Optimal Transmission Probabilities for Slotted ALOHA in Fading Channels

Srihari Adireddy
School of ECE
Cornell University
Ithaca, NY 14853
srihari@ece.cornell.edu

Lang Tong
School of ECE
Cornell University
Ithaca, NY 14853
ltong@ece.cornell.edu

Abstract — **We consider the uplink of a cellular system where M mobiles transmit over a common channel to a base station. We assume that the mobiles use a modified slotted ALOHA protocol to access the channel. The mobiles have access to the uplink SNR, and hence the transmission probability in each slot is chosen as a function (scheduler) of the uplink SNR (also called CSI for channel state information). For a symmetric system, we obtain the expression of maximum stable throughput for a general reception model. We then apply the theory to two particular reception models to find the stable throughput as a function of the scheduler. The scheduler is then optimized to the maximize this stable throughput. Surprisingly, it turns out that even though the reception model depends on the received SNR, the stable throughput does not depend heavily on the CSI.**

I. Introduction

The explosion in the demand for data rate over wireless channels has led to a rethinking of the traditional layered network architecture. Cross layer design is being explored as a viable alternative to the traditional design paradigm [4]. In this context, the combining of MAC and PHY layers seems natural, especially for wireless communication. The uplink of a cellular system where slotted ALOHA is used as the random access protocol, is one scenario where information from the physical layer can conceivably be used to enhance the performance of the MAC layer. As illustrated in Figure 1, differnet users experience different channel conditions and this knowledge can possibly be used to improve the throughput of the network.

We consider reception models like the capture model where the probability of success depends on the channel. The performance of Slotted ALOHA for uplink in fading channels both with and without capture has been previously explored in [6, 3, 5] and the references there in. But, in the previous works it was not assumed that the mobiles have access to CSI. Slotted ALOHA where mobiles have the knowledge of the uplink SNR was considered in [7]. In [7], this knowledge was used to vary the power of transmission and not the transmission probability. It was shown that because of power variation, the throughput increases with the number of users. The knowledge of the SNR was not used to design the transmission probability. In our work, we use the knowledge of CSI to design the transmission probability function (scheduler) and maximize the throughput.

*This work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564 and the ARL CTA on Communications and Networks.

In this paper we first obtain the expression for maximum stable throughput as a function of the scheduler, assuming a general reception model. The system is defined to be stable if for each node the buffer size does not go to infinite . In other words, every packet is transmitted with probability one. We then restrict ourselves to two particular reception models namely SNR threshold model and the capture model. In the SNR threshold model, a packet is successfully received if it does not collide with any other packet and the received SNR is greater than a given threshold. For the general capture model we assume that a packet is received successfully if the SINR (Signal to Interference Ratio) is above a given threshold. We then obtain optimal schedulers for each of the models. It turns out that obtaining the optimal scheduler for the general capture model is quite hard. We consider a particular special case of this capture model where if more than 3 stations transmit none of the packets is successfully received whereas if either one or two stations transmit, the packet with the larger SINR (Signal to Interference Ratio) is successfully received (special capture model). Intuitively, it seems that the optimal scheduler should depend on SNR, especially for the special capture model. However, it turns out that the optimal scheduler for the special capture model is independent of SNR. This result, though quite surprising, makes the scheduler implementation very simple.

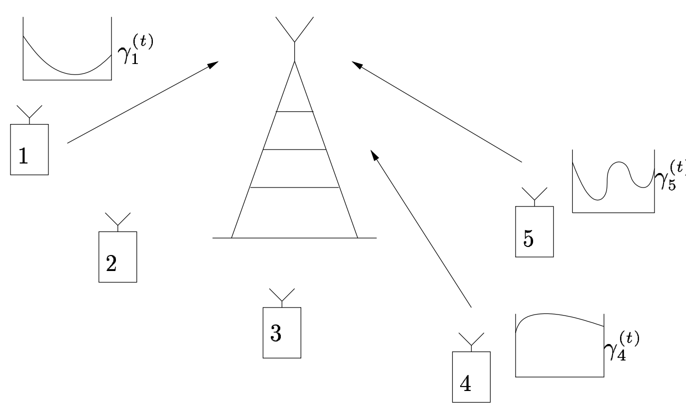

Fig. 1: Cellular Uplink

This paper is organized as follows. In Section II, we specify the system model and list the primary assumptions of the paper. In Section III, we obtain the maximum stable throughput of the system as a function of the scheduler function. In Section IV, we introduce the SNR threshold model and apply the results of the previous to obtain the expression of maximum stable throughput and optimize the scheduler by maximizing the throughput. In Section V, we do the same for capture model. We conclude in Section VI.

II. System Model

We consider a network where M users are trying to communicate with a base station over a common channel. Each user has a buffer of infinite length which is used to store the incoming packets until they are sent successfully to the base station. The incoming packets are all assumed to be of the same length. Time is slotted into intervals of length equal to the time required to transmit a packet. We make the slot time equal to one time unit and slot t is assumed to occupy the time $[t, t+1)$. We also assume that each user synchronizes his transmission with the slot boundaries. We denote by $X_m^{(t)}$ the number of incoming packets to user m during time slot t. It is assumed that $X_m^{(t)}$ for $t = 0, 1, \cdots$ are independent and identically distributed random variables. The packet arrival process for different $X_m^{(t)}$ for $m = 1, \cdots, M$ is assumed to be independent and identically distributed as well. It is assumed that the arrival process has a finite mean λ and finite variance σ^2. The above model for the arrival process is the same as that in [1] for a symmetric system.

The uplink channel between the m^{th} user and the base station during slot t is parametrized by the SNR $\gamma_m^{(t)}$. It is assumed that the quantities $\gamma_m^{(t)}$ for $m = 1, \cdots, M$ and $t = 0, 1, \cdots$ are independent and identically distributed with probability density $f(\gamma)$. Further, we assume that the user m has access to the uplink SNR (CSI) $\gamma_m^{(t)}$ at time t. This is conceivable in a network employing time division duplex access or in a network where in the base station continuously transmits a pilot signal in a control channel.

We define a very general reception model that is given by a set of M functions $g_k(\cdot)$ for $k = 1, \cdots, M$ where,

$$g_k(\cdot) : [0, \infty)^k \times \{0, 1\}^k \to [0, 1]. \tag{1}$$

Let θ_k be a binary k-tuple that describes the outcome of a slot. Let $\mathcal{A}(\theta_k)$ be the set of indexes at which θ_k is equal to one. That is, if $\theta_k = (b_1, \cdots, b_k)$ we have

$$\mathcal{A}(\theta_k) = \{1 \leq n \leq k : b_n = 1\}. \tag{2}$$

Given that $1 \leq k \leq M$ users are transmitting, and that their ordered k-tuple of SNRs given by $(\gamma_1, \cdots, \gamma_k)$, then $g_k(\gamma_1, \cdots, \gamma_k; \theta_k)$ gives the probability that only the users $\mathcal{A}(\theta_k)$ are successful. This reception model allows the reception of multiple packets at the base station. Special cases of this reception model are capture model, MPR matrix model [2] and the standard collision model.

We impose some constraints on the reception model functions that in fact hold for most practical scenarios. For each k, we assume that if we permute the SNRs $(\gamma_1, \cdots, \gamma_k)$ and apply the same permutation to the bits of θ_k, the value of $g_k(\cdots)$ does not change. Further, we assume that for any given $(\gamma_1, \cdots, \gamma_k)$, adding an extra user decreases the probability of packets success for each of the k users. That is, for all $\gamma_1 \geq 0, \cdots, \gamma_{k+1} \geq 0$, for all $1 \leq i \leq k$,

$$\sum_{\theta_k} L_i(\theta_k) g_k(\gamma_1, \cdots, \gamma_k; \theta_k) \leq \sum_{\theta_{k+1}} L_i(\theta_{k+1}) g_{k+1}(\gamma_1, \cdots, \gamma_{k+1}; \theta_{k+1}), \tag{3}$$

where $L_i(\theta_k)$ is equal to one only if the i^{th} bit in θ_{k+1} is equal to one.

In a conventional ALOHA system [1], if the user m has a packet to transmit, he transmits it with a probability p_m. We consider a more general random access scheme, where the probability of transmission for each user is allowed to be a function of his CSI. The function is called scheduler and denoted by $s(\cdot)$. Thus we assume that in slot t, user m transmits a packet with a probability $s(\gamma_m^{(t)})$.

At the end of slot t, the base station broadcasts the indexes of those users whose packets it was able to demodulate successfully. We assume that the base station does not differentiate between an empty slot and a slot in which there were some transmission but the base station failed to demodulate any packet successfully. The base station gives the same feedback for both these events.

III. Maximum Stable Throughput

In this section, we derive expressions for the maximum stable throughput of the system described in the previous section. Let the M-tuple $\mathbf{N}^{(t)} = (N_1^{(t)}, N_2^{(t)}, \cdots, N_M^{(t)})$ be the length of the buffers at each node at the beginning of slot t. We say that the system is stable for a particular arrival process, if for $\mathbf{x} \in \mathbb{N}_+^M$

$$\lim_{t \to \infty} \Pr\{\mathbf{N}^{(t)} < \mathbf{x}\} = F(\mathbf{x}) \quad \lim_{\mathbf{x} \to \infty} F(\mathbf{x}) = 1, \tag{4}$$

where $\mathbb{N}_+$ is the set of non-negative integers. This notion of stability is also used in [1]. We will later see that the stability of the system is characterized only by λ, the mean of the arrival process. This will allow us to define maximum stable throughput as the supremum of all input rates λ for which the system is stable.

The time evolution of the random variable $\mathbf{N}^{(t)}$ is given by

$$N_j^{(t+1)} = (N_j^{(t)} - Y_j^{(t)})^+ + X_j^{(t)}, \tag{5}$$

where $Y_j^{(t)}$ is equal to one if node j successfully transmits a packet during slot t and is equal to zero otherwise. Since the channel is independent from slot to slot, the M-dimensional process $\mathbf{N}^{(t)}$ is a Markov chain. It can be seen that the Markov chain is aperiodic and irreducible. Hence the stability of the system is equivalent to the ergodicity of the Markov chain.

In order to show the stability of this Markov chain we use techniques that are similar to the ones used in [1]. The principle of stochastic dominance is used to show that a fully loaded is more backed up than the original system. Thus, sufficient conditions for ergodicity are obtained by analyzing ergodicity conditions for the simpler fully loaded system. We denote by $Q_j^{(t)}$, the fully loaded version of $N_j^{(t)}$. That is, $Q_j^{(t)}$ is a Markov chain and

$$\Pr\{Q_j^{(t+1)} = k | Q_j^{(t)} = s\} = \Pr\{N_j^{(t+1)} = k | N_j^{(t)} = s, N_i^{(t)} > 0, i = 1, \cdots, M, i \neq j\}.$$

In order that we use stochastic dominance to analyze $\mathbf{N}^{(t)}$, we need to first show that [1], for all t, s, k, x,

$$\Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k\} \leq \Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s+1, Q_j^{(0)} = k\}.$$

In other words, the probability that the buffer goes above a certain level in slot $(t+1)$ is larger if the queue has more packets in slot t. It is obvious that this is indeed the case. The other property to be shown is

$$\Pr\{N_j^{(t+1)} > x | N_j^{(t)} = s, N_j^{(0)} = k\} \le \Pr\{Q_j^{(t+1)} > x | Q_j^{(t)} = s, Q_j^{(0)} = k\}.$$

In other words, the tendency of the buffer of the fully loaded system to exceed a level x is higher than that of the original system. In order to show this, we first observe that the evolution of the j^{th} buffer in the original system and the fully loaded system is given by

$$\begin{aligned} N_j^{(t+1)} &= (s - Y_j^{(t)})^+ + X_j^{(t)} \\ Q_j^{(t+1)} &= (s - Z_j^{(t)})^+ + X_j^{(t)}. \end{aligned} \quad (6)$$

Hence, in order to show (6), it is only necessary that we show that the probability of success is higher in the original system,or

$$\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\} \ge \Pr\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\}. \quad (7)$$

If $U_j^{(t)}$ is the number of nodes competing with node j to send packets in time slot t, we note that

$$\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k\} = \sum_{k=0}^{M-1} \Pr\{U_j^{(t)} = k\} \underbrace{\Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, U_j^{(t)} = k\}}_{f_k}, \quad (8)$$

whereas

$$\Pr\{Z_j^{(t)} = 1 | Q_j^{(t)} = s, Q_j^{(0)} = k\} = \Pr\{Y_j^{(t)} = 1 | N_j^{(t)} = s, N_j^{(0)} = k, L_j^{(t)} = M-1\}. \quad (9)$$

We show that the probability of success f_k is a decreasing function of k which will then imply (7) because of (8) and (9). We have the following formula for f_k :

$$f_k = \sum_{l=0}^{k} \binom{k}{l} (1-p)^{k-l} \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{l+1}) f(\gamma_1) \cdots f(\gamma_{l+1}) \sum_{\theta_{l+1}} L_1(\theta_{l+1}) g_{l+1}(\gamma_1, \cdots, \gamma_{l+1}; \theta_{l+1}) d\gamma_1 \cdots d\gamma_{l+1},$$

where

$$p = \int_0^\infty f(\gamma) s(\gamma) d\gamma. \quad (10)$$

Equivalently, f_k is the coefficient of x^k in

$$(1 + (1-p)x)^k \, h(x), \quad (11)$$

where $h(x) = h_0 + h_1 x + h_2 x^2 + \cdots$ and

$$h_l = \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{l+1}) f(\gamma_1) \cdots f(\gamma_{l+1}) \sum_{\theta_{l+1}} L_1(\theta_{l+1}) g_{l+1}(\theta_1, \cdots, \theta_{l+1}; \theta_{l+1}) d\gamma_1 \cdots d\gamma_{l+1}.$$

Therefore, $f_k - f_{k+1}$ is the coefficient of x^{k+1} in

$$(1 + (1-p)x)^k \left(pxh(x) - h(x)\right). \quad (12)$$

Hence the difference $f_k - f_{k+1}$ is a function of the coefficients of $x, \cdots, x^{k+1}$ in $(pxh(x) - h(x))$. The coefficient of $x^j, j = 1, \cdots, (k+1)$ is given by

$$p \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_j) f(\gamma_1) \cdots f(\gamma_j) \sum_{\theta_j} L_1(\theta_j) g_j(\gamma_1, \cdots, \gamma_j; \theta_j) - \int_0^\infty \cdots \int_0^\infty s(\gamma_1) \cdots s(\gamma_{j+1}) f(\gamma_1) \cdots f(\gamma_{j+1}) \sum_{\theta_{j+1}} L_1(\theta_{j+1}) g_{j+1}(\gamma_1, \cdots, \gamma_{j+1}; \theta_{j+1}).$$

Due to the condition (3) on the reception functions $g_k(\cdots)$, the coefficients of x^j for $j = 1, \cdots, (k+1)$ are greater than zero which implies that $f_k \ge f_{k+1}$. Hence the sufficient conditions for ergodicity can be derived by analyzing the fully loaded system. For, the fully loaded system, we can show that the system is stable if

$$\lambda < \sum_{k=0}^{M-1} \binom{M-1}{k} \left(1 - \int_0^\infty f(\gamma) s(\gamma) d\gamma\right)^{M-1-k} \left(\int_0^\infty \cdots \int_0^\infty f(\gamma_1) \cdots f(\gamma_{k+1}) s(\gamma_1) \cdots s(\gamma_{k+1}) \sum_{\theta_{k+1}} L_1(\theta_{k+1}) g_{k+1}(\gamma_1, \cdots, \gamma_{k+1}; \theta_{k+1})\right) d\gamma_1 \cdots d\gamma_{k+1}. \quad (13)$$

The arguments in [1] can be extended to obtain necessary conditions for stability. Thus,we have the following theorem about maximum stable throughput.

Theorem 1 *Given the density function of uplink SNR $f(\gamma)$, the scheduler $s(\gamma)$ and the reception functions $\{g_k(\cdots)\}_{k=1}^M$, the maximum stable throughput is given by*

$$\lambda^*(s(\cdot)) = \sum_{k=0}^{M-1} \binom{M-1}{k} \left(1 - \int_0^\infty f(\gamma) s(\gamma) d\gamma\right)^{M-1-k} \left(\int_0^\infty \cdots \int_0^\infty f(\gamma_1) \cdots f(\gamma_{k+1}) s(\gamma_1) \cdots s(\gamma_{k+1}) \sum_{\theta_{k+1}} L_1(\theta_{k+1}) g_{k+1}(\gamma_1, \cdots, \gamma_{k+1}; \theta_{k+1})\right) d\gamma_1 \cdots d\gamma_{k+1}. \quad (14)$$

IV. SNR Threshold Model

In this section, we apply the results derived in the previous section for the SNR threshold model and obtain the maximum stable throughput. We then optimize the scheduler by maximizing this stable throughput. The SNR threshold model is defined as follows. We assume that a user is successfully demodulated if no other user transmits and if his SNR is larger than a given threshold γ_0. The reception model for this is given by

$$g_1(\gamma; 1) = \begin{cases} 1 & \gamma \ge \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}. \quad (15)$$

The function $g_1(\gamma; 0)$ is of course equal to $1 - g_1(\gamma; 1)$. For $k \ge 2, g_k(\cdots)$ is identically equal to zero.

Given a scheduler $s(\gamma)$, the maximum stable throughput is given by

$$\lambda^*(s(\cdot)) = \left(1 - \int_0^\infty x(\gamma) d\gamma\right)^{M-1} \left(\int_{\gamma_0}^\infty x(\gamma) d\gamma\right), \quad (16)$$

where $x(\gamma) = f(\gamma) s(\gamma)$.

A Optimal Scheduler

The optimal scheduler is then obtained as

$$s^*(\cdot) = \arg\max_{s(\cdot)} \lambda^*(s(\cdot)). \quad (17)$$

We then have the following theorem.

Theorem 2 *Denote $p_{\gamma_0} = P\{\gamma \geq \gamma_0\}$. We then have the following theorem. If $p_{\gamma_0} \geq \frac{1}{M}$, the optimal scheduler is*

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ \frac{1}{Mp_{\gamma_0}} & \gamma \geq \gamma_0 \end{cases}, \quad (18)$$

and the corresponding throughput is

$$\lambda^*(s^*(\cdot)) = (1 - \frac{1}{M})^{M-1}\frac{1}{M}. \quad (19)$$

If $p_{\gamma_0} < \frac{1}{M}$, the optimal scheduler is

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ 1 & \gamma \geq \gamma_0 \end{cases}, \quad (20)$$

and the corresponding throughput is

$$\lambda^*(s^*(\cdot)) = (1 - p_{\gamma_0})^{M-1}p_{\gamma_0}. \quad (21)$$

Proof : Let $\Lambda(u)$ be defined as

$$\Lambda(u) = \left\{s(\gamma) : 0 \leq s(\gamma) \leq 1, \int_0^\infty x(\gamma)d\gamma = u\right\}. \quad (22)$$

For $s(\gamma) \in \Lambda(u)$, we have

$$\lambda^*(s(\cdot)) = (1-u)^{M-1}\left(\int_{\gamma_0}^\infty f(\gamma)s(\gamma)d\gamma\right). \quad (23)$$

We note that $\int_{\gamma_0}^\infty f(\gamma)s(\gamma)d\gamma \leq p_{\gamma_0}$. This leads to the follows upper bound on the maximum stable throughput.

$$\lambda^*(s(\cdot)) \leq \begin{matrix} (1-u)^{M-1}u. & u \leq p_{\gamma_0} \\ (1-u)^{M-1}p_{\gamma_0} & u > p_{\gamma_0} \end{matrix} \quad (24)$$

If $p_{\gamma_0} \geq \frac{1}{M}$, maximizing the upper bound by varying u between 0 and 1, we find that for all $s(\cdot)$,

$$\lambda^*(s(\cdot)) \leq (1 - \frac{1}{M})^{M-1}\frac{1}{M}. \quad (25)$$

Hence choosing the scheduler as

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ \frac{1}{Mp_{\gamma_0}} & \gamma \geq \gamma_0 \end{cases} \quad (26)$$

achieves the maximum and is hence optimal. If $p_{\gamma_0} < \frac{1}{M}$, then the above choice is not valid since $\frac{1}{Mp_{\gamma_0}} > 1$. If $p_{\gamma_0} < \frac{1}{M}$, we find that the upper bound is maximized at $u = p_{\gamma_0}$ and

$$\lambda^*(s(\cdot)) \leq (1 - p_{\gamma_0})^{M-1}p_{\gamma_0}. \quad (27)$$

Hence for $p_{\gamma_0} < \frac{1}{M}$, choosing the optimal choice for the scheduler is given by

$$s^*(\gamma) = \begin{cases} 0 & \gamma < \gamma_0 \\ 1 & \gamma \geq \gamma_0 \end{cases}. \quad (28)$$

□

We find that, as expected, if $\gamma < \gamma_0$, the mobiles do not transmit. The scheduler is a step function and hence does not depend "strongly" on SNR.

V. Capture Model

We first consider a general capture model where a user is demodulated successfully only if his SINR is larger than a given threshold γ_0. The user is received successfully only if his SINR ratio is larger than a given threshold γ_0. If n users transmit with SNRs given by $(\gamma_1, \gamma_2, \cdots, \gamma_n)$, user one is demodulated successfully only if

$$\frac{\gamma_1}{\gamma_2 + \cdots + \gamma_n} > \gamma_0. \quad (29)$$

We assume that the only interference is due to the transmitting users. Given the scheduler $s(\cdot)$, the maximum stable throughput for this model is given by

$$\begin{aligned} \lambda^*(s(\cdot)) &= \sum_{l=0}^{M-1}\binom{M-1}{l}(1-p)^{M-1-l}\int_0^\infty s(\gamma_1)f(\gamma_1) \\ &\quad \int_0^\infty \cdots \int_0^\infty I(\gamma_2 + \cdots + \gamma_{l+1} < \frac{\gamma_1}{\gamma_0}) \quad (30) \\ &\quad s(\gamma_2)\cdots s(\gamma_{l+1})f(\gamma_2)\cdots f(\gamma_{l+1})d\gamma_2\cdots d\gamma_{l+1}, \end{aligned}$$

where as before $p = \int_0^\infty s(\gamma)f(\gamma)d\gamma$ and $I(\cdot)$ is the indicator function. We denote $x(\gamma) = \frac{s(\gamma)f(\gamma)}{p}$ and note that we can interpret $x(\gamma)$ as a density function. We can now simplify the above expression as

$$\lambda^*(s(\cdot)) = \sum_{l=0}^{M-1}\binom{M-1}{l}(1-p)^{M-1-l}p^{l+1}\int_0^\infty G^l(\frac{\gamma}{\gamma_0})dX(\gamma), \quad (31)$$

where $X(\gamma)$ is probability distribution of $x(\gamma)$ and $G^l(\gamma)$ is the l fold convolution of $X(\gamma)$. It turns out that finding the optimal scheduler for this general capture model is quite hard. Instead we consider a special case of this capture model where if more than three users transmit the packets of all the users are destroyed but if one or two users transmit then the strongest user is successfully demodulated. For this model, the maximum stable throughput is obtained by replacing γ_0 by one in (31) and then considering the summation in (31) for only $l = 0, 1$. Hence the maximum stable throughput is given by

$$\begin{aligned} \lambda^*(s(\cdot)) &= (1-p)^{M-1}p + (1-p)^{M-2}p^2\int_0^\infty X(\gamma)dX(\gamma) \\ &= (1-p)^{M-1}p + (1-p)^{M-2}\frac{p^2}{2}, \quad (32) \end{aligned}$$

where $p = \int_0^\infty s(\gamma)f(\gamma)d\gamma$. As earlier, the optimum scheduler is obtained as

$$s^*(\cdot) = \arg\max_{s(\cdot)} \lambda^*(s(\cdot)). \quad (33)$$

A Optimal Scheduler

For the capture model, the following theorem gives the optimal scheduler.

Theorem 3 *The optimal scheduler for the capture model is*

$$s^*(\gamma) = \frac{-1 + \sqrt{1 + \frac{M(M-3)}{2}}}{\frac{M(M-3)}{2}} \quad \forall\gamma, M \geq 3 \quad (34)$$

For $M = 3$, $s(\gamma) = \frac{1}{2}$ and for $M = 2$, $s(\gamma) = 1$

Proof : Optimizing the expression in (32) with respect to p, we find that the throughput is maximized, for $M > 3$, when

$$p^* = \frac{-1 + \sqrt{1 + \frac{M(M-3)}{2}}}{\frac{M(M-3)}{2}} \quad \forall \gamma. \tag{35}$$

Since $p^* \leq 1$, the choice of

$$s(\gamma) = p^* \quad \forall \gamma > 0, \tag{36}$$

is valid and maximizes the throughput. □.

Since the reception model favors users with a higher SNR, it is natural to expect the optimal $s(\cdot)$ to be higher at larger γ. It is therefore quite surprising that the optimal $s(\cdot)$ is independent of γ. The optimal $s(\cdot)$ is also independent of the distribution of the fading process.

VI. Conclusions

We consider a network where M users are trying to communicate with the base station over a common channel using a slotted ALOHA random access protocol. Under the assumption that each user has access to his CSI, we let the transmission probability be a function (called scheduler) of this CSI. For this scenario, we obtain an expression for the maximum stable throughput as a function of the scheduler. We evaluate this expression for two particular reception models namely the SNR threshold model and the general capture. For SNR threshold model, we optimize the scheduler by maximizing the throughput. We also optimize the scheduler for a special case of the general capture model. We find that in this case, the optimal scheduler turns out to be independent of the channel state information. But, we do not expect this to be true in general. An interesting model that could bring this out is one where capture occurs only if the signal to interference ratio (SINR) is larger than some given threshold γ_0. It would also be interesting to see how the scheduler is affected when the transmitted rate is allowed to be a function of the CSI. The setup developed in this paper can also be used to analyze both the above problems.

References

[1] B.S.Tsybakov and V.A.Mikhailov. " Ergodicity of a Slotted ALOHA System". *Problemy Peredachi Informatsii*, 15(4):73–87, October 1979.

[2] S. Ghez, S. Verdú, and S. Schwartz. "Stability Properties of Slotted Aloha with Multipacket Reception Capability". *IEEE Trans. Automatic Control*, 33(7):640–649, July 1988.

[3] R.O. LaMaire, A. Krishna, and M. Zorzi. "On the Use of Transmitter Power Variations to Increase Throughput in Multiple Access Radio Systems ". *Wireless Networks*, 4(3):263–277, June 1998.

[4] L.Tong, Q.Zhao, and G.Mergen. "Multipacket reception in random access wireless networks: From signal processing to optimal medium access control ". *IEEE Communications Magazine*, 39(11):108–112, November 2001.

[5] M.Zorzi. "Mobile Radio Slotted ALOHA with Capture, Diversity and Retransmission Control in the Presence of Shadowing ". *Wireless Networks*, 4:379–388, August 1998.

[6] M.Zorzi and R.Rao. " Capture and Retransmission Control in Mobile Radio". *IEEE JSAC*, 12(8):1289–1298, October 1994.

[7] X.Qin and R.Berry. "Exploiting Multiuser Diversity in Wireless ALOHA Networks". In *Proc. Allerton Conf. on Communication, Control and Computing*, Allerton, IL, October 2001.