be a function of number of users in the network. As discussed previously, given a sequence of transmission controls $s_n(\gamma)$, the AST is defined as

$$\lambda_\infty(\{s_n(\cdot)\}) = \liminf_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_{s_n}(\cdot)). \tag{24}$$

We will first derive the AST for transmission control sequences that do not use CSI and then introduce a simple PDTC sequence that can improve significantly over this AST. The results in [17] can be directly used to show that if we use a transmission control $s_n(\gamma) = \min\left(\frac{x}{n}, 1\right)$, where x is an arbitrary positive real number, we achieve an AST equal to

$$e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)) \triangleq f(x, F). \tag{25}$$

The following proposition says that in fact the control above achieves all possible AST, and it is not possible to do better using a more complicated transmission control.

Proposition 3: If the sequence of transmission control $s_n(\gamma)$ is chosen to be independent of γ but as a function of n alone, then the maximum possible AST is given by $\sup_x f(x, F)$, where $F(\gamma)$ is the distribution of γ.

Proof: The proof follows directly from [17].

It is possible to construct a simple sequence of transmission controls that improves significantly upon the AST obtained above. Let $T(\cdot)$ be a distribution function such that $T(\cdot) \ll F(\cdot)$. From the Radon–Nikodym theorem, there exists a nonnegative function $\frac{dT}{dF}$ such that

$$\mu_T(A) = \int_A \frac{dT}{dF} dF. \tag{26}$$

The sequence of transmission controls is chosen as

$$s_n(\gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right). \tag{27}$$

The following proposition characterizes the achievable throughput.

Proposition 4: With the sequence of transmission controls chosen as

$$s_n(\gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right) \tag{28}$$

the AST is given by

$$\lambda_\infty(\{s_n(\cdot)\}) = e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) = f(x, T). \tag{29}$$

Proof: Refer to Appendix IV.

By comparing Proposition 3 with Proposition 4, it can be seen that the effect of the chosen transmission control sequence is to effectively change the CSI distribution from $F(\gamma)$ to $T(\gamma)$. From Propositions 4 and 1, we can see that the advantage of using PDTC is two-fold. First, the set of distributions that can be reached is larger than the set Λ_F because we do not need the target distribution to obey P3 listed earlier. Second, the performance is no longer limited by $C_\infty(T(\cdot))$ but is given by $\sup_x f(x, T)$. It can be shown using techniques in [17] that the function $f(x, T)$ has the property that

$$\lim_{x\to\infty} f(x, T) = C_\infty(T(\cdot)). \tag{30}$$

This implies that $\sup_x f(x, T) \geq C_\infty(T(\cdot))$.

The intuition behind choosing the particular sequence of transmission controls is that first we have $p_{s_n} \to 0$ while $np_{s_n} \to x$ and second we have the *a posteriori* distribution

$$G_{s_n}(\gamma) \to T(\gamma) \quad \text{point wise}. \tag{31}$$

The first condition ensures that the number of transmission attempts in any given slot converges to a Poisson random variable with mean x, and the second condition ensures that the *a posteriori* CSI distribution converges to $T(\gamma)$.

It can be seen that through a judicious choice of transmission control sequence, it is possible to achieve an AST of

$$\lambda_c^* = \sup_{x, T(\cdot)\ll F(\cdot)} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)) = \sup_{x, T(\cdot)\ll F(\cdot)} f(x, T). \tag{32}$$

The quantity λ_c^* is in some sense the capacity associated with the reception model, CSI distribution $F(\cdot)$, and the protocol proposed. For a given reception model, it is important to characterize λ_c^* and find distributions $T(\cdot)$ that achieve an AST that is close to λ_c^*.

For a given reception model and CSI distribution $F(\gamma)$, choosing a target distribution that guarantees improvement is in general not easy. The reason is that the AST is equal to $\sup_x f(x, F)$ which in turn depends on all of $C_k(F(\cdot))$. However, it is at times easy to characterize the value of $f(x, F)$ as $x \to \infty$ and this offers us a way of comparing different distributions. As mentioned previously, for any $F(\cdot)$ [17]

$$\lim_{x \to \infty} f(x, F) = C_\infty(F(\cdot)). \tag{33}$$

Hence, if for a distribution function $F(\cdot)$, we find a distribution $T(\cdot)$ such that $C_\infty(T(\cdot)) > \sup_x f(x, F)$, then improvement is guaranteed by using a transmission control that changes the distribution to $T(\cdot)$. However, it is important to note that $C_\infty(T(\cdot)) > C_\infty(F(\cdot))$ does not in general guarantee improvement.

V. Application to CDMA Networks

In this section, we apply the results derived in the previous section to the uplink of CDMA networks. This application illustrates the theory and also demonstrates the magnitude of gains possible through the use of CSI.

In order to apply the theory, we need to first select the parameter that will be used as the channel state. The choice of the channel state parameter might be influenced by issues like potential gain and ease of estimation. Once the channel state parameter is fixed, the distribution of the CSI should be determined. Then a reception model as described in Section II should be developed for the physical layer processing.

For the purposes of the current application, we will choose the propagation channel gain as the CSI. The possible models for the CSI and the distributions that arise due to these models are delineated as part of the section below on channel model. We analyze the CDMA network under two receiver structures; one where the receiver uses an MF and the other where the receiver uses a linear MMSE multiuser receiver. The two structures give rise to two different reception models. The results for the linear MMSE multiuser receiver are presented in considerable detail and the corresponding results for the matched filter are stated in brief because they are conceptually similar to the ones for the LMMSE multiuser receiver. For each reception model, the program is to first analyze the performance possible without transmission control. Since the use of transmission control essentially changes the underlying CSI distribution, the objective then is to find distributions that improve over the existing CSI distributions. In this connection, we will show that distributions with rolloff (see (36)) form "good" target distributions for PDTC and that it is possible to obtain large gains by using transmission controls that steer the underlying CSI distribution to this distribution.

A. Channel Model

The propagation channel gain from each user to the base station is selected as the channel state. Since we require that each user has access to his channel state, we imagine a time-division duplex (TDD) system where the base station is transmitting a pilot tone.

If the received power is modeled as

$$P_R = KR^2 P_T \tag{34}$$

where K is a constant, R is Rayleigh distributed, and the channel state is given by $\gamma = P_R$, then the underlying CSI distribution is exponential. This corresponds to the case when a slow power control is being employed. This model is also reasonable for modeling the propagation channel gain in the reachback problem because all the nodes are typically at the same distance from the collecting station and undergo the same propagation loss and shadow fading. Thus, the underlying CSI distribution for the reachback problem can be assumed to be exponential.

Another possible model for received power at the base station is

$$P_R = R^2 K_s e^{\xi} K r^{-\alpha} P_T \tag{35}$$

where R is Rician or Rayleigh distributed, ξ is Gaussian distributed, with zero mean and standard deviation γ_s and P_T is the constant transmitted power, r is the distance from base station, and α is the propagation constant that typically lies between 2 and 6. In this case, the CSI distribution is a complicated function of the distribution of r, the distance from the base station. A particular property of this distribution that turns out to be very crucial is the way in which the tail of the distribution rolls off. Given a distribution function $G(\cdot)$, define δ to be the rolloff of $G(\cdot)$, if there exists a c such that $0 < c < \infty$ and [37]

$$\lim_{\gamma \to \infty} (1 - G(\gamma))\gamma^\delta = c. \tag{36}$$

If there exists a positive constant a such that the cumulative distribution function (CDF) of the distance r of a station satisfies

$$\lim_{x \to 0} F_r(x) x^{-a} = c_r \tag{37}$$

where c_r is a positive constant. Then it can be shown that [37] the distribution of the received power P_R above has a rolloff $\delta = \frac{a}{\alpha}$. This model corresponds to the case when there is no power control. Different possible distributions for r are the so-called uniform distributions where

$$\mathrm{P}\{r < x\} = x^2, \qquad 0 \le x \le 1 \tag{38}$$

the quasi-uniform distribution for which the density of r is given by

$$f_r(x) = x e^{-\frac{\pi x^4}{4}}, \qquad 0 \le x < \infty \tag{39}$$

and bell-shaped distribution.

B. Linear MMSE Multiuser Receiver

In this subsection, we study the case when the receiver uses an LMMSE multiuser receiver. We start by describing the reception model to be used and then apply the results to this reception model.

We assume that each user is assigned a particular signature waveform that is used to modulate the data. Each packet starts with sufficient training symbols that the receiver can use to form an equalizer. The packet is assumed to be successfully demodulated if the signal-to-interference ratio (SIR) after the LMMSE multiuser receiver is greater than β. (The parameter β is a function of modulation, code, and quality of service required for the application.) For the LMMSE receiver structure, the SIR for each user is a complicated function of the received power and signature sequences of the transmitting users. However, if the signature sequences are random, the size of the network and the spreading gain are large, the SIR can be approximated as a simple function of the received powers [38]. Given that K users transmit, N is the processing gain of the system, and P_i is the power received from user i, user i goes through if

$$\frac{\gamma_i}{\sigma^2 + \frac{1}{N}\sum_{k=1,k\neq i}^{K}\frac{\gamma_i\gamma_k}{\beta\gamma_k+\gamma_i}} > \beta. \tag{40}$$

This condition can be used[1] as a reception model as defined in Section II. The above condition can be rewritten as

$$\frac{\beta N\sigma^2}{\gamma_i} + \sum_{k=1,k\neq i}^{K}\frac{\beta\gamma_k}{\gamma_i+\beta\gamma_k} < N. \tag{41}$$

This shows that the effective interference from other users is limited to at most 1. This is the advantage of using an MMSE multiuser detector over a matched filter. In deriving this condition, it is assumed in [38] that the receiver employs a true MMSE filter or equivalently that the receivers knows the spreading sequences of the transmitting users. This assumption is not a contradiction to the fact that we are considering a random-access protocol because we assume that each packet starts with training and these training symbols are used to obtain a least squares equalizer and if we have a sufficient number of training symbols present we can ensure that the least squares equalizer converges to the true LMMSE equalizer derived under the assumption that the receiver knows exactly who the transmitting users are.

1) PITC: For PITC, the AST without the use of CSI is $C_\infty(F(\cdot))$ and the AST with CSI is $C_\infty(G(\cdot))$ where $G(\cdot)\in\Lambda_F$. We first assume that the underlying CSI distribution $F(\cdot)$ is exponential and we evaluate C_∞ for the exponential distribution.

Proposition 5: Let $F(\cdot) = 1 - e^{-\frac{\gamma}{P_T}}$ and the noise variance be equal to σ^2, then

$$\lim_{k\to\infty} C_k(F(\cdot)) = 0. \tag{42}$$

Proof: Refer to Appendix V.

Thus, the AST for exponential distribution is equal to zero. The following proposition gives the AST for the set of distributions that can be reached from the exponential distribution.

Proposition 6: Let $F(\cdot) = 1 - e^{-\frac{\gamma}{P_T}}$ and the noise variance be equal to σ^2, and $G(\cdot)\in\Lambda_F$, then

$$\lim_{k\to\infty} C_k(G(\cdot)) = 0. \tag{43}$$

Proof: Refer to Appendix VI.

[1]Note that the signal-to-interference-noise ratio (SINR) condition in [38] may not be accurate for random access when K is small.

This proposition implies that it is not possible to improve the asymptotic throughput with PITC if the underlying distribution is exponential. Hence, the set Λ_F is not "large enough" to improve the throughput.

We now consider the case when the distribution of the received power has a rolloff δ. This corresponds to the case when there is no power control.

Proposition 7: If $F(\cdot)$ has a rolloff δ, then

$$\lim_{k\to\infty} C_k(F(\cdot)) = \begin{cases} \frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta} + e(N,\delta), & 0<\delta\le 1\\ 0, & \delta>1\end{cases} \tag{44}$$

where $e(N,\delta)$ satisfies

$$\lim_{N\to\infty} e(N,\delta) = \frac{\sin\pi\delta}{\pi\delta}\frac{1-\delta}{2\beta^\delta}. \tag{45}$$

Proof: Refer to Appendix VII.

Thus, for large N, we can neglect the quantity $e(N,\delta)$ and assume that AST is $\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}$. We conjecture that for any finite N

$$\max_\delta e(N,\delta) = e(N,0) = \frac{1}{2}. \tag{46}$$

For all the arguments that follow, we assume that the AST is given by $\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}$.

When the distribution of the received power has a rolloff, the asymptotic throughput is not equal to zero. In order to determine if the use of CSI can increase the AST, we consider the AST of the distributions in the set Λ_F.

Proposition 8: Let $F(\gamma)$ be a distribution function with rolloff δ, $G(\cdot)\in\Lambda_F$, and $G(\cdot)$ has a rolloff δ' then $\delta'\ge\delta$. Further, for all $\delta'>\delta$, there exists a $G(\cdot)\in\Lambda_F$ such that the rolloff of $G(\cdot)$ is δ'.

Proof: Refer to Appendix VIII.

If $\frac{1}{\beta}\le 1$ (which is typical), then the asymptotic throughput is a decreasing function of δ and if $\frac{1}{\beta}>1$, the asymptotic throughput reaches a maximum for some value of δ that lies between 0 and 1. This fact together with Proposition 8 has the following implications on possible improvements in AST. If $\frac{1}{\beta}\le 1$, the AST cannot be improved by steering to distributions with a rolloff. However, if $\frac{1}{\beta}\ge 1$, it can be shown quite easily that for a δ, the AST can be improved by PITC if

$$\left(1-\frac{\pi\delta}{\tan\pi\delta}\frac{1}{\delta}\right) < \log\frac{1}{\beta}. \tag{47}$$

To illustrate: for example, if $\frac{1}{\beta}>e^2$ and $\delta\le 0.5$, then improvement is possible.

Thus, for the reception model under consideration, if the transmission control is not allowed to use the size of the network, improvement in AST is not possible for most cases. As shown later, this will change quite significantly when the transmission control is allowed to use the size of the network.

2) PDTC: We now consider the use of CSI for PDTC when the underlying distribution is exponential. As shown in Proposition 3, the AST obtained without the use of CSI is given by

$$f(x,F) = e^{-x}\sum_{k=0}^{\infty}\frac{x^k}{k!}C_k(F(\cdot)). \tag{48}$$

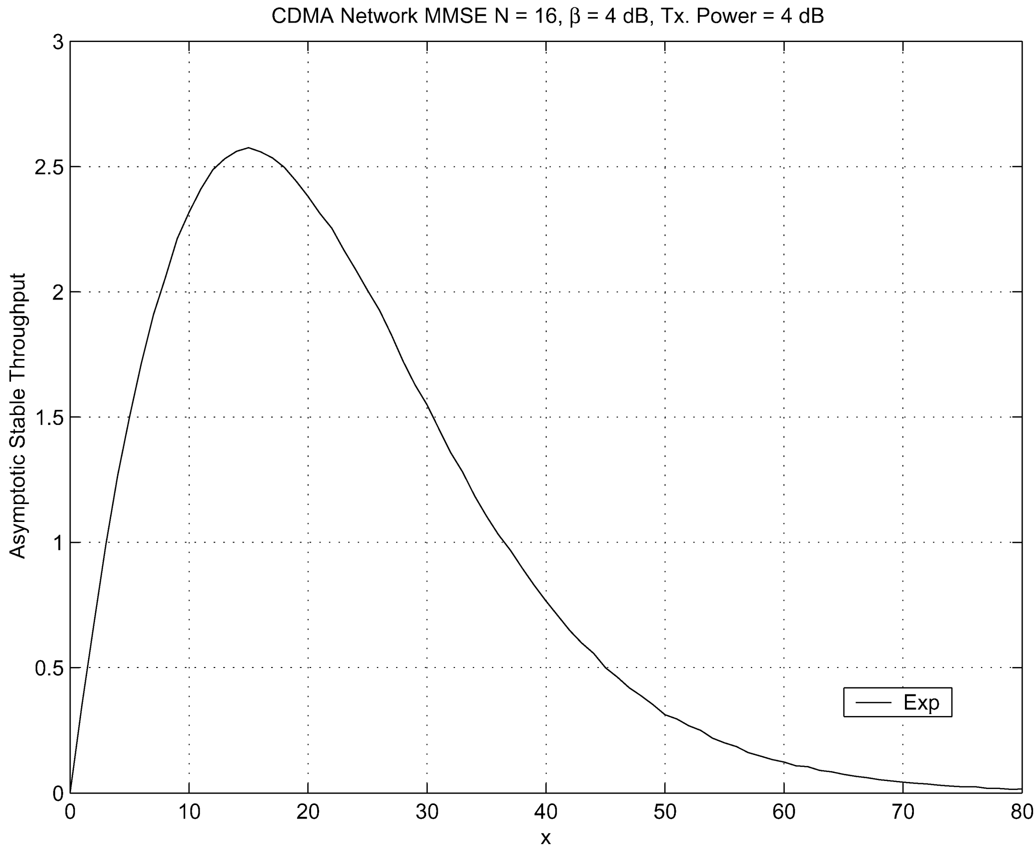

Fig. 5. AST with PDTC that does not use CSI.

Fig. 5 illustrates the AST for LMMSE when the underlying CSI distribution is exponential and CSI is not used for transmission control. The transmit power is 4 dB over noise, spreading gain $N = 16$ and $\beta = 4$ dB. The x-axis is the design variable x, the average number of transmissions in each slot. We see that it is possible to achieve an AST of approximately 2.6 packets per slot without using CSI by setting x to be approximately equal to 15 transmissions per slot. We should now find distributions $T(\cdot)$ such that $T \ll F$ and $f(x,T) > 2.6$ for some x. If $T(\cdot)$ is a distribution with a rolloff, Proposition 7 gives the value of $f(\infty, T)$, and we see that there are many distributions for which $f(\infty, T) > 2.6$. This implies that for these distributions, there do exist y such that $f(y,T)$ improves over $\sup_x f(x,F)$. We select distributions with rolloff 0.5 and 0.3 as the target distributions and Fig. 6 plots $f(x,T)$ for each of them. The solid line in Fig. 6 illustrates the AST when the underlying channel state is distributed exponentially with mean 4 dB. The dotted lines are the AST for distributions with a rolloff. We see that it is in fact possible to obtain significant gains over the maximum AST that can be obtained without the use of CSI. From Proposition 4, a transmission control that can be used to steer the underlying exponential distribution to a distribution with rolloff is given by

$$s_n(\gamma) = \min\left(\frac{e^{\gamma} P_T}{\gamma^{1+\delta}} \frac{x}{n}, 1\right) 1_{\gamma > \gamma_0} \tag{49}$$

where γ_0 is any fixed constant.

From Fig. 6, it can also be seen that for a given AST, the mean number of transmissions required is smaller for the rolloff distributions compared to those required for the exponential distribution. This implies that utilizing a transmission control that uses CSI decreases the required average number of transmissions in a slot. This has implications on network-wide power savings.

We now consider the importance of the use of CSI at low transmit power. Even for arbitrarily small power P_T, the distributions with rolloff are dominated by the underlying exponential CSI distribution. This implies that it is possible to steer to the rolloff distributions from an exponential distribution with an arbitrarily small mean. From Proposition 7, when $\beta > 1$ (typical), it is possible to achieve an AST of N using distributions with a rolloff (that corresponds to $\delta \to 0$). Thus, even if the CSI is exponential with an arbitrarily small mean it is possible to achieve an AST of N using CSI. However, without the use of CSI, the maximum achievable AST goes to zero. The following theorem summarizes the importance of CSI for the reception model under consideration at small powers.

Theorem 3: Assume $\beta > 1$ and $F(\gamma) = 1 - e^{-\frac{\gamma}{P_T}}$, then

$$\lim_{P_T \to 0} f(x, F) = 0. \tag{50}$$

However, for any given P_T, the maximum achievable AST with CSI satisfies

$$N \le \lambda_c^* \le N + \frac{N}{\beta}. \tag{51}$$

Proof: Refer to Appendix XI.

The preceding theorem implies that CSI can be used to achieve large asymptotic throughputs even in cases where each

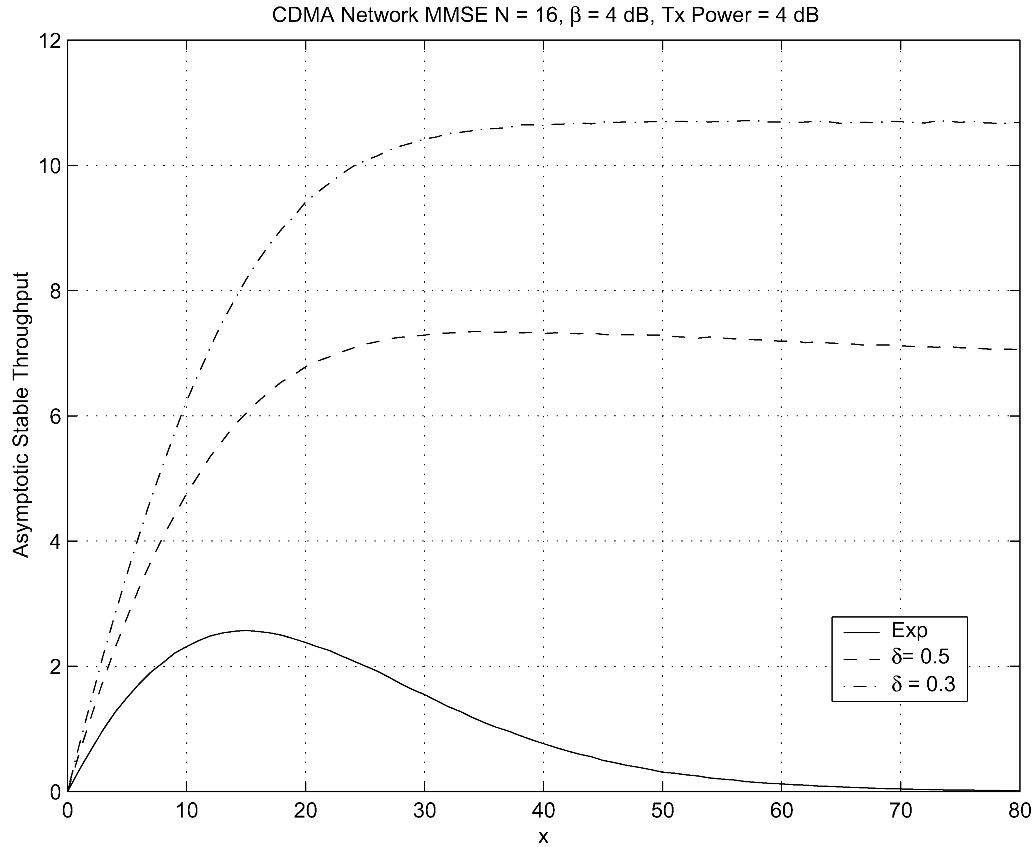

Fig. 6. AST with and without CSI for PDTC.

node is equipped with small power. This result is of relevance for the reachback problem because sensors are typically deployed in large numbers but each is capable of transmitting at a low power. However, with the use of the CSI, it is possible for the nodes to employ transmission control and achieve a large throughput.

As a final note, we would like to point out that the distributions that can be used to improve the AST beyond N and achieve the capacity λ_c^* are not known.

C. Matched Filter (MF)

In this subsection, we list the results that correspond to the MF reception model. We do not give detailed comments in this part, because the results are conceptually similar the ones obtained for the LMMSE reception model.

The reception model is as follows: given that K users transmit, P_i is the power received from user i, user i goes through if and only if the corresponding SINR is greater than β, that is,

$$\frac{P_i}{\sigma^2 + \sum_{k=1,k\neq i}^{K} P_k} > \beta. \tag{52}$$

This criterion follows from the heuristics [38] for networks with large N. It can be seen that criterion is quite similar to the capture model and is most popular for CDMA networks with matched filters.

1) PITC: We will now characterize the AST with PITC with and without using CSI when the underlying distribution is exponential.

Proposition 9: If $F(\gamma)$ is the distribution function of an exponential random variable with mean P_T, then

$$\lim_{k\to\infty} C_k(F(\cdot)) = 0. \tag{53}$$

Proof: Refer to Appendix IX.

Proposition 10: If $F(\gamma)$ is the distribution function of an exponential random variable with mean α, and $G(\cdot) \in \Lambda_F$ then

$$\lim_{k\to\infty} C_k(G(\cdot)) = 0. \tag{54}$$

Proof: Refer to Appendix X.

Propositions 9 and 10 imply that if the received power is distributed exponentially, then PITC does not improve the AST.

We now consider the case when the received power has a distribution with a rolloff. The following proposition follows from a straightforward application of the result in [37].

Proposition 11: If $G(\cdot)$ has a rolloff δ, then

$$\lim_{k\to\infty} C_k(G(\cdot)) = \begin{cases} \left(\frac{N}{\beta}\right)^{\delta} \frac{\sin \pi\delta}{\pi\delta}, & 0 < \delta \leq 1 \\ 0, & \delta > 1. \end{cases} \tag{55}$$

We see that in this case it is possible to obtain nonzero asymptotic throughput with constant transmission control. In order to determine if the use of CSI can increase the AST, we consider the AST of the distributions in the set Λ_F. From Proposition 8, we have that if we start with a distribution with a rolloff, we can go to distributions that have a larger rolloff but we cannot go to distributions with a smaller rolloff. This fact has the following implications on the possible improvements in AST. If

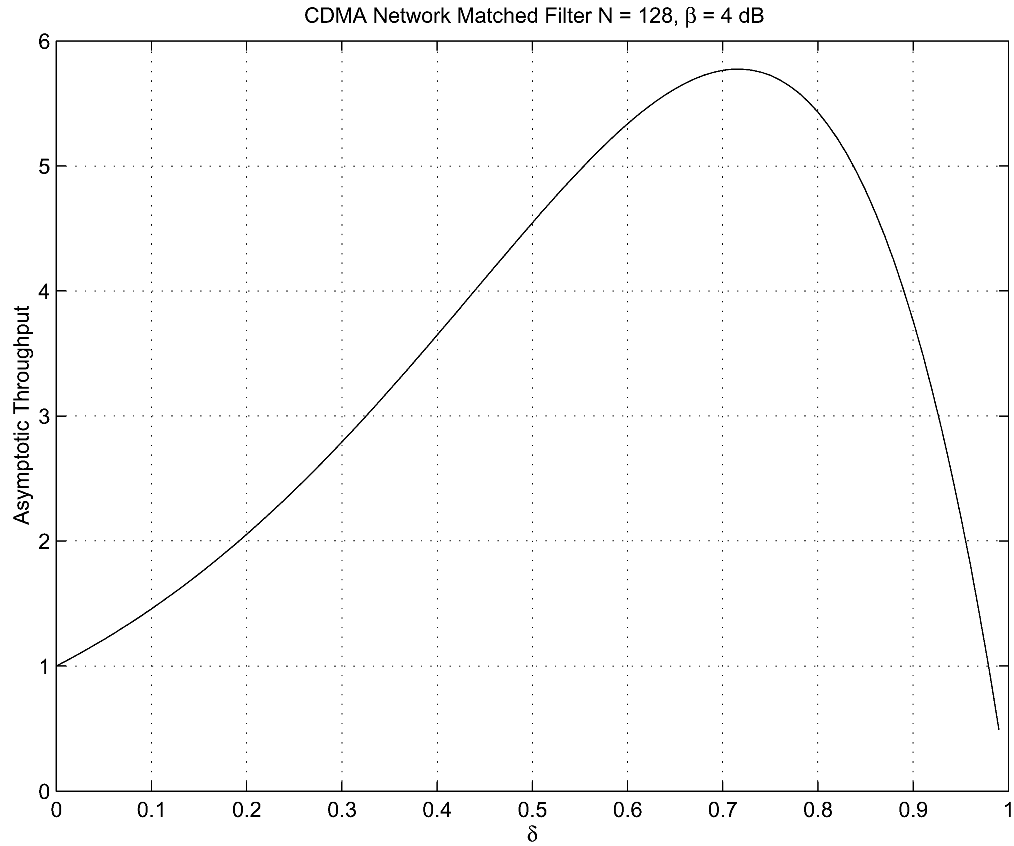

Fig. 7. Asymptotic throughput versus δ.

$\frac{N}{\beta} \leq 1$, then the asymptotic throughput is a decreasing function of δ, therefore, the AST cannot be improved if by steering to distributions with a rolloff. However, if $\frac{N}{\beta} > 1$ (typical), the asymptotic throughput reaches a maximum for some value of δ that lies between 0 and 1. Hence, it is possible that decreasing δ increases the throughput. It can be shown quite easily that for a δ the AST can be improved by PITC if

$$\left(1 - \frac{\pi\delta}{\tan \pi\delta}\frac{1}{\delta}\right) < \log \frac{N}{\beta}. \tag{56}$$

To illustrate: for example, if $\frac{N}{\beta} > e^2$ and $\delta \leq 0.5$ then improvement is possible. Fig. 7 shows the variation of asymptotic throughput with δ for $N = 20$ and $\beta = 4$ dB. It can be seen that significant gains are possible if we start with a δ is less than 0.5.

2) PDTC: We now consider the use of CSI for PDTC. As shown in Proposition 3, the AST obtained without the use of CSI is given by

$$f(x, F) = e^{-x} \sum_{k=0}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)). \tag{57}$$

Fig. 8 illustrates the AST for matched filter when the underlying CSI distribution is exponential. The transmit power is 4 dB over noise, spreading gain $N = 16$, and $\beta = 4$ dB. The x-axis is the design variable x. We see that it is possible to achieve an AST of approximately one packet per slot without using CSI by setting x to be approximately equal to seven transmissions per slot. We would like to find if there exist distributions $T(\cdot)$ such that, for some x, $f(x, T) > 1$ and $T \ll F$. If $T(\cdot)$ is a distribution with a rolloff, Proposition 11 gives the value of $f(\infty, T)$, and we see that there exists a distribution with a rolloff for which $f(\infty, T) > 1$. Fig. 9 plots $f(x, T)$ for distributions with a rolloff 0.5 and 0.3. We see that it is in fact possible to improve over the AST that was possible without CSI. From Proposition 4, we know that a transmission control that can be used to steer to a distribution with rolloff is given by

$$s_n(\gamma) = \min\left(\frac{e^{\gamma} P_T}{\gamma^{1+\delta}} \frac{x}{n}, 1\right) 1_{\gamma > \gamma_0} \tag{58}$$

where γ_0 is any fixed constant.

VI. Conclusion and Future Directions

In this paper, we studied the use of decentralized CSI for random access. To perform this study, we first proposed a reception model for the physical layer that takes into account the channel states of the transmitting users. A variant of Slotted ALOHA where the transmit probability is a function of the channel state was used for random access. We then obtained expressions for the maximum stable throughput of the network as a function of the transmission control used and the reception model. Determining optimal transmission controls for a reception model is in general a hard problem.

We then considered the regime of large networks and introduced the notion of asymptotic stable throughput (AST). AST

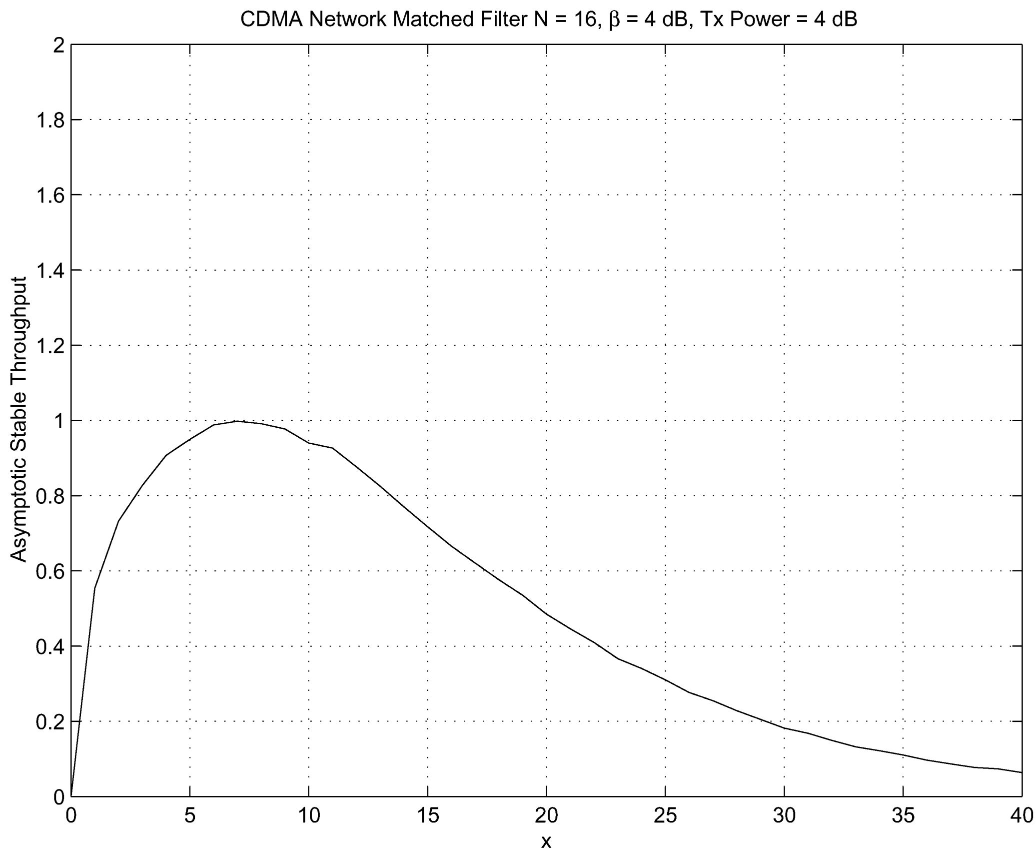

Fig. 8. AST with PDTC that does not use CSI.

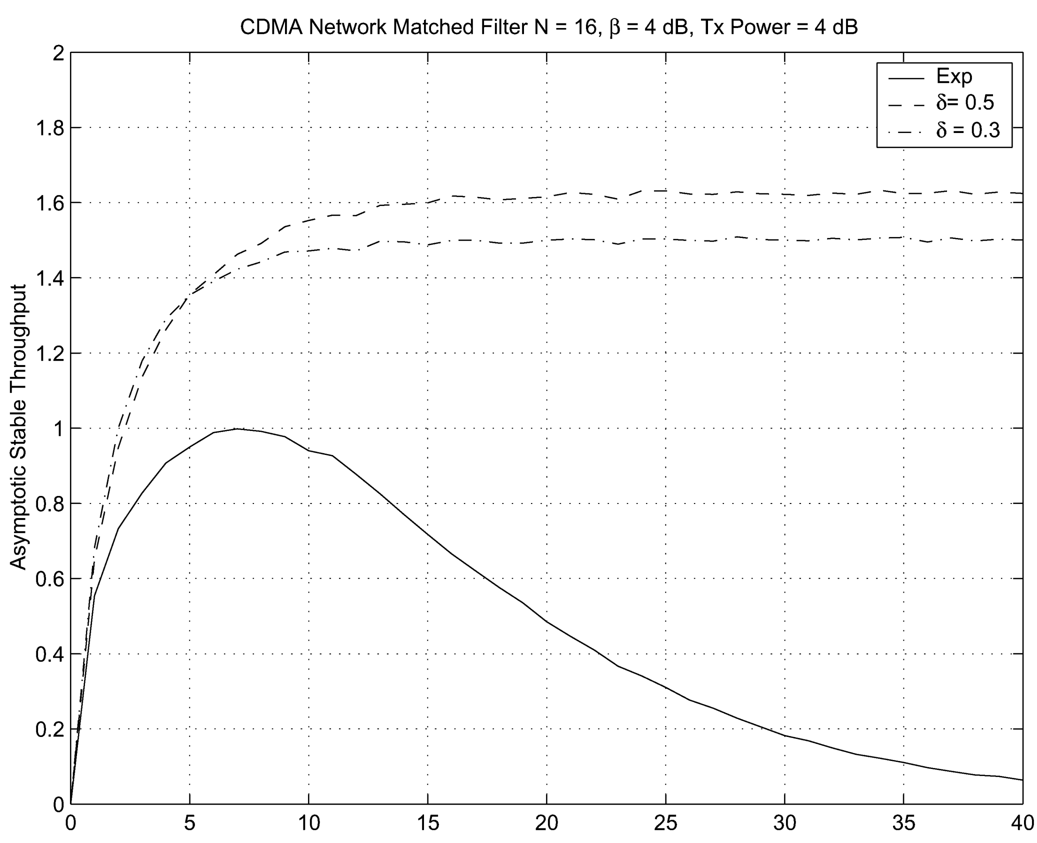

Fig. 9. AST with PDTC that uses CSI.

is the maximum stable throughput of the network as the number of users goes to infinity. PITC (transmission control is not a function of the size of the network) was considered and the AST was derived for it. It was shown that the effect of transmission control is to effectively change the underlying CSI distribution and the set of distributions that can be reached through PITC was characterized.PDTC (transmission control is a function of the size of the network) was then studied. If transmission control is not used, then the maximum possible AST is given by

$$\sup_x e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(F(\cdot)) \tag{59}$$

where $F(\cdot)$ is the CSI distribution. We showed that if the transmission control sequence is chosen as

$$s_n(\gamma) = \min\left(\frac{dT}{dF}\frac{x}{n}, 1\right) \tag{60}$$

where F is the underlying distribution, T is a target distribution that is dominated by F, n is the size of the network, and x is a design variable which is equal to the average number of attempts per slot then the AST is given by

$$e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T(\cdot)). \tag{61}$$

The problem then is one of identifying the right target distributions to use for a given reception model. We note that the transmission control scheme derived using AST provides significant gain even for moderate network size n; see [46].

The theory was then applied to the uplink of CDMA networks with LMMSE multiuser detectors and MF receivers. In either case, propagation channel gain was used as the channel state. Two different models leading to two different distributions were considered for the propagation channel gain. It was shown that if the channel state distribution is exponential, there is no gain to be achieved from PITC. However, with PDTC, if the target distribution is chosen as a distribution with a rolloff, it is possible to obtain significant gains. For the LMMSE receiver, it was shown that if the nodes do not use CSI then the AST tends to zero as transmit power decreases but with the use of CSI the achievable AST is lower-bounded by the spreading gain of the network. This outcome has important implications for the reachback problem in sensor networks where the number of nodes is large but each is equipped with small transmit power capabilities.

We now discuss some possible further research directions that arise from this study. The theory can be applied to a variety of reception models with different channel state parameters. In this paper, we have primarily considered the case when the propagation channel gain is chosen as the channel state. Other possibilities include position of the mobile, etc. This leads to interesting problems in development of reception models for different signal processing and physical layer architectures assuming different channel state parameters. Once the reception model has been developed, then it is important to determine good target distributions and then evaluate the possible gains from transmission control. See [47] for related discussions.

The results presented in this paper are mostly asymptotic in nature and there are different transmission control algorithms that give the same AST. But, these different choices might have different performance in terms of convergence to the asymptotic value. We feel that convergence will depend on how "different" the target distribution is from the current distribution. Hence, more work needs to be performed to characterize the rate of convergence. We suspect that this will have a bearing on the delay of the network.

For the LMMSE and MF reception model, we have only characterized the AST for two types of probability distributions (exponential and rolloff distributions). An interesting direction is to determine the AST for other distributions and the related problem of the capacity of both reception models is open.

For the case of CDMA networks, it is interesting to compare the strategy of transmission control with the strategy of power control. Both of them require only decentralized CSI. The comparison between the two strategies is currently under investigation. It should be noted that transmission control is in general easier to implement than power control because power control might require a large dynamic range for the power amplifier.

In this paper, we have assumed that the distribution of the channel state is the same across users and that the reception model is invariant to permutation of channel states. The reception model considered in this paper thus cannot capture long-run asymmetry in the users of the network. Addressing the problem after relaxing these assumptions is definitely interesting. We have assumed that the channel state is independent from slot to slot, and we have restricted ourselves to stationary policies. Other important models which we believe might lead to interesting results are when the channel state is independent from user to user but correlated in time and we are allowed to use nonstationary policies. The model where the channel state is correlated between users is also quite interesting and might lead to different solutions. The results in this paper are quite surprising because we have demonstrated that CSI can be used to improve the performance of the network even when it is i.i.d. and the users are restricted to stationary policies. However, we conjecture that our results go through if the channel states are independent from user to user and ergodic and the user is restricted to stationary policies. In this case, the proofs might be more involved because the theory of Markov chains cannot be used to analyze the queue lengths.

Appendix I
Proof of Theorem 1

The time evolution of the random variable $\boldsymbol{N}^{(t)}$ is given by

$$N_j^{(t+1)} = \left(N_j^{(t)} - Y_j^{(t)}\right)^+ + X_j^{(t)} \tag{62}$$

where $Y_j^{(t)}$ is equal to one if node j successfully transmits a packet during slot t and is equal to zero otherwise, and $X_j^{(t)}$ is the number of newly arrived packets in slot t. Since the channel is independent from slot to slot and the transmission probability depends only on the current channel state, the n-dimensional process $\boldsymbol{N}^{(t)}$ is a Markov chain. We assume that the arrival process and the reception model are such that the Markov chain is aperiodic and irreducible. This is a mild requirement that is satisfied for most nontrivial arrival processes and reception models.

The stability of the system, which is equivalent to the existence of a limiting distribution for the Markov chain is therefore also equivalent to the ergodicity of the Markov chain.

In order to show the stability of this Markov chain, we borrow the techniques that were used in [23]. We state a key lemma from [23] that will be used to obtain a sufficient condition for stability.

Lemma 1: Assume that $\xi(t)$ and $\chi(t), t \in \mathbb{N}$, are two random sequences with values from the set $\{0,1,2,\ldots\}$, while $\mathcal{A}$ is some event associated with them. If for any x, t, k, s

$$\begin{aligned}\Pr\{\chi(t) > x|\chi(t) = s, \chi(0) = k\} \\ \leq \Pr\{\chi(t) > x|\chi(t) = s+1, \chi(0) = k\} \quad (63)\end{aligned}$$

$$\begin{aligned}\Pr\{\xi(t) > x|\xi(t) = s, \xi(0) = k, \mathcal{A}\} \\ \leq \Pr\{\chi(t) > x|\chi(t) = s, \chi(0) = k\} \quad (64)\end{aligned}$$

then

$$\Pr\{\xi(t) > x|\xi(0) = k, \mathcal{A}\} \leq \Pr\{\chi(t) > x|\chi(0) = k\}. \quad (65)$$

This lemma says that the stability of $\chi(t)$ implies the stability of $\xi(t)$. The properties listed in the lemma are commonly notated as $\chi(t)$ stochastically dominates $\xi(t)$ [39]–[41]. Given the random sequence $N_i(t)$, the key is to identify a sequence that stochastically dominates $N_i(t)$ and whose stability is easy to analyze. As in [23], we define a one-dimensional Markov chain $Q_j^{(t)}$ which is the fully loaded version of $N_j^{(t)}$. That is, $Q_j^{(t)}$ is a Markov chain and

$$\begin{aligned}\Pr\left\{Q_j^{(t+1)} = k|Q_j^{(t)} = s\right\} = \Pr\Big\{N_j^{(t+1)} = k|N_j^{(t)} = s, \\ N_i^{(t)} > 0, i = 1,\ldots,n, i \neq j\Big\}. \quad (66)\end{aligned}$$

In order to use stochastic dominance to analyze $\boldsymbol{N}^{(t)}$, we need to first show that the random sequence defined above satisfies the properties listed in Lemma 1.

Lemma 2: The Markov chain $Q_j(t)$ stochastically dominates $N_j(t)$.

Proof: Refer to Appendix II.

For the fully loaded system, an application of Pakes' lemma [42], which gives a sufficient condition on drift

$$D_i \triangleq \mathrm{E}\{Q_j(t+1) - Q_j(t)|Q_j(t) = i\}$$

can be used to obtain a sufficient condition for stability. For the sake of completeness, we state Pakes' lemma as follows.

Lemma 3: Suppose that the drift $D_i < \infty$ for all i, and that for some scalar $\delta > 0$ and integer $\bar{i} \geq 0$ we have $D_i \leq -\delta$, for all $i > \bar{i}$. Then the Markov chain has a stationary distribution.

It is easy to see that the drift D_i for the fully loaded system is independent of i and is given by

$$\begin{aligned}D_i &= \frac{\lambda}{n} - \sum_{k=0}^{n-1}\binom{n-1}{k}\left(1 - \int_0^\infty s(\gamma)dF(\gamma)\right)^{n-1-k} \\ &\quad\times\Bigg(\int_0^\infty\cdots\int_0^\infty s(\gamma_1)\cdots s(\gamma_{k+1}) \\ &\quad\quad\times\sum_{\Theta_{k+1}}\theta_{k+1}^{(1)}\Phi^{(k+1)}(\gamma_1,\ldots,\gamma_{k+1};\Theta_{k+1}) \\ &\quad\quad\times dF(\gamma_1)\cdots dF(\gamma_{k+1})\Bigg) \quad (67) \\ &= \frac{1}{n}\Bigg(\lambda - \sum_{k=1}^{n}\binom{n}{k}\left(1 - \int_0^\infty s(\gamma)dF(\gamma)\right)^{n-k} \\ &\quad\times\Bigg(\int_0^\infty\cdots\int_0^\infty s(\gamma_1)\cdots s(\gamma_k) \\ &\quad\times\Psi^{(k)}(\gamma_1,\ldots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k)\Bigg)\Bigg). \quad (68)\end{aligned}$$

The second equality follows from the symmetry of the reception model that was assumed in Section II. The above equation gives a sufficient condition for the stability of $Q_j(t)$, which due to Lemma 2 is also a sufficient condition for the stability of $\boldsymbol{N}(t)$.

We obtain necessary conditions for stability in a straightforward way by following the arguments in [23]. We now state a key result that is proved in [23] in a slightly more general form.

Lemma 4: Let Markov chain $\boldsymbol{N}(t)$ defined over $\mathbb{Z}_+^n$ possess the following property of bounded homogeneity with respect to its states: for any $\boldsymbol{s} \in \mathbb{Z}_+^n$ and $\boldsymbol{r} \in \mathbb{Z}_+^n$, such that for every i, either $s_i = r_i = 0$ or $s_i > 0, r_i > 0$ and for any $\boldsymbol{k} \in \mathbb{Z}_+^n$, we have

$$\begin{aligned}\Pr\{\boldsymbol{N}(t+1) = \boldsymbol{s} + \boldsymbol{k}|\boldsymbol{N}(t) = \boldsymbol{s}\} \\ = \Pr\{\boldsymbol{N}(t+1) = \boldsymbol{r} + \boldsymbol{k}|\boldsymbol{N}(t) = \boldsymbol{r}\}. \quad (69)\end{aligned}$$

Then for $j = 1,\ldots,n$, $\mathrm{E}\{Q_j(t+1) - Q_j(t)|Q_j(t) > 0\} > 0$ implies that $N_j(t) \to \infty$ as $t \to \infty$ with probability 1 for all j.

It is easy to see that the bounded homogeneity property holds for the Markov chain under consideration. Thus, Lemma 4 implies that the following condition is necessary for stability:

$$\begin{aligned}\lambda \leq \sum_{k=1}^{n}\binom{n}{k}\left(1 - \int_0^\infty s(\gamma)dF(\gamma)\right)^{n-k} \\ \times\Bigg(\int_0^\infty\cdots\int_0^\infty s(\gamma_1)\cdots s(\gamma_k)\Psi^{(k)} \\ \times(\gamma_1,\ldots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k)\Bigg). \quad (70)\end{aligned}$$

Thus, the theorem about maximum stable throughput follows. □

APPENDIX II
PROOF OF LEMMA 2

We need that for all t, s, k, and x

$$\Pr\left\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k\right\} \leq \Pr\left\{Q_j^{(t+1)} > x | Q_j^{(t)} = s+1, Q_j^{(0)} = k\right\}.$$

In other words, the probability that the buffer goes above a certain level in slot $(t+1)$ is larger if the queue has more packets in slot t. It is obvious that this is indeed the case. The other property to be shown is

$$\Pr\left\{N_j^{(t+1)} > x | N_j^{(t)} = s, \boldsymbol{N}^{(0)} = \boldsymbol{k}\right\} \leq \Pr\left\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k_j\right\}$$

where $\boldsymbol{k} = (k_1, \ldots, k_n)$. In other words, the tendency of the buffer of the fully loaded system to exceed a level x is higher than that of the original system. In order to show this, we first observe that the evolution of the jth buffer in the original system and the fully loaded system is given by

$$\begin{aligned} N_j^{(t+1)} &= \left(s - Y_j^{(t)}\right)^+ + X_j^{(t)} \\ Q_j^{(t+1)} &= \left(s - Z_j^{(t)}\right)^+ + X_j^{(t)}. \end{aligned} \tag{71}$$

Hence, in order to show (71), it is only necessary that we show that the probability of success is higher in the original system, or

$$\Pr\left\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\right\} \geq \Pr\left\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\right\}. \tag{72}$$

If $U_j^{(t)}$ is the number of nodes competing with node j to send packets in time slot t (the nodes with nonempty queues), we note that

$$\Pr\left\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\right\} = \sum_{k=0}^{n-1} \Pr\left\{U_j^{(t)} = k\right\} \times \underbrace{\Pr\left\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, U_j^{(t)} = k\right\}}_{f_k} \tag{73}$$

whereas

$$\Pr\left\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\right\} = \underbrace{\Pr\left\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, U_j^{(t)} = n-1\right\}}_{f_{n-1}}. \tag{74}$$

We show that the probability of success f_k is a decreasing function of k which will then imply (72) because of (73) and (74). We have the following formula for f_k:

$$f_k = \sum_{l=0}^{k} \binom{k}{l} (1-p)^{k-l} \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{l+1}) \sum_{\Theta_{l+1}} \theta_{l+1}^{(1)} \times \Phi^{(l+1)}(\gamma_1, \ldots, \gamma_{l+1}; \Theta_{l+1}) dF(\gamma_1) \cdots dF(\gamma_{l+1})$$

where

$$p = \int_0^\infty s(\gamma) dF(\gamma). \tag{75}$$

Equivalently, f_k is the coefficient of x^k in

$$(1 + (1-p)x)^k \, h(x) \tag{76}$$

where $h(x) = h_0 + h_1 x + h_2 x^2 + \cdots$ and

$$h_l = \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{l+1}) \sum_{\Theta_{l+1}} \theta_{l+1}^{(i)} \Phi^{(l+1)} \times (\gamma_1, \ldots, \gamma_{l+1}; \Theta_{l+1}) dF(\gamma_1) \cdots dF(\gamma_{l+1}). \tag{77}$$

Therefore, $f_k - f_{k+1}$ is the coefficient of x^{k+1} in

$$(1 + (1-p)x)^k (xh(x)) - (1 + (1-p)x)^{k+1} h(x) \tag{78}$$

$$= (1 + (1-p)x)^k (pxh(x) - h(x)). \tag{79}$$

The difference $f_k - f_{k+1}$ is a function of the coefficients of $x, \ldots, x^{k+1}$ in $(pxh(x) - h(x))$, which we will show are all positive. The coefficient of $x^j, j = 1, \ldots, (k+1)$ is given by

$$p \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_j) \sum_{\Theta_j} \theta_j^{(1)} \Phi^{(j)} \times (\gamma_1, \ldots, \gamma_j; \Theta_j) dF(\gamma_1) \cdots dF(\gamma_j) - \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{j+1}) \sum_{\Theta_{j+1}} \theta_{j+1}^{(1)} \Phi^{(j+1)}(\gamma_1, \ldots, \gamma_{j+1}; \Theta_{j+1}) \times dF(\gamma_1) \cdots dF(\gamma_{j+1}).$$

Due to the condition (4) on the reception functions $\Phi^{(k)}(\cdot)$, the coefficients of x^j for $j = 1, \ldots, (k+1)$ are greater than zero which implies that $f_k \geq f_{k+1}$. Hence, the Markov chain $Q_j(t)$ stochastically dominates $N_j(t)$. □

APPENDIX III
PROOF OF THEOREM 2

Let $\Lambda(u)$ be defined as

$$\Lambda(u) = \left\{s(\gamma) : 0 \leq s(\gamma) \leq 1, \int_0^\infty s(\gamma) dF(\gamma) = u\right\}. \tag{80}$$

For $s(\gamma) \in \Lambda(u)$, we have

$$\lambda_n(s(\cdot)) = n(1-u)^{n-1}\left(\int_{\gamma_0}^{\infty} s(\gamma)dF(\gamma)\right). \tag{81}$$

We note that $\int_{\gamma_0}^{\infty} s(\gamma)dF(\gamma) \leq p_{\gamma_0}$. This leads to the following upper bound on the maximum stable throughput:

$$\lambda_n(s(\cdot)) \leq \begin{cases} n(1-u)^{n-1}u, & u \leq p_{\gamma_0} \\ n(1-u)^{n-1}p_{\gamma_0}, & u > p_{\gamma_0}. \end{cases} \tag{82}$$

If $p_{\gamma_0} \geq \frac{1}{n}$, maximizing the upper bound by varying u between 0 and 1, we find that for all $s(\cdot)$

$$\lambda_n(s(\cdot)) \leq \left(1 - \frac{1}{n}\right)^{n-1}. \tag{83}$$

Hence, choosing the transmission control as

$$s^*(\gamma) = \begin{cases} 0, & \gamma < \gamma_0 \\ \frac{1}{np_{\gamma_0}}, & \gamma \geq \gamma_0 \end{cases} \tag{84}$$

achieves the maximum and is hence optimal. If $p_{\gamma_0} < \frac{1}{n}$, then the preceding choice is not valid since $\frac{1}{np_{\gamma_0}} > 1$. If $p_{\gamma_0} < \frac{1}{n}$, we find that the upper bound is maximized at $u = p_{\gamma_0}$ and

$$\lambda_n(s(\cdot)) \leq n(1-p_{\gamma_0})^{n-1}p_{\gamma_0}. \tag{85}$$

Hence, for $p_{\gamma_0} < \frac{1}{n}$, choosing the optimal choice for the transmission control is given by

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ 1 & \gamma \geq \gamma_0. \end{cases} \tag{86}$$

□

APPENDIX IV
PROOF OF PROPOSITION 4

For convenience, we define the function $H_n(s_n(\cdot))$ as

$$\begin{aligned} H_n(s_n(\cdot)) \triangleq \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int s_n dF\right)^{n-k} \\ \times \int \cdots \int s_n(\gamma_1) \cdots s_n(\gamma_k)\Psi^{(k)}(\gamma_1, \ldots, \gamma_k) \\ \times dF(\gamma_1) \cdots dF(\gamma_k). \end{aligned} \tag{87}$$

We assume that given a distribution function $T(\cdot)$ that is dominated by $F(\cdot)$, the sequence of transmission controls is chosen as $t_n(T, x, \gamma)$, where

$$t_n(T, x, \gamma) = \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right). \tag{88}$$

We claim that for this choice of transmission control sequence

$$\liminf_{n\to\infty} H_n(t_n(\cdots)) = e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T(\cdot)). \tag{89}$$

Due to assumption *A1*, we have

$$e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T(\cdot)) < \infty \tag{90}$$

which implies that for all $\epsilon > 0$, $\exists M$ such that

$$\sum_{k=M+1}^{\infty}\frac{x^k}{k!}C_k(T(\cdot)) < \epsilon. \tag{91}$$

Therefore, for $n > M$, we have

$$\begin{aligned} &\sum_{k=M+1}^{n} \binom{n}{k}\left(1 - \int t_n dF\right)^{n-k} \\ &\quad \times \int \cdots \int t_n(x, T, \gamma_1) \cdots t_n(x, T, \gamma_k) \\ &\quad \times \Psi^{(k)}(\gamma_1, \ldots, \gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) && (92)\\ &\quad \leq \sum_{k=M+1}^{n} \frac{n!}{(n-k)!k!}\frac{x^k}{n^k}C_k(T(\cdot)) && (93)\\ &\quad \leq \sum_{k=M+1}^{n}\frac{x^k}{k!}C_k(T(\cdot)) && (94)\\ &\quad < \epsilon. && (95) \end{aligned}$$

The second inequality follows because for all γ

$$t_n(x, T, \gamma) \leq \frac{x}{n}\frac{dT}{dF}. \tag{96}$$

Hence,

$$\begin{aligned} &\liminf_{n\to\infty} H_n(t_n(x, T, \gamma)) \\ &\leq \liminf_{n\to\infty}\sum_{k=1}^{M}\binom{n}{k} \\ &\quad \times \left(1 - \int t_n dF\right)^{n-k}\int\cdots\int t_n(x, T, \gamma_1) \\ &\quad \cdots t_n(x, T, \gamma_k)\Psi^{(k)}(\gamma_1, \ldots, \gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) + \epsilon. \end{aligned} \tag{97}$$

For each k we have

$$\begin{aligned} &\binom{n}{k}\left(1 - \int t_n dF\right)^{n-k} \\ &\quad \times \int\cdots\int t_n(x, T, \gamma_1)\cdots t_n(x, T, \gamma_k) \\ &\quad \times \Psi^{(k)}(\gamma_1, \ldots, \gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) \\ &= \frac{1}{k!}\left(1 - \frac{1}{n}\int nt_n dF\right)^{n-k}\frac{n!}{(n-k)!n^k} \\ &\quad \times \int\cdots\int n^k t_n(x, T, \gamma_1)\cdots t_n(x, T, \gamma_k) \\ &\quad \times \Psi^{(k)}(\gamma_1, \ldots, \gamma_k)dF(\gamma_1)\cdots dF(\gamma_k). \end{aligned} \tag{98}$$

Since $nt_n(x, T, \gamma) \uparrow x\frac{dT}{dF}(\gamma)$ for all γ, we have using monotone convergence theorem

$$\begin{aligned} \lim_{n\to\infty}\int nt_n(x, T, \gamma)dF &= \int x\frac{dT}{dF}dF && (99)\\ &= x. && (100) \end{aligned}$$

Similarly, we also have

$$\lim_{n\to\infty}\int n^k t_n(x,T,\gamma_1)\cdots t_n(x,T,\gamma_k)\Psi^{(k)} \times(\gamma_1,\dots,\gamma_k)dF(\gamma_1)\cdots dF(\gamma_k) = x^k\int\cdots\int\Psi^{(k)}(\gamma_1,\dots,\gamma_k)dT(\gamma_1)\cdots dT(\gamma_k) = x^kC_k(T(\cdot)). \quad (101)$$

Define $f_n(y)$ as

$$f_n(y)\triangleq\left(1-\frac{y}{n}\right)^{(n-k)}. \quad (102)$$

We know that

$$\lim_{n\to\infty}f_n(y)=e^{-y}. \quad (103)$$

In fact, for any $A>0$, the sequence of functions $f_n(y)$ converges uniformly to e^{-y} over the range $[0,A]$. This implies that if the sequence $x_n\to x$, where $x<A$, then

$$\lim_{n\to\infty}f_n(x_n)=e^{-x}. \quad (104)$$

Hence, taking the limit of (98), we have

$$\lim_{n\to\infty}\frac{x^k}{k!}\left(1-\frac{x}{n}\right)^{n-k}C_k(T(\cdot))\prod_{i=0}^{k-1}\left(1-\frac{i}{n}\right) = e^{-x}\frac{x^k}{k!}C_k(T(\cdot)). \quad (105)$$

Therefore,

$$\liminf_{n\to\infty}H_n(t_n(\cdots))=e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T(\cdot)). \quad (106)$$

□

APPENDIX V
PROOF OF PROPOSITION 5

If $G(\cdot)=1-e^{-\frac{\gamma}{P_T}}$, it is easy to see that

$$C_{k+1}(G(\cdot))=\frac{k+1}{P_T}\int_0^\infty\cdots\int_0^\infty\Pr\left\{\frac{\beta N\sigma^2}{\gamma}+\sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\}\times e^{-\frac{\gamma}{P_T}}d\gamma d\gamma_1\cdots d\gamma_k \le\frac{k+1}{P_T}\int_0^\infty\cdots\int_0^\infty\Pr\left\{\sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\}\times e^{-\frac{\gamma}{P_T}}d\gamma d\gamma_1\cdots d\gamma_k. \quad (107)$$

We obtain an upper bound on the inner probability using Chernoff's bound as follows. Given an $h>0$, we have

$$\Pr\left\{\sum_{i=1}^{k}\frac{\beta\gamma_i}{\beta\gamma_i+\gamma}<N\right\}\le\mathrm{E}\left\{\exp\left(hN-\sum_{i=1}^{k}\frac{h\beta\gamma_i}{\beta\gamma_i+\gamma}\right)\right\} = e^{hN}\mathrm{E}^k\left\{\exp\left(-\frac{h\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\}. \quad (108)$$

We let $h=1$ and define $\mu(\gamma)$ as the characteristic function

$$\mathrm{E}\left\{\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\}=\frac{1}{P_T}\int_0^\infty\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\times\exp\left(-\frac{\gamma_1}{P_T}\right)d\gamma_1. \quad (109)$$

Therefore,

$$C_{k+1}(G(\cdot))\le\frac{(k+1)e^N}{P_T}\int_0^\infty\mu^k(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma. \quad (110)$$

We now show that there exists $c>0$ such that

$$\lim_{\gamma\to\infty}\gamma(1-\mu(\gamma))>c \quad (111)$$

which will imply that there exists γ^* such that $\gamma>\gamma^*$ implies

$$\mu(\gamma)\le 1-\frac{c}{\gamma}. \quad (112)$$

We have

$$\gamma(1-\mu(\gamma))=\frac{1}{P_T}\int_0^\infty\gamma\left(1-\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right)\times\exp\left(-\frac{\gamma_1}{P_T}\right)d\gamma_1 \ge\frac{1}{2P_T}\int_0^\infty\frac{\beta\gamma\gamma_1}{\gamma+\beta\gamma_1}\exp\left(-\frac{\gamma_1}{P_T}\right)d\gamma_1.$$

The inequality follows because

$$1-e^{-x}\ge\frac{x}{2},\qquad 0\le x\le 1. \quad (113)$$

Using the monotone convergence theorem, we have

$$\lim_{\gamma\to\infty}\gamma(1-\mu(\gamma))\ge\frac{1}{2P_T}\int_0^\infty\lim_{\gamma\to\infty}\frac{\beta\gamma\gamma_1}{\gamma+\beta\gamma_1}\times\exp\left(-\frac{\gamma_1}{P_T}\right)d\gamma_1 =\frac{\beta}{2}. \quad (114)$$

We therefore have

$$\frac{1}{e^N}C_{k+1}(G(\cdot))\le\frac{k+1}{P_T}\int_0^\infty\mu^k(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma \le\frac{1}{P_T}\int_0^{\gamma^*}(k+1)\mu^k(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma +\frac{1}{P_T}\int_{\gamma^*}^\infty(k+1)\left(1-\frac{c}{\gamma}\right)^ke^{-\frac{\gamma}{P_T}}d\gamma. \quad (115)$$

The first integral goes to zero as $k\to\infty$, since $\mu(\gamma)\uparrow 1$ as $\gamma\to\infty$ which implies that there exists an $r<1$ such that for

$0 \leq \gamma \leq \gamma^*$ implies $\mu(\gamma) < r$. The second integral can be shown to go to zero by dominated convergence theorem since for γ large enough

$$(k+1)\left(1-\frac{c}{\gamma}\right)^k \leq \gamma, \qquad \forall\, k \tag{116}$$

and

$$\int_{\gamma^*}^{\infty} \gamma e^{-\frac{\gamma}{P_T}} d\gamma < \infty. \tag{117}$$

We therefore have that

$$\lim_{k\to\infty} C_{k+1}(G(\cdot)) = 0. \tag{118}$$

□

APPENDIX VI
PROOF OF PROPOSITION 6

The proof of this proposition is similar to the previous one. Let $F(\gamma) = 1 - e^{-\frac{\gamma}{P_T}}$ and $G(\cdot) \in \Lambda_F$. If $s(\gamma)$ is the transmission control used then f

$$C_{k+1}(G(\cdot)) = \frac{k+1}{p_s P_T}\int_0^\infty \Pr\left\{\frac{\beta N \sigma^2}{\gamma} + \sum_{i=1}^{k} \frac{\beta\gamma_i}{\beta\gamma_i + \gamma} < N\right\} \times s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma \tag{119}$$

$$\leq \frac{k+1}{p_s P_T}\int_0^\infty \Pr\left\{\sum_{i=1}^{k} \frac{\beta\gamma_i}{\beta\gamma_i + \gamma} < N\right\} \times s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma. \tag{120}$$

We obtain an upper bound on the inner probability using Chernoff's bound as follows. Given an $h > 0$, we have

$$\Pr\left\{\sum_{i=1}^{k} \frac{\beta\gamma_i}{\beta\gamma_i + \gamma} < N\right\} \leq \mathrm{E}\left\{\exp\left(hN - \sum_{i=1}^{k} \frac{h\beta\gamma_i}{\beta\gamma_i + \gamma}\right)\right\} = e^{hN}\mathrm{E}^k\left\{\exp\left(-\frac{h\beta\gamma_1}{\beta\gamma_1 + \gamma}\right)\right\}. \tag{121}$$

We let $h = 1$ and define $\mu(\gamma)$ as the characteristic function

$$\mathrm{E}\left\{\exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right\} = \frac{1}{p_s P_T}\int_0^\infty \exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right) \times s(\gamma_1)\exp\left(-\frac{\gamma_1}{P_T}\right) d\gamma_1. \tag{122}$$

As before, we show below that there exists $c > 0$ such that

$$\lim_{\gamma\to\infty} \gamma(1-\mu(\gamma)) > c, \tag{123}$$

which will imply that there exists γ^* such that $\gamma > \gamma^*$ implies

$$\mu(\gamma) \leq 1 - \frac{c}{\gamma}. \tag{124}$$

We have

$$\gamma(1-\mu(\gamma)) = \frac{1}{p_s P_T}\int_0^\infty \gamma\left(1 - \exp\left(-\frac{\beta\gamma_1}{\beta\gamma_1+\gamma}\right)\right) \times s(\gamma_1)\exp\left(-\frac{\gamma_1}{P_T}\right) d\gamma_1$$
$$\geq \frac{1}{2p_s P_T}\int_0^\infty \frac{\beta\gamma\gamma_1}{\gamma+\beta\gamma_1} s(\gamma_1)\exp\left(-\frac{\gamma_1}{P_T}\right) d\gamma_1.$$

The inequality follows because

$$1 - e^{-x} \geq \frac{x}{2}, \qquad 0 \leq x \leq 1. \tag{125}$$

Using the monotone convergence theorem, we have

$$\lim_{\gamma\to\infty} \gamma(1-\mu(\gamma)) \geq \frac{1}{2p_s P_T}\int_0^\infty \lim_{\gamma\to\infty} \frac{\beta\gamma\gamma_1}{\gamma+\beta\gamma_1} s(\gamma_1) \times \exp\left(-\frac{\gamma_1}{P_T}\right) d\gamma_1 = \frac{\beta}{2}\mathrm{E}\{\gamma_1\}. \tag{126}$$

It is easy to see that $0 < \mathrm{E}\{\gamma_1\} < \infty$.

We therefore have

$$\frac{1}{e^N} C_{k+1}(G(\cdot)) \leq \frac{k+1}{p_s P_T}\int_0^\infty \mu^k(\gamma) s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma$$
$$\leq \frac{1}{p_s P_T}\int_0^{\gamma^*} (k+1)\mu^k(\gamma) s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma + \frac{1}{p_s P_T}\int_{\gamma^*}^\infty (k+1)\left(1-\frac{c}{\gamma}\right)^k \times s(\gamma) e^{-\frac{\gamma}{P_T}} d\gamma. \tag{127}$$

The first integral goes to zero as $k \to \infty$, since $\mu(\gamma) \uparrow 1$ as $\gamma \to \infty$ which implies that there exists an $r < 1$ such that for $0 \leq \gamma \leq \gamma^*$ implies $\mu(\gamma) < r$. The second integral can be shown to go to zero by dominated convergence theorem since for γ large enough

$$(k+1)\left(1-\frac{c}{\gamma}\right)^k \leq \gamma, \qquad \forall\, k \tag{128}$$

and

$$\int_{\gamma^*}^{\infty} \gamma e^{-\frac{\gamma}{P_T}} d\gamma < \infty. \tag{129}$$

We therefore have that

$$\lim_{k\to\infty} C_{k+1}(G(\cdot)) = 0. \tag{130}$$

□

APPENDIX VII
PROOF OF PROPOSITION 7

From an extension of the arguments in [37], the asymptotic MMSE throughput for a distribution of rolloff δ is given by

$$C(\beta, N, \delta) = \int_0^\infty \mathrm{P}\left[\sum_{i=1}^{\infty} \frac{\beta S_i^{-\frac{1}{\delta}}}{\beta S_i^{-\frac{1}{\delta}} + s^{-\frac{1}{\delta}}} < N\right] ds \tag{131}$$

where $\{S_i\}_{i=1}^\infty$ are points of a homogeneous Poisson process of rate 1. We then have

$$C(\beta, N, \delta) = \int_0^\infty \mathrm{P}\left[\sum_{i=1}^{\infty} \frac{1}{N + \frac{N}{\beta} S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] ds = \left(\frac{N}{\beta}\right)^\delta \int_0^\infty \mathrm{P}\left[\sum_{i=1}^{\infty} \frac{1}{N + S_i^{\frac{1}{\delta}} s^{-\frac{1}{\delta}}} < 1\right] ds. \tag{132}$$

The second equality follows from a simple substitution. Note that

$$\mathrm{P}\left[\sum_{i=1}^{\infty}\frac{1}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}<1\right] = \mathrm{P}\left[-1<\sum_{i=1}^{\infty}\frac{1}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}<1\right]. \quad (133)$$

We also have [43] (p. 346) that

$$\mathrm{P}\left[-1<\sum_{i=1}^{\infty}\frac{1}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}<1\right] = \lim_{M\to\infty}\int_{-M}^{M}\frac{e^{j\omega}-e^{-j\omega}}{j\omega}\mu^{\infty}(\omega)d\omega \quad (134)$$

where

$$\mu^{\infty}(\omega)\triangleq \mathrm{E}\left[\exp\left(\sum_{i=1}^{\infty}\frac{j\omega}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)\right]. \quad (135)$$

Therefore,

$$C(\beta,N,\delta)=\int_0^{\infty}\lim_{M\to\infty}\int_{-M}^{M}\frac{e^{j\omega}-e^{-j\omega}}{j\omega}\mu^{\infty}(\omega)d\omega ds. \quad (136)$$

It is easy to see that

$$\begin{aligned}\mu^{\infty}(\omega) &= \mathrm{E}\left[\lim_{T\to\infty}\exp\left(\sum_{i=1}^{N(T)}\frac{j\omega}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)\right] &(137)\\ &= \lim_{T\to\infty}\mathrm{E}\left[\exp\left(\sum_{i=1}^{N(T)}\frac{j\omega}{N+S_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)\right] &(138)\\ &= \lim_{T\to\infty}\sum_{k=1}^{\infty}\frac{e^{-T}T^k}{k!}\mathrm{E}\left[\exp\left(\sum_{i=1}^{k}\frac{j\omega}{N+U_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)\right] &(139)\\ &= \lim_{T\to\infty}\sum_{k=1}^{\infty}\frac{e^{-T}T^k}{k!}\mu^k(\omega,T) &(140)\\ &= \lim_{T\to\infty}\exp\left(T\mu(\omega,T)-T\right) &(141)\end{aligned}$$

where $N(T)$ is the number of Poisson points in $[0,T)$, $\{U_i\}_{i=1}^{\infty}$ are independent and uniformly distributed between 0 and T, and $\mu(\omega,T)$ is the characteristic function of $\frac{1}{N+U_i^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}$. The first and third equalities follow due to the properties of the Poisson process. The second equality follows due to bounded convergence theorem. We have

$$\mu(\omega,T)=\frac{1}{T}\int_0^T\exp\left(\frac{j\omega}{N+u^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)du. \quad (142)$$

Therefore,

$$\begin{aligned}\mu^{\infty}(\omega) &= \lim_{T\to\infty}\exp\left(\int_0^T\left(\exp\left(\frac{j\omega}{N+u^{\frac{1}{\delta}}s^{-\frac{1}{\delta}}}\right)-1\right)du.\right) &(143)\\ &= \exp\left(s\int_0^{\infty}\left(\exp\left(\frac{j\omega}{N+p^{\frac{1}{\delta}}}\right)-1\right)dp.\right). &(144)\end{aligned}$$

Consider the inner integral. It turns out that

$$\int_0^{\infty}\left(\exp\left(\frac{j\omega}{N+p^{\frac{1}{\delta}}}\right)-1\right)dp = \frac{j\omega}{N}N^{\delta}\frac{\pi\delta}{\sin\pi\delta}F_1\left(1-\delta,2;\frac{j\omega}{N}\right) \quad (145)$$

where $F_1(a,b;z)$ is a confluent hyper-geometric function [44]. Therefore, the original integral can be written as

$$C(\beta,N,\delta)=\left(\frac{N}{\beta}\right)^{\delta}\frac{1}{2\pi}\int_0^{\infty}\lim_{M\to\infty}\int_{-M}^{M}\frac{2\sin\omega}{\omega} \times\exp\left(s\frac{j\omega}{N}N^{\delta}\frac{\pi\delta}{\sin\pi\delta}F_1\left(1-\delta,2;\frac{j\omega}{N}\right)\right)d\omega ds. \quad (146)$$

After a simple change of variables, this becomes

$$C(\beta,N,\delta)=\frac{N}{\beta^{\delta}}\frac{\sin\pi\delta}{\pi\delta}\frac{1}{\pi}\int_0^{\infty}\lim_{M\to\infty} \times\int_{-M}^{M}\underbrace{\frac{\sin\omega}{\omega}\exp\left(sj\omega F_1\left(1-\delta,2;\frac{j\omega}{N}\right)\right)}_{F(\omega,s)}d\omega ds. \quad (147)$$

We now consider some properties of the function

$$j\omega F_1\left(1-\delta,2;\frac{j\omega}{N}\right)\triangleq F_R(\omega)+jF_I(\omega) \quad (148)$$

that are crucial for the evaluation of the integral. The above function can also be written as [44]

$$\int_0^1 j\omega\exp\left(\frac{j\omega t}{N}\right)\left(\frac{1}{t}-1\right)^{\delta}dt. \quad (149)$$

We therefore have

$$\begin{aligned}F_R(\omega) &= -\int_0^1\omega\sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t}-1\right)^{\delta}dt &(150)\\ &\geq -\frac{\omega^2}{N}\int_0^1 t^{1-\delta}(1-t)^{\delta}dt &(151)\\ &= -k\omega^2 &(152)\end{aligned}$$

where k is a positive constant. The second inequality follows because $\sin(\omega t)\leq\omega t$. It is also easy to see that the function $F_R(\omega)\leq 0$ for all ω. Further, the Taylor series expansion of $F_R(\omega)$ is given by

$$F_R(\omega)=-\frac{\omega^2}{N}\frac{1-\delta}{2}+\frac{\omega^4}{4!N^3}\frac{3-\delta}{3}\frac{2-\delta}{2}\frac{1-\delta}{1}-\dots. \quad (153)$$

It follows that the function $F_R(\omega)$ is even. Similarly, the Taylor series expansion of $F_I(\omega)$ is given by

$$F_I(\omega)=\omega-\frac{\omega^3}{3!N^2}\frac{2-\delta}{2}\frac{1-\delta}{1} +\frac{\omega^5}{5!N^4}\frac{4-\delta}{4}\frac{3-\delta}{3}\frac{2-\delta}{2}\frac{1-\delta}{1}-\dots. \quad (154)$$

More importantly, we have $F_I(\omega)=\omega+O(\omega^3)$ as $\omega\to 0$. It can also be seen that the function $F_I(\omega)$ is odd.

Let $\epsilon > 0$. In order to find $C(\beta, N, \delta)$, we need to evaluate the following integrals:

$$\int_0^\infty \lim_{M\to\infty}\left(\int_{-M}^{-\epsilon} F(\omega,s)d\omega + \int_{-\epsilon}^{\epsilon} F(\omega,s)d\omega + \int_0^\infty \int_\epsilon^M F(\omega,s)d\omega\right)ds. \quad (155)$$

Even though these integrals are difficult to evaluate, it turns out that the middle integral contains most of the mass. So let ϵ tend to zero and consider the middle integral which can be written as

$$\begin{aligned}
&\lim_{\epsilon\to 0}\int_0^\infty \int_{-\epsilon}^{\epsilon} F(\omega,s)d\omega ds\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} F(\omega,s)d\omega ds\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \mathrm{Re}\{F(\omega,s)\}d\omega ds\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \frac{\sin\omega}{\omega}\exp(sF_R(\omega))\cos(sF_I(\omega))d\omega ds\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \exp(sF_R(\omega))\cos(sF_I(\omega))d\omega ds.
\end{aligned} \quad (156)$$

The last equality follows because ϵ can be made as small as possible and hence $\frac{\sin\omega}{\omega}$ can be made as close as possible to 1. Since $F_R(\omega) \le 0$, an upper bound on the above integral is

$$\lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \cos(sF_I(\omega))d\omega ds. \quad (157)$$

Since $F_R(\omega) \ge -k\omega^2$, where $k \ge 0$, a lower bound on the integral is

$$\lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \exp(-ks\omega^2)\cos(sF_I(\omega))d\omega ds \quad (158)$$

due to the property of $F_R(\omega)$ given in (152). We will show that both these integrals are equal to π and thus the required integral would have been evaluated. Consider

$$\begin{aligned}
&\lim_{\epsilon\to 0}\lim_{t\to\infty}\int_0^t \int_{-\epsilon}^{\epsilon} \cos(sF_I(\omega))d\omega ds\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_{-\epsilon}^{\epsilon} \int_0^t \cos(sF_I(\omega))ds d\omega\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_{-\epsilon}^{\epsilon} \frac{\sin tF_I(\omega)}{F_I(\omega)}d\omega \quad (159)\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \frac{\sin\theta}{\theta}\frac{1}{g(\frac{\theta}{t})}d\theta \quad (160)
\end{aligned}$$

where

$$g(\omega) \triangleq F_I'(F_I^{-1}(\omega)). \quad (161)$$

The last inequality follows after the substitution $F_I(\omega)t = \theta$. For convenience, we define the function $f(\omega)$ as

$$f(\omega) \triangleq F_I^{-1}(\omega). \quad (162)$$

Since $F_I'(\omega) = 1 + O(\omega^2)$, we can choose ϵ small enough such that the inverse function of $F_I(\omega)$ is defined. This also implies, from the inverse function theorem [45], that the function F_I^{-1} is continuously differentiable. It is therefore simple to show that the function $F_I^{-1}(\omega) = O(\omega)$ as $\omega \to 0$. It is also easy to show that the function $F_I'(F_I^{-1}(\omega)) = 1 + O(\omega^2)$ as $\omega \to 0$. Thus, $f(\omega) = O(\omega)$ and $g(\omega) = 1 + O(\omega^2)$. For simplicity, we define $h(\omega) = \frac{1}{g(\omega)}$. It is easy to see that $h(\omega) = 1 + O(\omega^2)$. This implies that the above integral is in fact equal to π.

We now consider the lower bound which after interchange of integrals becomes

$$\begin{aligned}
&\lim_{\epsilon\to 0}\lim_{t\to\infty}\int_\epsilon^\epsilon \exp(-kt\omega^2)\frac{\sin F_I(\omega)t}{F_I(\omega)}\frac{F_I^2(\omega)}{F_I^2(\omega)+k^2\omega^4}d\omega\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_\epsilon^\epsilon \exp(-kt\omega^2)\frac{\sin F_I(\omega)t}{F_I(\omega)}d\omega\\
&= \lim_{\epsilon\to 0}\lim_{t\to\infty}\int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)\frac{\sin\theta}{\theta}\frac{1}{g(\frac{\theta}{t})}d\theta.
\end{aligned} \quad (163)$$

The last inequality follows after the substitution $F_I(\omega)t = \theta$ and the function $f(\omega)$ and $g(\omega)$ are as defined previously. We know that

$$\lim_{\epsilon\to 0}\lim_{t\to\infty}\int_{F_I(-\epsilon)t}^{F_I(\epsilon)t} \frac{\sin\theta}{\theta}d\theta = \pi \quad (164)$$

and hence, we consider the difference between the two integrals and bound the difference. Set $F_I(\epsilon) = \delta$ and note that by making ϵ small δ can be made as small as possible. Now

$$\begin{aligned}
&\left|\int_{-\delta t}^{\delta t} \frac{\sin\theta}{\theta}\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)d\theta - \int_{-\delta t}^{\delta t}\frac{\sin\theta}{\theta}d\theta\right|\\
&= \left|\int_{-\delta t}^{\delta t} \frac{\sin\theta}{\theta}\left(\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h(\frac{\theta}{t}) - 1\right)\right|\\
&= \left|-\cos\theta\frac{\exp(-tf^2(\frac{\theta}{t}))h\left(\frac{\theta}{t}\right)-1}{\theta}\right|_{-\delta t}^{\delta t} - \int_{-\delta t}^{\delta t} -\cos\theta\\
&\quad \times\left\{\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)-1}{-\theta^2}\right.\\
&\qquad + \frac{1}{\theta}\left(\frac{h'\left(\frac{\theta}{t}\right)\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)}{t}\right.\\
&\qquad \left.\left. + \exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)\frac{-2tf\left(\frac{\theta}{t}\right)f'\left(\frac{\theta}{t}\right)}{t}\right)\right\}d\theta\Bigg|\\
&\le \frac{c_1}{\delta t} + \left|\int_{-\delta t}^{\delta t}\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)-1}{\theta^2}d\theta\right|\\
&\quad + \left|\frac{1}{t}\int_{-\delta t}^{\delta t}\frac{h'\left(\frac{\theta}{t}\right)\exp\left(-tf^2(\frac{\theta}{t})\right)}{\theta}d\theta\right|\\
&\quad + \left|2\int_{-\delta t}^{\delta t}\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)f\left(\frac{\theta}{t}\right)f'\left(\frac{\theta}{t}\right)}{\theta}d\theta\right|\\
&\le \frac{c_1}{\delta t} + \int_{-\delta t}^{\delta t}\left|\frac{h'\left(\frac{\theta}{t}\right)}{\theta}\right|d\theta + 2\int_{-\delta t}^{\delta t}\left|\frac{h\left(\frac{\theta}{t}\right)f(\frac{\theta}{t})f'\left(\frac{\theta}{t}\right)}{\theta}\right|d\theta\\
&\quad + \int_{-\delta t}^{\delta t}\left|\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)-1}{\theta^2}\right|d\theta\\
&\le \frac{c_1}{\delta t} + \frac{c_2\delta}{t} + c_3\delta + 4\int_0^{\sqrt{\delta t}}\left|\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)-1}{\theta^2}\right|d\theta
\end{aligned} \quad (165)$$

$$+ 4\int_{\sqrt{\delta t}}^{\delta t}\left|\frac{\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right)h\left(\frac{\theta}{t}\right)-1}{\theta^2}\right|d\theta$$
$$\le \frac{c_1}{\delta t}+\frac{c_2\delta}{t}+c_3\delta+c_4\sqrt{\frac{\delta}{t}}+\frac{c_5}{\sqrt{\delta t}}. \quad (166)$$

The second equality follows from integration by parts. The first inequality follows because $h(\omega) = 1 + O(\omega^2)$ and $\exp(-tf^2(\frac{\theta}{t})) \le 1$. The second inequality follows because $\exp\left(-tf^2\left(\frac{\theta}{t}\right)\right) \le 1$. The final inequality follows because in 0 to $\sqrt{\delta t}$, $tf^2\left(\frac{\theta}{t}\right)$ is small and hence $\exp\left(tf^2\left(\frac{\theta}{t}\right)\right)$ is close to $1 - tf^2\left(\frac{\theta}{t}\right)$).

The rest of the integral is given by

$$\lim_{\epsilon\to 0^+}\int_0^\infty \lim_{M\to\infty}\left(\int_{-M}^{-\epsilon}F(\omega,s)dw+\int_{\epsilon}^{M}F(\omega,s)dw\right)ds. \quad (167)$$

It is easy to see that this is equal to

$$\lim_{\epsilon\to 0^+}\int_0^\infty \lim_{M\to\infty}\int_\epsilon^M 2\mathrm{Re}\left(F(\omega,s)\right)d\omega ds. \quad (168)$$

We now show that the integrals can be interchanged due to Fubini's theorem. Consider

$$\int_0^\infty\int_\epsilon^\infty |\mathrm{Re}\left(F(\omega,s)\right)|\,d\omega ds$$
$$\le \int_0^\infty\int_\epsilon^\infty \exp(sF_R(\omega))d\omega ds \quad (169)$$
$$= \int_\epsilon^\infty -\frac{1}{F_R(\omega)}d\omega \quad (170)$$
$$= \int_\epsilon^\infty \frac{1}{\omega\int_0^1 \sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t}-1\right)^\delta dt}d\omega \quad (171)$$
$$< \infty. \quad (172)$$

The last inequality follows because for ω large the integral

$$\int_0^1 \sin\left(\frac{\omega t}{N}\right)\left(\frac{1}{t}-1\right)^\delta dt \quad (173)$$

increases faster than ω^δ. Hence, the interchange of integrals is justified. Due to this, the rest of the integral can now be written as

$$C(\beta,N,\delta)=\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}+\left(\frac{N}{\beta}\right)^\delta\frac{2}{\pi}$$
$$\times\left(\lim_{\epsilon\to 0}\int_\epsilon^\infty\frac{\sin\omega}{\omega}\mathrm{Re}\left\{\frac{-1}{F_R(\omega)+jF_I(\omega)}\right\}d\omega\right). \quad (174)$$

Therefore, we need to evaluate

$$\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}\frac{2}{\pi}\left(\mathrm{Re}\left\{\int_0^\infty\frac{\sin\omega}{\omega}\frac{1}{j\omega F_1\left(1-\delta,2;\frac{j\omega}{N}\right)}d\omega\right\}\right). \quad (175)$$

This integral can be rewritten with a change of variables as

$$\frac{1}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}\frac{2}{\pi}\left(\mathrm{Re}\left\{\int_0^\infty\frac{\sin N\omega}{\omega}\frac{1}{j\omega F_1(1-\delta,2;j\omega)}d\omega\right\}\right). \quad (176)$$

It is difficult to evaluate the inner integral and it is bounded as follows. Due to the properties of delta functions, the limit of the integral as $N\to\infty$ is given by

$$\frac{\pi}{2}\lim_{\omega\to 0}\mathrm{Re}\left\{\frac{1}{j\omega F_1(1-\delta,2;j\omega)}d\omega\right\} \quad (177)$$

which is equal to $\frac{\pi(1-\delta)}{4}$. Therefore, we have shown that

$$C(\beta,N,\delta)=\frac{N}{\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}+e(N,\delta) \quad (178)$$

where

$$\lim_{N\to\infty}e(N,\delta)=\frac{1-\delta}{2\beta^\delta}\frac{\sin\pi\delta}{\pi\delta}. \quad (179)$$

We conjecture that for any finite N

$$\max_\delta e(N,\delta)=e(N,0)=\frac{1}{2}. \quad (180)$$

We take the value of hyper-geometric function as $\delta = 0$. We have $F_1(1,2,j\omega)=\frac{e^{j\omega}-1}{j\omega}$. Making this substitution, we have

$$\frac{2}{\pi}\mathrm{Re}\left\{\lim_{\epsilon\to 0^+}\int_{[\epsilon,\infty)}\frac{\sin N\omega}{\omega}\frac{1}{e^{j\omega}-1}d\omega\right\}$$
$$=\frac{2}{\pi}\left(\lim_{\epsilon\to 0^+}\int_{[\epsilon,\infty)}\frac{1}{2}\frac{\sin N\omega}{\omega}d\omega\right)$$
$$=\frac{1}{2}. \quad (181)$$

□

Appendix VIII
Proof of Proposition 8

Since $F(\gamma)$ has rolloff δ, we have

$$\lim_{\gamma\to\infty}F^c(\gamma)\gamma^\delta=c \quad (182)$$

where $0 < c < \infty$. Let $G(\cdot)\in\Lambda_F$ and if $G(\cdot)$ has a rolloff smaller than δ, this implies

$$\lim_{\gamma\to\infty}G^c(\gamma)\gamma^\delta=\infty. \quad (183)$$

Therefore,

$$\lim_{\gamma\to\infty}\frac{G^c(\gamma)}{F^c(\gamma)}=\infty. \quad (184)$$

But, we also have that

$$\frac{G^c(\gamma)}{F^c\gamma}=\frac{\int_\gamma^\infty\frac{dG}{dF}dF}{\int_\gamma^\infty dF} \quad (185)$$
$$\le C\frac{\int_\gamma^\infty dF}{\int_\gamma^\infty dF} \quad (186)$$
$$=C. \quad (187)$$

The second inequality follows because $G(\cdot)\in\Lambda_F$ and P3. Equation (187) is clearly in contradiction with (184) which implies that the rolloff of $G(\cdot)$ cannot be smaller than δ.

If $\delta' > \delta$, we show that there exists a transmission control $s(\gamma)$ such that the rolloff of $G_s(\gamma)$ is equal to δ'. Since the rolloff of $F(\gamma)$ is equal to δ, for all $\epsilon > 0$ there exists a γ^* such that $\gamma > \gamma^*$ implies that

$$\frac{c-\epsilon}{\gamma^\delta} \leq F^c(\gamma) \leq \frac{c+\epsilon}{\gamma^\delta}. \tag{188}$$

Choose $s(\gamma)$ as

$$s(\gamma) = \begin{cases} 0, & 0 \leq \gamma < \gamma^* \\ \min\left(1, \frac{1}{\gamma^{\delta'-\delta}}\right), & \gamma \geq \gamma^*. \end{cases} \tag{189}$$

For γ large enough, we have

$$G^c(\gamma) = \frac{1}{p_s}\int_\gamma^\infty s(x)dF(x) \tag{190}$$
$$= \frac{1}{p_s}s(x)F(x)\Big|_\gamma^\infty - \frac{1}{p_s}\int_\gamma^\infty s'(x)F(x)dx \tag{191}$$
$$= \frac{s(\gamma)F(\gamma)}{p_s} + \frac{\delta'-\delta}{p_s}\int_\gamma^\infty \frac{F(x)}{x^{\delta'-\delta+1}}dx. \tag{192}$$

Using (188), the second term can be bounded as

$$\int_\gamma^\infty \frac{c-\epsilon}{x^{\delta'+1}}dx \leq \int_\gamma^\infty \frac{F(x)}{x^{\delta'-\delta+1}}dx \leq \int_\gamma^\infty \frac{c+\epsilon}{x^{\delta'+1}}. \tag{193}$$

Therefore, we have

$$G^c(\gamma) \leq \frac{c+\epsilon}{p_s}\left(\frac{1}{\gamma^{\delta'}} + \frac{\delta'-\delta}{\delta'}\frac{1}{\gamma^{\delta'}}\right) \tag{194}$$

and

$$G^c(\gamma) \geq \frac{c-\epsilon}{p_s}\left(\frac{1}{\gamma^{\delta'}} + \frac{\delta'-\delta}{\delta'}\frac{1}{\gamma^{\delta'}}\right). \tag{195}$$

Hence, the rolloff of $G(\gamma)$ is δ'. □

APPENDIX IX
PROOF OF PROPOSITION 9

It is easy to see the asymptotic throughput is given by

$$\lim_{k\to\infty} C_k(F(\cdot)) = \lim_{k\to\infty} k \int\cdots\int F^c\left(\frac{\beta}{N}\sum_{i=1}^{k-1}\gamma_i + \beta\sigma^2\right) \times dF(\gamma_1)\cdots dF(\gamma_{k-1})$$
$$= \lim_{k\to\infty}\frac{k}{P_T^{k-1}}e^{-\frac{\beta\sigma^2}{P_T}}\int e^{-\frac{\beta\gamma_1}{NP_T}}e^{-\frac{\gamma_1}{P_T}}d\gamma_1 \cdots \int e^{-\frac{\beta\gamma_{k-1}}{NP_T}}e^{-\frac{\gamma_{k-1}}{P_T}}d\gamma_{k-1}$$
$$= e^{-\frac{\beta\sigma^2}{P_T}}\lim_{k\to\infty}\frac{k}{(1+\frac{\beta}{N})^{k-1}}$$
$$= 0. \tag{196}$$

□

APPENDIX X
PROOF OF PROPOSITION 10

Let $G(\cdot) \in \Lambda_F$, then there exists a transmission control $s(\cdot)$ such that $\int s dF > 0$ and

$$G(\gamma) = \frac{\int_0^\gamma s(x)dF(x)}{\int s dF}.$$

We therefore have

$$\lim_{k\to\infty} C_k(G(\cdot))$$
$$= \lim_{k\to\infty}\frac{k}{\left(\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma\right)^k}\int s(\gamma_1)e^{-\frac{\gamma_1}{P_T}}d\gamma_1\cdots \int s(\gamma_{k-1})e^{-\frac{\gamma_{k-1}}{P_T}}d\gamma_{k-1}$$
$$\times\int s(\gamma)e^{-\frac{\gamma}{P_T}}I\left(\gamma > \left(\frac{\beta}{N}\sum_{i=1}^{k-1}\gamma_i + \beta\sigma^2\right)\right)d\gamma$$
$$\leq \lim_{k\to\infty}\frac{ke^{-\frac{\beta\sigma^2}{P_T}}}{\left(\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma\right)^k}\int s(\gamma_1)e^{-\frac{\gamma_1}{P_T}}e^{-\frac{\beta\gamma_1}{NP_T}}d\gamma_1\cdots \int s(\gamma_{k-1})e^{-\frac{\gamma_{k-1}}{P_T}}e^{-\frac{\beta\gamma_{k-1}}{NP_T}}d\gamma_{k-1}$$
$$= \frac{e^{-\frac{\beta\sigma^2}{P_T}}}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma}\lim_{k\to\infty}k\left(\frac{\int s(\gamma)e^{-\frac{\left(1+\frac{\beta}{N}\right)\gamma}{P_T}}d\gamma}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma}\right)^{k-1}$$
$$= 0. \tag{197}$$

The second inequality follows because $s(\gamma) \leq 1$ and the last equality follows because

$$\frac{\int s(\gamma)e^{-\frac{\left(1+\frac{\beta}{N}\right)\gamma}{P_T}}d\gamma}{\int s(\gamma)e^{-\frac{\gamma}{P_T}}d\gamma} < 1. \tag{198}$$

□

APPENDIX XI
PROOF OF THEOREM 3

If $F(\gamma) = 1 - e^{-\frac{\gamma}{P_T}}$, we have

$$C_{k+1}(F(\cdot)) = \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma} + \sum_{i=1}^k\frac{\beta\gamma_i}{\beta\gamma_i+\gamma} < N\right\} \times e^{-\frac{\gamma}{P_T}}d\gamma \tag{199}$$
$$\leq \frac{k+1}{P_T}\int_0^\infty \Pr\left\{\frac{\beta N\sigma^2}{\gamma} < N\right\}e^{-\frac{\gamma}{P_T}}d\gamma$$
$$= (k+1)e^{-\frac{\beta\sigma^2}{P_T}}. \tag{200}$$

From (109) and (118), we have that for every $\epsilon > 0$ there exists a k^* such that $k > k^*$ implies that

$$C_{k+1}(F(\cdot)) < \epsilon. \tag{201}$$

It is also easy to see that this k^* does not depend on P_T. This is because of the interference-limited nature of the system. For $k < k^*$, we use the upper bound on $C_k(F(\cdot))$ due to the noise limited nature of the system and for larger k we use the upper bound due to the interference-limited nature of the system. Therefore,

$$\begin{aligned} f(x,F) &= e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(F(\cdot)) \\ &\le e^{-x}\sum_{k=1}^{k^*}\frac{x^k}{k!}ke^{-\frac{\beta\sigma^2}{P_T}} + \epsilon e^{-x}\sum_{k=k^*}^{\infty}\frac{x^k}{k!} \qquad (202) \\ &\le e^{-\frac{\beta\sigma^2}{P_T}}k^* + \epsilon. \qquad (203) \end{aligned}$$

Thus,

$$\sup_x f(x,F) \le e^{-\frac{\beta\sigma^2}{P_T}}k^* + \epsilon \qquad (204)$$

which in turn implies that

$$\lim_{P_T\to 0}\sup_x f(x,F) = 0. \qquad (205)$$

We have a lower bound of N on λ_c^* because for any given P_T, it is possible to use a control that changes the CSI distribution to one with a rolloff as close to 0 as possible and achieve an AST of at least N. The upper bound of $N + \frac{N}{\beta}$ is easily obtained due to the reception model for MMSE. □

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