Repeatable Quasi Periodic Placement Schemes

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Abstract — **The problem of placing known symbols in a continuous data stream is considered from an information-theoretic perspective. Given the amount of redundancy associated with known symbols and under the assumptions that the transmit shaping filter is linear and time invariant with a finite impulse response and that the known symbols are placed in clusters of length α that is greater than or equal to the order of the composite channel, placement schemes that maximize the asymptotic mutual information between the channel input and output are obtained. It is found that these schemes, referred to as Repeatable Quasi Periodic Placements, do not depend on the channel coefficients.**

I. Introduction and System Model

Known symbols are introduced into a data stream for various purposes like channel estimation and synchronization. The placement of known symbols affects the throughput through channel estimation and detection. The focus of this paper is the problem of optimal placement of known symbols in an infinitely long data stream.

The system model is shown in Fig 1. The channel h has a finite impulse response of order L. The propagation channel h is preceded by a linear and time invariant pulse shaping filter that has a finite impulse response of length $(\nu+1)$. We assume that the transmitter has a unit power constraint. The channel output is corrupted by the additive, zero mean, white Gaussian noise n_k with variance σ_n^2. We assume that the

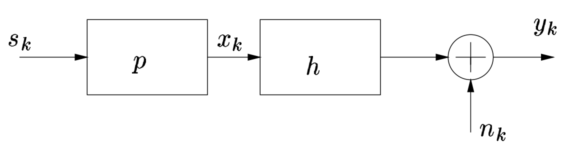

Fig. 1: System Model

fraction of known symbols in the data stream is equal to $\frac{u}{v}$. The channel coefficients $\{h_l\}_{l=0}^{l=L}$ are known to the receiver. We define as $\mathcal{P}_\alpha$, the set of placement schemes for which known symbol come in clusters of length at least $\alpha \geq (L+\nu)$ and further

$$\lim_{k\to\infty} \frac{P(k)}{k} = \frac{u}{v}, \tag{1}$$

where $P(k)$ is the number of known symbols in the first k symbols of the frame. We restrict ourselves to the class of placement schemes in $\mathcal{P}_\alpha$.

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II. Problem Statement

Given any placement scheme $\mathbf{r} \in \mathcal{P}_\alpha$ and a shaping filter $\mathbf{p}$, if $\mathbf{s}^k$ is the vector containing the first k symbols in the frame and $\mathbf{y}^k$ is the corresponding output, the asymptotic mutual information per channel use is defined as

$$I(\mathbf{r},\mathbf{p}) = \limsup_{k\to\infty} \max_{f_{i.i.d}(\mathbf{s}^k)} \frac{I(\mathbf{y}^k;\mathbf{s}^k)}{k}, \tag{2}$$

where $f_{i.i.d}(\mathbf{s}^k)$ is an i.i.d probability distribution satisfying the power constraint. Our objective is to perform the following optimization :

$$\mathbf{r}^*(\mathbf{p}) = \arg \sup_{\mathbf{r}\in\mathcal{P}_\alpha} I(\mathbf{r},\mathbf{p}). \tag{3}$$

The problem is motivated by the transmission of data in broadcast scenario. In broadcast systems, it is necessary that the a positive fraction of the data stream be known since the users may start reception at any point in time. For all the placement schemes in $\mathcal{P}_\alpha$, the number of known symbols in the data stream after any point in time is infinite. Since almost all the good channel estimation algorithms are consistent, it is thus reasonable to assume that the receiver estimates the channel perfectly.

III. Main Results

Given α and the fraction of known symbols $\frac{u}{v}$, a placement scheme $\mathbf{r}^\alpha$ is said to belong the class RQPP-α if

1. Every known symbol cluster size is exactly equal to α.
2. The placement scheme is periodic with the period of repetition M and if the number of known symbol clusters in each period is m then we have
$$\frac{m\alpha}{M} = \frac{u}{v}. \tag{4}$$
3. Every period starts with an unknown symbol cluster and ends with a known symbol cluster.
4. In each period, the $(m-1)$ known symbol clusters are placed such that the unknown symbol block sizes are as equal as possible.

The following theorem shows that for any transmit filter $\mathbf{p}$, RQPP-α schemes are optimal.

Theorem 1 *If the placement scheme $\mathbf{r}^\alpha$ is in RQPP-α, and $I^*(\mathbf{p})$ is defined as*

$$I^*(\mathbf{p}) = \sup_{\mathbf{r}\in\mathcal{P}_\alpha} I(\mathbf{r},\mathbf{p}) \tag{5}$$

then $I(\mathbf{r}^\alpha,\mathbf{p}) = I^*(\mathbf{p})$.

Proof : Refer to [1].

References

[1] S.Adireddy and L.Tong, "Optimal Placement of Known Symbols", *Submitted to IEEE Trans. Information Theory*, See also http://www.ee.cornell.edu/ ltong, July 2000.