Optimal Embedding of Known Symbols

Srihari Adireddy
School of Electrical Engineering
Cornell University
Ithaca, NY 14853
srihari@ee.cornell.edu

Lang Tong
School of Electrical Engineering
Cornell University
Ithaca, NY 14853
ltong@ee.cornell.edu

Abstract — **The problem of placing known symbols optimally is considered. We first approach the problem from information theoretic perspective. We obtain placement schemes that maximize the mutual information between the input and the output of the channel under the constraint that we use only finite impulse response linear time invariant transmit filters. We also examine how the placement of known symbol clusters affects the performance of the block DFE. The performance criterion used is geometric SNR. Simulation results indicate that considerable gain can be obtained by placing the known symbols optimally.**

I. Introduction

In data communications, especially in packet transmission schemes, usually there are known symbols embedded in the data stream. Figure 1 shows a typical data stream. These known symbols serve various purposes such as synchronization, training of receivers and packet identification. For example, in the packet structure of GSM, there are 26 consecutive known symbols in the middle of every packet and 3 known symbols at each end. For transmissions over a high frequency (HF) channel, known symbol sequences as long as the duration of the channel are placed periodically in the data stream. Known symbols are introduced to avoid inter block interference in multi-carrier systems using modulation schemes like DMT and OFDM and in vector coding. [12, 10, 8]

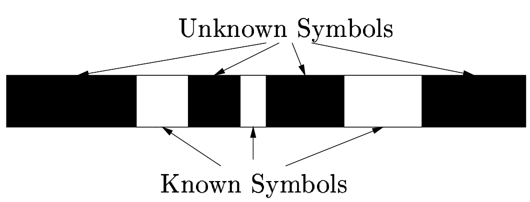

Fig. 1: Typical data stream.

The insertion of known symbols leads to loss of capacity. But inserting known symbols is mandatory to simplify the task of the receiver. Hence the insertion must be done in a way such that the capacity lost is minimized. Also for a given receiver structure, the placement of known symbols may affect the receiver performance. This raises the following questions: given the percentage of known symbols to be embedded in the data stream,

1. What is the optimal placement of known symbols?
2. Does the placement of known symbols depend on the transmission channel?
3. How does the placement affect the performance of the receiver?

Very little work has been reported regarding the optimization of the position of known symbols . We have previously considered the problem of joint optimization of symbol placement and equalizer for a symbol by symbol decision feedback receiver [11]. The performance criterion used was average MSE.It turns out that the optimal symbol placement is to separate the known symbols by d, the detection delay of the decision feedback receiver. It was shown that a lot of gain could be achieved by spreading the known symbols in the data stream instead of clustering them. The problem of maximizing the information rate in the presence of known symbols has been studied previously[3, 12, 2]. However in all these cases, the positions of known symbols in the stream was not optimized. This is implied in the assumption of selecting equal sized blocks. All of them lead to some variant of the water filling solution. [6]. The problem of equalization with clusters of known symbols has also been studied previously.[7, 5] The starting work point once again is the assumption of a fixed placement for known symbols. Again there is some gain to obtained by optimizing the position of known symbols as indicated initially in [11].

The main objective of this paper is to optimize the known symbol placement It turns out that the conventional strategy of placing known symbols together in big clusters carries a penalty in performance. For some cases, the loss of performance can be quite substantial. We first prove that the information rate is maximized by making the blocks of unknown symbols statistically independent. Then under the constraint of LTI FIR transmit filters, we give known symbol placement schemes that are optimal in the sense of maximizing the information rate. We also consider the problem of known symbol placement with a Block DFE as the receiver. We find that surprisingly the optimal known symbol placement does not depend on the actual channel coefficients. Because the placement of known symbols is a transmitter technique, the proposed strategy is particularly attractive for broadcast applications.

*This work was supported in part by the Philips Research and by the National Science Foundation under Contract CCR-9804019.

This paper is organized as follows. In Section 2, we introduce the notation, list the primary assumptions and state the problem. In Section 3, we obtain placement schemes that maximize the mutual information between the input and the output of the channel. We restrict ourselves to the case that we have a FIR LTI transmit filter. In Section 4, we consider the effect of known symbol placement on the performance of Block DFE. Using geometric SNR as the performance criterion we obtain optimal known symbol placement schemes. In Section 5, a simulation example is used to demonstrate the potential gain in performance with a Block DFE when the optimal symbol placement is used in place of the conventional clustered symbol placement

II. Problem Statement

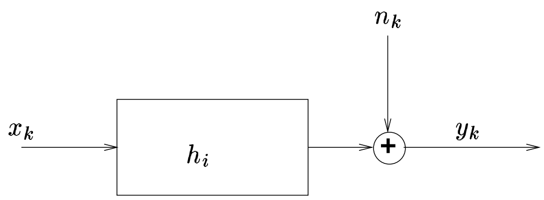

Figure 2: System Model

The system model is as shown in Figure 2. The channel h has a finite impulse response of order L. The input to the channel, as illustrated in Figure 3, consists of unknown symbols and chunks of known symbols of length at least as big as L. We assume that the input stream has totally P known and N unknown symbols. Also we assume that the transmission starts and ends with zeros (or equivalently known symbols). The noise is assumed to be additive, white, gaussian and of variance σ_n^2. The transmitter is assumed to have a power constraint of P_x. This model is valid for many single carrier systems that insert known symbols for purposes like channel estimation, synchronization etc and for avoiding inter block interference. It is in fact also valid for multi carrier systems like DMT, OFDM and vector coding where known symbols of length L are appended to each block of unknown symbols.[8, 10]

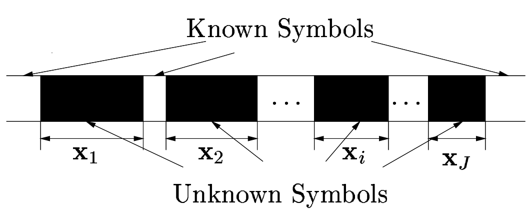

Figure 3: Typical Segmentation of the Input Sequence

We refer to the symbols between any two consecutive known symbol clusters as unknown symbol blocks. Let $\mathbf{x}_1$ denote the column vector of the first block of unknown symbols, so that $\mathbf{x}_1 = (x_0, x_1, \cdots, x_{m_1})^t$. In general, let $\mathbf{x}_i$ denote the i^{th} block of unknown symbols. The total number of unknown symbol blocks J depends on how many blocks the known symbols are divided into. We use $\mathbf{x}$ to denote the column vector having all the unknown symbols. So we have $\mathbf{x} = (\mathbf{x}_1^t, \mathbf{x}_2^t, \cdots, \mathbf{x}_J^t)^t$. We also assume that the i^{th} block of unknown symbols is of length m_i.

The constraint that the known symbol clusters are of length at least as big as L implies that there is no inter symbol interference between the symbols belonging to different unknown symbol blocks. As far as mutual information or detection is concerned we can assume that the known symbols are all zeros. This is because from the received signal we can always subtract the contribution of known symbols as it does not contain any information about the unknown symbols. If $\mathbf{y}_i$ is the output corresponding to the i^{th} block of unknown symbols, then we have

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad \text{for } i = 1, 2, \cdots, J \tag{1}$$

where

$$\mathbf{H}_i = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & & \vdots \\ \vdots & h_1 & & 0 \\ h_L & \vdots & & h_0 \\ 0 & h_L & & h_1 \\ \vdots & 0 & & \vdots \\ 0 & \cdots & \cdots & h_L \end{bmatrix}_{(m_i+L)\times m_i} \tag{2}$$

We denote the total output as $\mathbf{y}$ where $\mathbf{y} = (\mathbf{y}_1^t, \mathbf{y}_2^t, \cdots, \mathbf{y}_J^t)^t$. Since the role played by known symbol clusters is only to divide the input stream into blocks of different lengths, we can represent any placement scheme by the tuple $\mathbf{m} = (m_1, \cdots, m_i, \cdots, m_J)$. Note that the total number of elements in the tuple is a variable and depends on the placement scheme.

Our objective is to examine the following joint optimization.

$$\mathbf{m}^* = \max_{\mathbf{m}\in\mathcal{P}, f(\mathbf{x})} I(\mathbf{y}; \mathbf{x}) \tag{3}$$

where $f(\mathbf{x})$ is a probability distribution that satisfies the input power constraint. We do not however consider this optimization problem for all the possible distribution functions and $\mathbf{m}$. We restrict ourselves to a smaller class that can be generated with a LTI FIR transmit filter.

III. Optimal Symbol Placement

We first show that the mutual information $I(\mathbf{y}; \mathbf{x})$ is maximized by making the blocks of unknown symbols statistically independent of each other. Formally we state this as

Lemma 1 *For any placement scheme* $\mathbf{m}$*, we have* $I(\mathbf{y}; \mathbf{x}) \leq \sum_{i=1}^{i=J} I(\mathbf{y}_i; \mathbf{x}_i)$ *with equality occurring if and only if the unknown symbol blocks are statistically independent.*

Proof: For any $\mathbf{m}$, consider $I(\mathbf{y}_1, \mathbf{y}_2; \mathbf{x}_1, \mathbf{x}_2)$. We have

$$I(\mathbf{y}_1, \mathbf{y}_2; \mathbf{x}_1, \mathbf{x}_2) = H(\mathbf{y}_1, \mathbf{y}_2) - H(\mathbf{y}_1, \mathbf{y}_2/\mathbf{x}_1, \mathbf{x}_2) \tag{4}$$

Using the chain rule for entropy we find

$$\begin{aligned} I(\mathbf{y}_1, \mathbf{y}_2; \mathbf{x}_1, \mathbf{x}_2) &= H(\mathbf{y}_1, \mathbf{y}_2) - H(\mathbf{y}_1/\mathbf{x}_1, \mathbf{x}_2) \\ &\quad - H(\mathbf{y}_2/\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1) \end{aligned} \tag{5}$$

Due to the insertion of known symbols in clusters, we have that given the block $\mathbf{x}_i$, the block $\mathbf{y}_i$ is statistically independent of all other symbols. Using this we can simplify (5) as

$$\begin{aligned} I(\mathbf{y}_1, \mathbf{y}_2; \mathbf{x}_1, \mathbf{x}_2) &= H(\mathbf{y}_1, \mathbf{y}_2) - H(\mathbf{y}_1/\mathbf{x}_1) - H(\mathbf{y}_2/\mathbf{x}_2) \\ &\leq H(\mathbf{y}_1) + H(\mathbf{y}_2) \\ &\quad - H(\mathbf{y}_1/\mathbf{x}_1) - H(\mathbf{y}_2/\mathbf{x}_2) \end{aligned} \tag{6}$$

The above equation can be now written as

$$I(\mathbf{y}_1, \mathbf{y}_2; \mathbf{x}_1, \mathbf{x}_2) \leq I(\mathbf{y}_1; \mathbf{x}_1) + I(\mathbf{y}_2; \mathbf{x}_2) \tag{7}$$

Equality is achieved in (7) if and only if the blocks $\mathbf{x}_1$ and $\mathbf{x}_2$ are made independent. This procedure extends to arbitrary number of blocks.□

Note that we did not assume anything about either $\mathbf{m}$ or the distribution of $\mathbf{x}$, so this lemma is quite general in nature. This lemma mandates that we have to make the input block independent in order to maximize the mutual information. While deriving optimum transceivers for block transmission, both [3, 2] make this assumption implicitly.

We now address the problem of obtaining the positions of the known symbols clusters that maximize the mutual information. Lemma 1 assures us that for each tuple $\mathbf{m}$, we can consider each block independently and maximize the mutual information. It is well known [2, 3] that for the finite dimensional vector model $\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}$ the mutual information is maximized if the input is gaussian. If the covariance of $\mathbf{y}_i$ is ${\mathbf{R}_{yy}}^{(i)}$ then the mutual information is given by †

$$I(\mathbf{y}_i; \mathbf{x}_i) = \log \frac{|{\mathbf{R}_{yy}}^{(i)}|}{|\sigma_n^2 \mathbf{I}|} \tag{8}$$

If the covariance of $\mathbf{x}_i$ is ${\mathbf{R}_{xx}}^{(i)}$ then we have

$${\mathbf{R}_{yy}}^{(i)} = \sigma_n^2 \mathbf{I} + \mathbf{H}_i {\mathbf{R}_{xx}}^{(i)} \mathbf{H}_i^H \tag{9}$$

Substituting this in (8) we have

$$I(\mathbf{y}_i; \mathbf{x}_i) = \log |\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H}_i {\mathbf{R}_{xx}}^{(i)} \mathbf{H}_i^H| \tag{10}$$

Using the fact that $|\mathbf{I} + \mathbf{BC}| = |\mathbf{I} + \mathbf{CB}|$ this reduces to

$$\begin{aligned} I(\mathbf{y}_i; \mathbf{x}_i) &= \log |\mathbf{I} + \frac{1}{\sigma_n^2} {\mathbf{R}_{xx}}^{(i)} \mathbf{H}_i^H \mathbf{H}_i| \\ &\triangleq G(m_i, {\mathbf{R}_{xx}}^{(i)}) \end{aligned}$$

By making the input unknown symbol blocks independent and gaussian, we have that

$$I(\mathbf{y}; \mathbf{x}) = \sum_{i=1}^{i=J} G(m_i, {\mathbf{R}_{xx}}^{(i)}) \tag{11}$$

†$|\mathbf{A}|$ denotes the determinant of the matrix $\mathbf{A}$

We constrain ${\mathbf{R}_{xx}}^{(i)}$ for all blocks to those autocorrelations (same as covariance since we assume that the input symbols are zero mean) that can be obtained with a fixed FIR LTI transmit filter of order ν. Equivalently we constrain the input sequences to those that can be obtained by passing a white gaussian sequence s with known symbols through an FIR LTI transmit filter. In effect we limit ourselves to the system model shown in figure 4 This translates

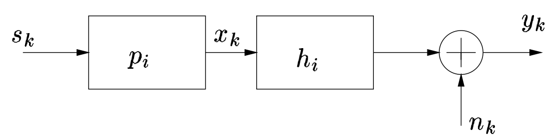

Figure 4: FIR LTI transmit filter

to constraining ${\mathbf{R}_{xx}}^{(i)} = \mathbf{P}_i \mathbf{P}_i^H$ where

$$\mathbf{P}_i = \begin{bmatrix} p_0 & 0 & \cdots & 0 \\ p_1 & p_0 & & \vdots \\ \vdots & p_1 & & 0 \\ p_\nu & \vdots & & p_0 \\ 0 & p_\nu & & p_1 \\ \vdots & 0 & & \vdots \\ 0 & \cdots & \cdots & p_\nu \end{bmatrix}_{m_i \times (m_i - \nu)} \tag{12}$$

This model is valid for many single carrier systems. For broadcast scenario too, usually a fixed FIR transmit filter is used. A by product of this model is that we cannot achieve all $\mathbf{m}$ in $\mathcal{P}$ because it is not possible to generate input sequences that have unknown symbol blocks smaller than ν, the order of the transmit filter. The constraint that in $\mathbf{x}$, we have known symbols in clusters of at least L implies that we have known symbols in blocks of length at least $\nu + L$ long in $\mathbf{s}$. We define a set $\mathcal{P}_c$ which has all the tuples that can be achieved with the model in figure 4 as

$$\mathcal{P}_c = \{\mathbf{m} : \min(\mathbf{m}) \geq \nu\} \tag{13}$$

Our optimization then becomes

$$\mathbf{m}^* = \max_{\mathbf{m} \in \mathcal{P}_c, p} \sum_{i=1}^{i=J} G(m_i, \mathbf{P}_i \mathbf{P}_i^H) \tag{14}$$

The fact that unknown symbol blocks cannot be smaller than ν gives an upper bound for J. $\lfloor \frac{N}{\nu} \rfloor$. There is another upper bound on J because of the constraint that known symbols can only be placed in clusters of length at least L which is equal to $\lfloor \frac{P}{L} \rfloor + 1$. If $J^* = \min(\lfloor \frac{N}{\nu} \rfloor, \lfloor \frac{P}{L} \rfloor + 1)$ we have $J \leq J^*$ We can now state the theorem about optimal placement schemes

Theorem 1 *For the class of FIR LTI transmit filters of order ν, a placement $\mathbf{m}$ maximizes the mutual information*

$I(\mathbf{y};\mathbf{x})$ *if and only if* $\mathbf{m} \in \mathcal{M}$ *where*

$$\mathcal{M} = \{(m_1, \cdots, m_{J^*}) : \sum_i m_i = N \ \& \ m_i \in \{r, (r+1)\}\} \tag{15}$$

where $r = \lfloor \frac{N}{J^*} \rfloor$.

Proof : To prove the above statement it is sufficient if we prove that

$$2G(m_i, \mathbf{P}_i\mathbf{P}_i^H) \geq G(m_i + 1, \mathbf{P}_i\mathbf{P}_i^H) + G(m_i - 1, \mathbf{P}_i\mathbf{P}_i^H) \forall m_i \tag{16}$$

because the set $\mathcal{M}$ is formed by making the number of symbols in each block as "equal" as possible and the total number of blocks as large as possible.

We have

$$\begin{aligned} G(m_i, \mathbf{P}_i\mathbf{P}_i^H) &= \log|\mathbf{I} + \frac{1}{\sigma_n^2}(\mathbf{H}_i\mathbf{P}_i)^H(\mathbf{H}_i\mathbf{P}_i)| \\ &\triangleq \log|D_{m_i}|. \end{aligned} \tag{17}$$

D_{m_i} is a toeplitz matrix of size $m_i \times m_i$ and the problem that we are trying to solve is

$$\mathbf{m}^* = \max_{\mathbf{m} \in \mathcal{P}, p} \sum_{i=1}^{i=J} \log|D_{m_i}|. \tag{18}$$

D_{m_i} is the m_i^{th} order principal sub-matrix of D_N. It is known that if K is a positive definite toeplitz matrix and K_n denotes the n^{th} order principal minor of K, then $\frac{|K_n|}{|K_{n-1}|}$ is decreasing in n [4]. Using this we can show that

$$2\log|D_n| \geq \log|D_{n+1}| + \log|D_{n-1}| \tag{19}$$

Using above with (16) the theorem is proved.□

Upon examining (17), we can see that a similar statement can be proved if we constrain $\mathbf{P}_i$ to be an orthogonal matrix and $\mathbf{H}_i$ is a either a matrix of form given in (2) or a circulant matrix. For a system using OFDM, $\mathbf{H}$ is a circulant matrix and $\mathbf{P}$ is an orthogonal matrix.

Conventionally known symbols have been placed in big clusters. Theorem 1 indicates that there is some gain to be achieved by spreading them. Figure 5 gives an example for an optimal distribution. The algorithm for placing the known symbols is also quite simple. The optimal placement is independent of the channel coefficients. This property makes the scheme highly attractive for the broadcast scenario where the transmitter sees a variety of channels.

IV. Block DFE

In this section we consider the effect of varying the placement of known symbols on the performance of a particular receiver structure namely the Block DFE which is a generalization of the conventional DFE. The MMSE Block DFE has emerged as an effective receiver structure for broadband packet transmission systems[9]. Figure 6 illustrates the system model the Block DFE. For a conventional DFE the matrices $\mathbf{F}$ and $\mathbf{B}$ are constrained to be full windowed toeplitz.

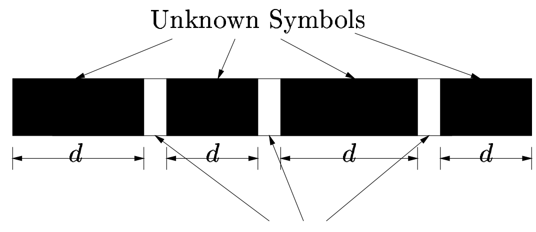

Figure 5: Example of a distribution that is Optimal

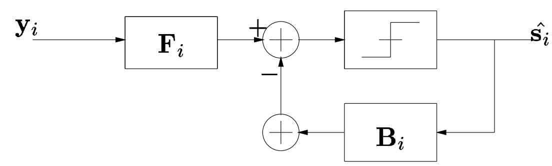

Figure 6: Block DFE

The only change to the assumptions about the system model used before is that the input symbols are now iid and of variance σ_s^2. As in [2] we use the geometric SNR as our performance criterion. Our objective hence is the following joint optimization

$$\mathbf{m}^* = \max_{\mathbf{m},\mathbf{F},\mathbf{B}} (\Pi_{i=1}^N SNR_i)^{\frac{1}{N}} \tag{20}$$

where SNR_i is defined as $\frac{\sigma_s^2}{MMSE_i}$. Because of the insertion of known symbol chunks, the detection of the unknown symbol blocks can be done independently. The forward filters and feedback filters are determined separately for each unknown block. For the i^{th} block of unknown symbols, that is of length m_i the forward filter $\mathbf{F}_i$ and the feedback filter $\mathbf{B}_i$ are determined as follows[7, 1]. We perform the cholesky factorization of the matrix

$$\mathbf{I} + \frac{\sigma_s^2}{\sigma_n^2}\mathbf{H}_i^H\mathbf{H}_i = \mathbf{L}_i diag(\lambda_1^{(i)}, \cdots, \lambda_{m_i}^{(i)})\mathbf{L}_i^H \tag{21}$$

We then have

$$\begin{aligned} \mathbf{B}_i &= \mathbf{L}_i \\ \mathbf{F}_i &= \mathbf{R}_{yy}{}^{-1}\mathbf{H}_i\mathbf{B}_i \\ MMSE_{(k,i)} &= \frac{1}{\lambda_k^{(i)}} \end{aligned} \tag{22}$$

$MMSE_{(k,i)}$ is the MMSE for the symbol detected at instant k in block i. For the i^{th} block we then have

$$\Pi_{i=1}^{m_i} SNR_i = |\mathbf{\Lambda}_i| = |\mathbf{I} + \frac{\sigma_s^2}{\sigma_n^2}\mathbf{H}_i^H\mathbf{H}_i| \tag{23}$$

We see that the performance of the MMSE-Block DFE for the block i depends only on the size of the block. If D_{m_i} is the matrix $\mathbf{I} + \frac{\sigma_s^2}{\sigma_n^2}\mathbf{H}_i^H\mathbf{H}_i$, the objective can be re-framed as

$$\mathbf{m}^* = \max_{\mathbf{m}} (\Pi_{i=1}^{i=J}|D_{m_i}|)^{\frac{1}{N}} \tag{24}$$

On comparing this equation with (14) we realize that this is the same problem that we had before, only that it arises in a different context. This is hardly surprising since the MMSE-DFE structure has shown to be canonical by Al-Dhahir, Cioffi and Forney .[9]

For this problem the constraint on J is only because of known symbol clustering because we have not assumed any transmit filter. We can redefine $\mathcal{M}$ by making $J^* = \lfloor \frac{P}{L} \rfloor + 1$. We therefore have the following theorem

Theorem 2 *For a Block-DFE receiver, a placement* $\mathbf{m}$ *is optimal as defined in (20) if and only if* $\mathbf{m} \in \mathcal{M}$.

Once again the optimal strategy is quite simple and is independent of the channel coefficients. Another advantage of this placement strategy is the reduction in computational complexity because of the smaller size of the forward and feedback filters $\mathbf{F}$ and $\mathbf{B}$. The optimal placement scheme also has advantages from the point of view of tracking.

V. Simulation

We test the joint optimization strategy by comparing the performance of the Block DFE with known symbols placed as in the GSM standard with the proposed placement scheme The packet structure of a normal burst in GSM was used for the simulation. The data structure within a normal burst consists of 148 bits out of which 116 are information bearing bits. There is a 26 bit training sequence in the middle of the packet. Also there are 3 start bit and 3 stop bits present in the packet.

The channel used for the simulation was $h = [0.407\ 0.815\ 0.407]$. For the symbol placement of GSM, the feed-forward filter $\mathbf{F}$ was chosen to be of the order 58×60 and the feedback filter $\mathbf{B}$ was of the order 58×58. For the optimal placement schemes that mandates that we have 12 blocks of unknown symbols of size 7 and 4 blocks of unknown symbols of size 8, we used $\mathbf{F}$ and $\mathbf{B}$ of orders 8×10 and 8×8 respectively. Figure 7 gives the performance of Block DFE for the GSM packet, the performance of the Block DFE if the known symbols were distributed as suggested by Theorem 2 and also the performance of Block-DFE if the packet had 50% known symbols that were distributed optimally.

VI. Conclusions

We obtained the optimal known symbol placement schemes that maximize mutual information under the assumption of LTI FIR transmit filters. We also derive the optimal known symbol placements for a particular receiver structure namely the Block DFE. Simulations indicate that there is potential gain in performance of the receiver from the optimization of placement of known symbols. We also find that the capacity lost due to the insertion of known symbols can be minimized by placing the known symbols judiciously. The optimal symbol placement strategy favors transmitting data in small but approximately equal blocks. Interestingly the optimal strategy is channel independent.

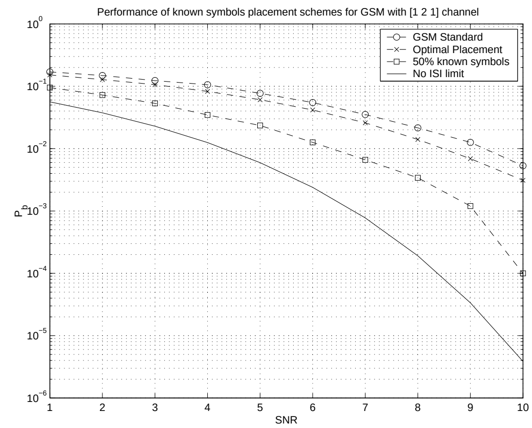

Fig. 7: Block-DFE with Proposed Known Symbol Distribution and Known Symbol Distribution given by the GSM Standard

References

[1] N. Al-Dhahir and J.M. Cioffi. "MMSE Decision-Feedback Equalizers: Finite-Length Results". *IEEE Trans. Information Theory*, pages 961–975, July 1995.

[2] N. Al-Dhahir and J.M. Cioffi. "Block Transmission over Dispersive Channels: Transmit Filter Optimization and Realization, and MMSE-DFE Receiver Performance". *IEEE Trans. Information Theory*, pages 137–160, January 1996.

[3] A.Scaglione, S.Barbarossa, and G.B.Giannakis. "Filterbank Transceivers Optimizing Information Rate in BlockTransmissions over Dispersive Channels". *IEEE Trans. Inform. Theory*, 45(3):1019–1032, April 1999.

[4] T.M. Cover and J.A.Thomas. *Elements of Information Theory.* John Wiley and Sons, Inc., New York NY, 1991.

[5] F.M.Hsu. "Data directed estimation techniques for single-tone HF modems' '. In *Proc. IEEE MILCOM'85*, pages 12.4.1–12.4.10, 1985.

[6] Robert G. Gallager. *Information Theory and Reliable Communication.* John Wiley and Sons, Inc., New York, NY, 1968.

[7] G.K.Kaleh. "Channel equalization for block transmission systems". *IEEE J. Sel. Areas Comms.*, 13(1):110–121, January 1995.

[8] L.J.Cimini. "Analysis and Simulation of a Digital Mobile Channel using Orthogonal Frequency Division Multiple Access ". *IEEE Trans. Communication*, COM-33(7):665–675, July 1985.

[9] N.AlDhahir, J.M.Cioffi, and G.D.Forney. "A Canonical Finite-Length MMSE-DFE". In *Proc. Thirty-Third Annual Allerton Conference on Communication, Control, and Computing*, pages 598–607, Monticello, IL, October 1995.

[10] A. Ruiz, J.M. Cioffi, and S. Kasturia. "Discrete multiple tone modulation with coset coding for the spectrally shaped channel". *IEEE Trans. Commun.*, 40(6):1012–1029, June 1992.

[11] S.Adireddy and L.Tong. "Detection with Embedded Known Symbols : Optimal Symbol Placement and Equalization". In *ICASSP to be Published*, Istanbul, Turkey, June 2000.

[12] S.Kasturia, J.T.Aslanis, and J.M.Cioffi. "Vector Coding for Partial Response Channels". *IEEE Trans. on Info. Theory*, 36(4):741–762, July 1990.