Optimal Placement of Known Symbols for Nonergodic Broadcast Channels

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Abstract — **The problem of placing known symbols in a data stream for a nonergodic broadcast inter symbol interference (ISI) channel is considered from an information-theoretic perspective. Given the amount of redundancy associated with known symbols, placement schemes that minimize the outage probability are derived under various constraints. We consider codewords that consist of packets that have the same placement. Under the assumption that each known symbol cluster is at least as large as $\alpha \geq L$(the channel order), we show that the optimal placement is obtained by arranging the known symbols into as many clusters as possible and placing them such that the unknown symbol blocks are as equal as possible. It is shown that the optimal placement of known symbol clusters in does not depend on the probability density of the channel when the known symbol clusters are constrained to be at least as large as the length of the channel response. Numerical examples are used to illustrate the ideas and potential gains of using optimal known symbol placement.**

I. Introduction

A broadcast system† as shown in Fig.1 typically transmits data to users with disparate channel characteristics. If the channel to a user cannot accommodate the attempted rate of transmission, the user experiences outage. The demand for larger coverage motivates us to investigate into techniques that minimize the probability of outage for a given rate.

In a broadcast system, known symbols are usually embedded into the data stream. These known symbols serve various purposes such as synchronization, training of receivers. Since a receiver may tune in at a random time, it is necessary that some known symbols be transmitted through out the broadcast. For example, in the ATSC standard for High Definition TeleVision (HDTV), a known symbol cluster of length 832 is inserted periodically in the transmitted data [1]. In order to be fair to all the users it is necessary that known symbols occupy a fraction of the data stream. Thus insertion of known symbols implies that there is a penalty in the transmission rate. An important question that should be addressed by a system designer is how should one place these known symbols in the data stream. In this paper, our objective is hence to obtain optimal placement schemes that minimize the probability of outage.

†We consider a broadcast system in which the the transmitter wants to send the same information to all the receivers. This model can be analyzed using single user compound channel ideas.

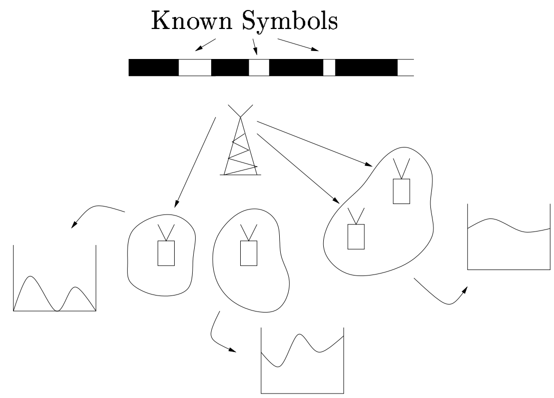

Fig. 1: Typical Broadcast Scenario.

We assume a nonergodic fading model where the channel between the transmitter and a user is governed by a probability density. The channel stays constant for all the uses of the channel. This model does not admit a Shannon capacity in general [5]. We therefore choose to use outage probability as the performance metric.

We assume that the codewords are constrained to consist of packets with the same placement. This model is defined as the packetized codeword (PCW) model. The PCW model turns out to be quite simple to analyze, and we obtain optimal placement schemes for known symbols within the packet. We also assume that the placement scheme is kept fixed within the codebook.

We assume that the receivers make an estimate of the channel based on all the available information until current time and pass this estimate to the decoder. The decoder then uses the channel estimate as if it were perfect. We show that when the channel estimator is consistent and the decoder used is a maximum likelihood decoder, the probability of error is almost surely the same as that of a receiver which knows the channel perfectly. Hence we can obtain outage probability for the model under consideration by assuming that the receiver knows the channel perfectly.

For the PCW case, we show that under the constraint that known symbols are placed in clusters of length at least $\alpha \geq L$ where L is the order of the channel, the outage probability is minimized by the family of placement schemes referred to as QPP-α. In this family, the known symbols are broken into as many clusters as possible under the constraint that each of them is at least α and they are placed such that the lengths of unknown symbol clusters are as equal as possible. It turns out that the conventional strategy of placing known symbols together in big clusters carries a penalty in performance. Surprisingly, the optimal known symbol placement does not depend on the depend on the probability density of the channel.

There has been some work reported on the effect of training and channel estimation errors on mutual information that does not consider the placement issue[8, 2, 6]. Medard [8] has obtained lower and upper bounds on mutual information that are a function of the variance of the error in the channel estimate formed at the receiver. Hassibi and Hochwald [2] have optimized training in multiple-antenna systems with quasi-static flat fading by maximizing a tight lower bound on the ergodic capacity. The training issues for quasi-static frequency selective fading were addressed in [6].

We have previously considered the problem of joint optimization of known symbol placement and equalizer for a symbol-by-symbol decision feedback receiver [11]. The performance criterion used was Average Mean Square Error (AMSE). It was shown that substantial gain could be achieved by spreading the known symbols in the data stream instead of clustering them. We have addressed the problem of optimal placement of known symbols for ergodic block frequency selective fading with i.i.d Gaussian taps in [13]. The framework derived in [2] was used to obtain a tight lower bound on the channel capacity and this lower bound was maximized to obtain optimal placement schemes for both OFDM and single carrier systems. We show that in OFDM placing known symbols periodically in frequency is optimal where as periodic placement in time (QPP-α placement schemes) turns out to be optimal for single carrier systems.

Training issues for tracking in Gauss-Markov channels was considered in [9]. The optimal placement problem has been explored for other metrics and models too. Optimal placement of known symbols for minimizing the variance of the error in channel estimate for OFDM systems has been addressed in [10]. The optimal placement for the more general setting of block precoded transmission systems with cyclic prefix was addressed with the channel estimate as the metric in [15] and at high SNR, with block length going to infinity, and with ergodic capacity as the metric in [14]. All these papers showed that periodic placement in frequency is optimal. Placement issues for multiple-antenna systems employing orthogonal space-time codes with the CRLB for channel estimation as the metric has been considered in [3]. The placement that minimizes the Cramer-Rao Lower Bound (CRLB) for semi-blind channel estimators was found in [4]. It was shown here that QPP-α placement is optimal under some constraints. It is quite surprising that the QPP-α placement schemes turn out to be optimal for a variety of metrics.

This paper is organized as follows. In Section II, we introduce the nonergodic channel model, the PCW model for codebook and define outage probability for the PCW model. We also show that if the channel estimator is consistent and the decoder used is an ML decoder, we can assume that receiver knows the channel perfectly for evaluating the outage probability. In Section III we formulate the optimization problem for the PCW model. In Section IV, we obtain optimal placement schemes for packetized codebooks. Section V illustrates the ideas proposed in the paper through different simulations. We finally conclude in Section VI.

II. Non-Ergodic Broadcast System

In this section we first define the channel model for a nonergodic broadcast system. We then describe in detail the PCW model. We introduce the metric that is used for optimizing placement. We also give the structure of the receiver employed by each user and its implication on the performance metric.

A Channel Model

We assume that the channel $\mathbf{h} = [h_0, h_1, \cdots, h_L]^t$ to a user is random, time invariant and is governed by the density function $p_{\mathbf{h}}(\cdot)$. We assume that neither the receiver nor the transmitter knows the propagation coefficients. A user can start reception at any point in time. The channel output to each user is corrupted by the additive, zero mean, white Gaussian noise w_k with variance $1/\rho$.

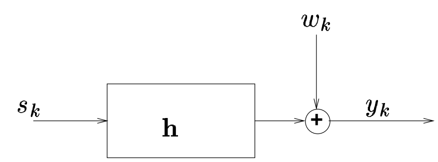

Fig. 2: System Model

B Outage Probability as Performance Metric

Since the information to be transmitted to all the users is the same, the broadcast channel reduces to a single user compound channel. But it is not possible to assign a Shannon capacity to this channel since, given a transmission rate, there is always a non-zero probability that the channel realization cannot support it. We hence use outage probability as the performance metric. The interpretation of outage probability is the one given in [5] and the references therein. Given a transmitted SNR ρ and rate R, $\Theta(R, \rho)$ is the largest possible set for which C_Θ, the capacity of the compound channel with the parameter $\mathbf{h} \in \Theta(R, \rho)$, satisfies $C_\Theta \geq R$. The outage probability is then defined as $\mathrm{P}_{out}(R, \rho) = \Pr(\mathbf{h} \notin \Theta(R, \rho))$. Hence there exists a sequence of codes $\mathcal{C}_n$ of rate R that satisfies the power requirement ρ, and for which the supremum of the probability of error over all channels $\mathbf{h} \in \Theta(R, \rho)$ tends to zero. In this paper, we consider outage probability for only i.i.d inputs. This makes the problem tractable and it also implies that the capacity achieving codes will have a flat transmit spectrum.

C Receiver Structure and Performance

We assume that the transmitter uses a codebook $\mathcal{C} = \{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}, \cdots, \mathbf{S}^{(M)}\}$ to transmit the data, and every codeword contains known symbols that enable the receiver to estimate the channel. Fig. (3) illustrates the re-

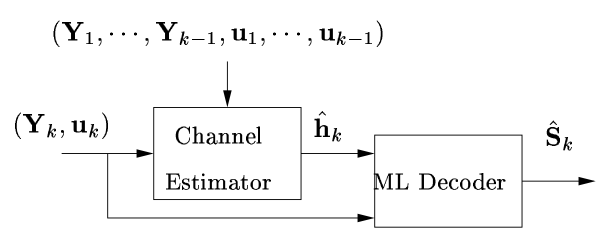

Fig. 3: Receiver Structure

ceiver structure, the channel estimator is given by $\hat{\mathbf{h}}_k = g_k(\mathbf{Y}_1, \cdots, \mathbf{Y}_k, \mathbf{u}_1, \cdots, \mathbf{u}_k)$ where $\mathbf{Y}_i$ is the vector containing the received output due to the codeword sent at the i^{th} instant and $\mathbf{u}_i$ is the vector containing the known symbols in the codeword sent at the i^{th} instant. Hence $\hat{\mathbf{h}}_k$, the channel estimate for decoding the codeword sent at k^{th} instant, is formed using the output from the first k codewords. We assume that the channel estimator is consistent in the sense that $\hat{\mathbf{h}}_k \triangleq g_k(\mathbf{Y}_1, \cdots, \mathbf{Y}_k) \to \mathbf{h}$ a.s. The codeword is decoded as

$$\hat{\mathbf{S}}_k = \arg \min_{\mathbf{S} \in \mathcal{C}} f(\mathbf{Y}_k, \mathbf{S}; \hat{\mathbf{h}}_k), \quad (1)$$

where $f(\cdot, \cdot, \cdot)$ is the maximum likelihood (ML) metric under the assumption that the channel estimate is perfect. We denote as $\hat{\epsilon}_{\mathbf{h}}$ the probability of error conditioned on the fact that the channel to the user is $\mathbf{h}$. Formally,

$$\hat{\epsilon}_{\mathbf{h}} = \lim_{k \to \infty} \frac{\sum_{i=1}^{k} P(\hat{\mathbf{S}}_i \neq \mathbf{S}_i | \text{ channel to the user is } \mathbf{h})}{k}. \quad (2)$$

In Lemma 1, we show that this quantity is well defined.

If the user knows the channel perfectly, the codewords are decoded as

$$\hat{\mathbf{S}}_k = \arg \min_{\mathbf{S} \in \mathcal{C}} f(\mathbf{Y}_k, \mathbf{S}; \mathbf{h}), \quad (3)$$

where $\mathbf{Y}$ is the received data corresponding to the codeword. We assume that the probability of error for this decoder conditioned on the fact that the channel to the user is $\mathbf{h}$ is equal to $\epsilon_{\mathbf{h}}$, that is defined in a way similar to (2). Then we have the following lemma.

Lemma 1 *Under the assumption that* $\hat{\mathbf{h}}_k$ *converges almost surely to* $\mathbf{h}$, *we have* $\hat{\epsilon}_{\mathbf{h}} = \epsilon_{\mathbf{h}}$ *almost surely.*

Proof : Refer to [12].

The above lemma tells us that if our performance metric is related to showing the existence of a code with a prescribed probability of error, we can assume that the channel is known at the receiver in order to analyze the problem.

D Packetized Codeword Structure

We assume that every codeword in the codebook has the same placement. For PCW, we assume that the codeword consists of packets that belong to the class $\mathcal{P}_\alpha$. A packet is in the class $\mathcal{P}_\alpha$ if

- A1: The length of each packet is $(N+P+L)$ where N is the number of unknown symbols and $(P+L)$ is the number of known symbols.
- A2: The known symbols come in clusters of length equal to at least $\alpha \geq L$.
- A3: Each packet starts with at least L known symbols.

The assumption A2 is introduced primarily from the point of view of channel estimation. A2 makes it possible to employ channel estimators based on training only. A3 implies that there is no inter-packet interference.

As shown in Fig. 4, every placement in the packet can be specified by $\mathbf{r} = (\mathbf{m}, \mathbf{n})$, that is, two tuples $\mathbf{m} = (m_1, \cdots, m_J)$ and $\mathbf{n} = (n_1, \cdots, n_{J+1})$ where $\mathbf{m}$ gives the lengths of unknown symbol blocks and $\mathbf{n}$ the lengths of known symbol clusters. For placements that end with unknown symbols we have $n_{J+1} = 0$. Further A3 implies that $n_1 \geq L$. We also have $\sum_{i=1}^{J} m_i = N$ and $\sum_{i=1}^{J+1} n_i = (P+L)$. It should be noted that the number

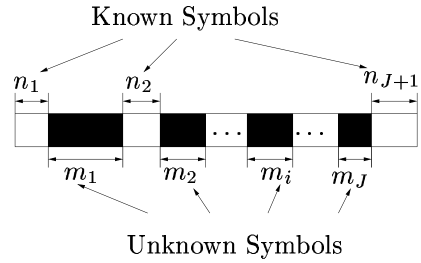

Fig. 4: Representation of Placement Schemes

of elements in each of these tuples is a variable and depends on the placement scheme. We refer to the symbols between any two consecutive known symbol clusters as unknown symbol blocks.

A code of rate R whose codewords consist of k packets each is denoted as (k, R). A rate R is said to be achievable with a placement $\mathbf{r}$ if there exists a sequence of codes $\{(k, R)\}, k = 1, 2 \cdots$ such that the placement of known symbols in each packet is $\mathbf{r}$ and the probability of error tends to zero.

III. Problem Formulation

The use of packetized codewords turns the model into the vector discrete memoryless channel shown in Fig.5 Since the

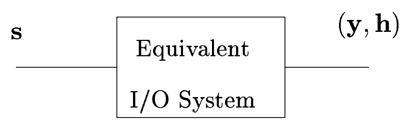

Fig. 5: Equivalent System

first L symbols of a packet are known, the contribution due to these symbols can be subtracted without any loss of generality. The column vector $\mathbf{s} = [s_1, \cdots, s_{N+P}]^t$ contains the symbols in the packet excluding the first $(L+1)$ and $\mathbf{y}$ is the output obtained after subtracting the contribution due to the starting $(L+1)$ known symbols. The relation between $\mathbf{s}$ and $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{H}_{N+P}\mathbf{s} + \mathbf{w}, \quad (4)$$

where

$$\mathbf{H}_{N+P} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & & \vdots \\ \vdots & h_1 & & 0 \\ h_L & \vdots & & h_0 \\ 0 & h_L & & h_1 \\ \vdots & 0 & & \vdots \\ 0 & \cdots & \cdots & h_L \end{bmatrix}_{(N+P+L)\times(N+P)}. \quad (5)$$

The probability of outage for this model is a function of the placement scheme $\mathbf{r}$ in the packet. The expression for $\mathrm{P}_{out}(R, \rho, \mathbf{r})$ is given by [7]

$$\mathrm{P}_{out}(R, \rho, \mathbf{r}) = P(\Psi(\mathbf{r}, \rho, \mathbf{h}) < R), \quad (6)$$

where

$$\Psi(\mathbf{r}, \rho, \mathbf{h}) = \log\,\det(\mathbf{I} + \rho \mathbf{H}_{N+P}\mathbf{J}_{\mathbf{r}}\mathbf{H}_{N+P}^{H}). \quad (7)$$

The matrix $\mathbf{J_r}$ is a square diagonal selection matrix of order $(N+P)$. It has ones in those positions that have unknown symbols and zeros in those that have known symbols. If $\mathbf{r} = (\mathbf{m}, \mathbf{n})$, then

$$\Psi(\mathbf{r}, \rho, \mathbf{h}) = \sum_{i=1}^{J} \log \det(\mathbf{I} + \rho \mathbf{H}_{m_i} \mathbf{H}_{m_i}^H) \quad (8)$$

$$\triangleq \sum_{i=1}^{J} G(m_i, \rho, \mathbf{h}) \quad (9)$$

This is possible due to A2 (each non-zero element of $\mathbf{n}$ is at least as large as L, which implies that there is no inter-symbol interference between two unknown symbol blocks). We see that the outage probability depends only on $\mathbf{m}$. The placements corresponding to all the permutations of the elements of the tuple $\mathbf{m}$ have the same outage probability. We can thus conclude that the order in which the unknown symbol blocks or the known symbol clusters are transmitted is immaterial. In fact the exact values of $\mathbf{n}$ are also irrelevant as long as A1 is satisfied and the channel estimator is consistent.

The objective is to minimize the outage probability with respect to the size of unknown symbol blocks $\mathbf{m}$. Formally, our objective is to examine the following optimization :

$$\mathbf{m}^* = \arg \min_{\mathbf{m} \in \mathcal{P}^\alpha} \mathrm{P}_{out}(R, \rho, \mathbf{r}). \quad (10)$$

IV. Quasi Periodic Placement and its Optimality

In this section, we define a family of placement schemes for the PCW model called Quasi Periodic Placement (QPP) and prove that they are optimal.

The family of QPP placement schemes is divided into different classes on the basis of the smallest allowable length of any known symbol cluster. The class of schemes for which α is the smallest allowable known symbol cluster length is denoted as QPP-α . Formally, we define $\mathcal{Q}_\alpha$ as the set of all placement schemes belonging to the class QPP-α .

Definition 1 *Given an α and a packet with N unknown symbols and $(P+L)$ known symbols, let $J = \lfloor \frac{P+L}{\alpha} \rfloor$. A placement scheme belongs to $\mathcal{Q}_\alpha$ if and only if*

1. $\mathbf{n} \in \mathcal{N}$ *where* $\mathcal{N} = \{(n_1, \cdots, n_J, 0) : \sum_i n_i = (P+L), \min(\{n_1, \cdots, n_J\}) \geq \alpha\}$
2. $\mathbf{m} \in \mathcal{M}$ *where* $\mathcal{M} = \{(m_1, \cdots, m_J) : \sum_i m_i = N, m_i \in \{\lfloor \frac{N}{J} \rfloor, (\lfloor \frac{N}{J} \rfloor + 1)\}\}$

In other words, in a QPP-α placement scheme, the known symbols are divided into as many clusters as possible under the constraint that each of them is no less than α, and these clusters are placed such that the unknown symbol blocks are as "equal" as possible. An element in $\mathcal{Q}_\alpha$ is denoted as $\mathbf{r}^\alpha$.

We state some fairly straight forward properties of QPP-α schemes without proof.

- P1: All QPP schemes have at most two different unknown symbol block sizes. If there are two distinct unknown symbol block sizes, they differ by one.
- P2: Under assumptions A1 and A2, given $\alpha \geq L$, all the placement schemes in $\mathcal{Q}_\alpha$ have the same outage probability. For all other α, such a claim is not true in general.

The following theorem states that all the placement schemes in QPP-α minimize the outage probability.

Theorem 1 *If $\mathbf{r}^\alpha \in \mathcal{Q}_\alpha$, then under A1-A3, for any given $p_{\mathbf{h}}(\cdot)$, R and ρ*

$$P_{out}(R, \rho, \mathbf{r}^\alpha) = \min_{\mathbf{r} \in \mathcal{P}^\alpha} P_{out}(R, \rho, \mathbf{r}). \quad (11)$$

Furthermore, $P_{out}(R\rho, \mathbf{r}^\alpha)$ is a monotonically increasing function of α. Hence

$$P_{out}(R, \rho, \mathbf{r}^L) = \min_{\alpha \geq L, \mathbf{r} \in \mathcal{P}^\alpha} P_{out}(R, \rho, \mathbf{r}). \quad (12)$$

Proof : Refer to [12].

Theorem 1 shows that if we allow all possible $\alpha \geq L$, the placement schemes belonging to QPP-L are optimal. Conventionally known symbols have been placed in big clusters. Theorem 1 indicates that there is some gain to be achieved by spreading them. The algorithm for placing the known symbols is also quite simple. The optimal placement is independent of the probability density of the channel coefficients. This property makes the scheme highly attractive for the broadcast scenario. The intuition in placing the known symbols in small clusters is that known symbol clusters reduce the inter-symbol interference (ISI) and one should thus maximize the number of known symbol clusters in the data stream.

V. Simulations

We assume that the receiver might belong to one of three different geographical locations, each of which has a different multipath structure. Each geographical region is assumed to have line of sight but distinctly different kind of ISI channel. The specular component in region A is assumed to be flat. The specular component in region B is assumed to have nulls on the unit circle where as the specular component in region C is assumed to have a deep null in the spectrum. The goal of the transmitter is to minimize the outage probability for a given rate.

We assume that the channel in each region has $L+1$ taps. The channel in region A is given by

$$h_A(l) = \sqrt{\beta} g_A(l) + \sqrt{1-\beta} r_A(l) \quad l = 0, 1, \cdots, L \quad (13)$$

where β gives the power of the specular component. If $g_A(z)$ denotes the z-transform of $g_A(l)$, then we have $g_A(z) = 1$. Hence the specular component is just the delta function. The channel $\{r_A(l)\}_{l=0}^L$ is generated from i.i.d. complex Gaussian with zero mean and variance equal to $\frac{1}{L+1}$. We have analogous equations for channels for region B and region C. For region B, the z-transform of the specular component is given by $g_B(z) = k_1 \Pi_{l=0}^{L-1} \left(z^{-1} - \exp(\frac{j2\pi l}{L})\right)$. The constant k_1 is selected so that the norm of the channel is equal to one. For region C, the z-transform of the specular component is given by $g_C(z) = k_2 \left(z^{-1} + 1\right)^L$ where once again the constant k_2 is selected so that the norm of the channel is equal to one.

Fig. 6 compares the performance of the QPP-L placement scheme with placing all the known symbols at the beginning of the packet (preamble scheme). We assume that $N = 112, (P+L) = 48$, $L = 3$ and the transmitted snr is equal to 20 dB. The user is assumed to belong to one of regions with equal probability. As expected the QPP-L scheme is better than the preamble scheme at every rate. We find that the QPP-1 scheme performs better than the QPP-L scheme at low rates but the QPP-L scheme performs better at high rates. We therefore conjecture that it is not possible to find a placement scheme that is uniformly better if we remove the assumption that known symbols come in cluster of length at least $\alpha \geq L$. We also find that the gain of the QPP-L scheme is higher at lower outage probabilities. This is because at lower outage probabilities, the bottle neck channels belong to region C, which is where the ISI is greatest. The optimal known symbol placement, being primarily a measure to decrease ISI, provides the maximum gain for these channels.

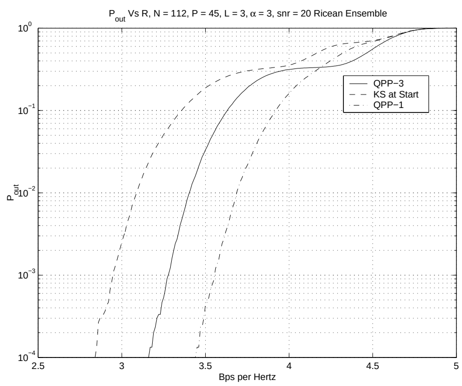

Fig. 6: Performance of Placement Schemes at SNR = 20dB, L=3

Fig. 7 shows the variation of the outage probability with SNR for the rate $R = 3.5$ bps per hertz and $L = 1$. We see that it is possible to obtain large gains in outage probability using the optimal placement scheme. This has a direct implication on the coverage that can be obtained at a given SNR.

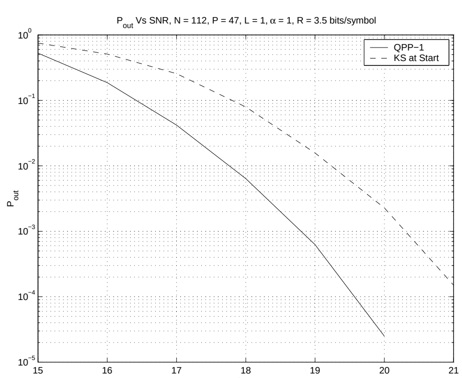

Fig. 7: Performance of Placement Schemes at R = 3.5 ,L = 1

VI. Conclusions

In this paper we studied the optimization of placement of redundancy in the data stream for a nonergodic broadcast fading model. The performance metric used is outage probability with i.i.d inputs. We examined the optimization of the position of known symbols in the data stream under the assumption that each codeword consists of packets and each packet contains the same number of known symbols and the same placement. We show that under the constraint each known symbol cluster is at least of length $\alpha > (L+1)$ the outage probability is minimized by breaking the known symbols into as many clusters as possible and by doing the best for placing these clusters periodically in the data stream. In particular the placement schemes belonging to the class QPP-α are optimal.

Simulations indicate that there is gain to be obtained by optimizing the position of known symbols. The gain in optimization is higher in channel ensembles that are more severely affected by ISI since the optimizing the placement is primarily a tool to decrease ISI. The placement schemes are optimal given any probability density $p_{\mathbf{h}}(\cdot)$ governing the channel realization. This property makes them attractive for broadcast applications.

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