Carrier Sense Multiple Access Communications on Multipacket Reception Channels: Theory and Applications to IEEE 802.11 Wireless Networks

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***Abstract*—Multipacket reception (MPR) refers to physical layers where receivers can decode multiple simultaneously transmitted packets. In this paper we investigate the resulting performance of conjoining carrier sense multiple access (CSMA) communications with MPR. We report on its maximum achievable stable throughput with decentralized control and show there can be throughput gain over slotted ALOHA (S-ALOHA), the non-channel-sensing protocol of choice. However, this gain diminishes as the physical layer's MPR strength increases, thereby diminishing the need for channel sensing. Nonetheless, for systems evolving from a single-user (SU) to a multiple-user (MU) channel, CSMA can furnish significantly more efficient utilization of MPR capacity than S-ALOHA. This is meaningful in practice because the emerging generation of the widely deployed IEEE 802.11 wireless local area networks (WLAN) — 802.11ac — is adapting MPR and will operate in said region. In that regard, we also discuss the effective usage of a channel's resources for MPR and highlight the advantages multiuser-MIMO (MU-MIMO), an MPR technique, can offer to WLANs. Using early design specifications of 802.11ac, we show that the existing SU-oriented 802.11 MAC parameters can under-utilize the MPR capacity offered by a MU-oriented physical layer.**

***Index Terms*—Multiple access theory, CSMA, multipacket reception, IEEE 802.11ac, wireless local area networks, cross-layer design, slotted ALOHA.**

I. Introduction

TRADITIONALLY, practical design and theoretical analysis of random multiple access protocols have assumed the *classical collision channel* model — namely, a transmitted packet is considered successfully received as long as it does not overlap or "collide" with another. Although this model is analytically amenable and reflected the state of technology when networking was an emerging field, the classical collision model does not represent the capabilities of today's transceivers. In particular, present transceiver technologies enable users to correctly receive multiple simultaneously transmitted data packets. With proper design, this capability — commonly referred to as *multiple packet reception* (MPR) [1][2] — can significantly enhance network performance.

Many fundamental ideas behind MPR already have been well researched and widely applied [3, §1]. Such examples include code division multiple access (CDMA) and frequency division multiple access (FDMA). But while MPR is applied therein as the basis for enabling multiple users to share a channel, for networks that employ packet-by-packet random multiple access, MPR represents a paradigm shift from the typical single-user (SU) to multiple-user (MU) models — something that is only beginning to happen in practice. Employing MPR in the physical (PHY) layer also expands the set of parameters for the media access control (MAC) layer to consider in its scheduling process, thus underscoring the importance for cross-layer design that strives to jointly optimize the functions of these layers [4]. Providing further insights for doing so in random multiple access communications with MPR — both in theory and in practice — is the main goal of the present paper.

A. Development of MPR in CSMA and 802.11 WLANs

The concept of random access on MPR channels was introduced by Ghez, Verdú and Schwartz in [1] and [2], where they studied the performance of slotted ALOHA (S-ALOHA) with MPR. Since then there has been much research on multiple access with MPR. However, until only recently, there has been no research published on carrier sense multiple access (CSMA) with MPR.

CSMA refers to a family of random multiple access schemes wherein a station (STA) with packets to transmit will attempt to do so only when the channel is detected to be idle. It was first shown by Kleinrock and Tobagi [5] that this provides significant performance improvement over S-ALOHA on the collision channel. CSMA have become the foundations of many networking technologies. For example, CSMA and CSMA with collision avoidance (CSMA/CA) are employed in the MAC layers of IEEE 802.11-based wireless local area networks (WLAN) [6]. However, none of these technologies have deployed MPR yet.

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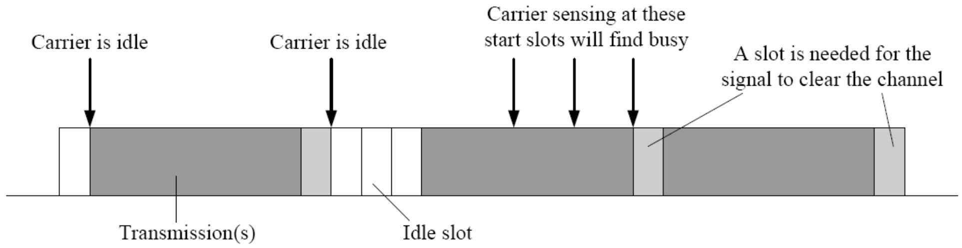

Fig. 1. Typical realization of channel activities with the CSMA protocol based on our network model.

The first substantial research on CSMA with MPR did not appear until 2004 when we reported in [7] on conjoining the two concepts and studied the resulting performance. Recognizing the benefits that MPR can bring to WLANs, in 2005 we presented the idea in a proposal [8] to the 802.11 WLAN Working Group for a future WLAN standard that incorporates MPR. Two years later, the notion of adapting a MU model in WLAN began garnering more consideration from members of the Working Group (eg. [9]). In late 2008, the 802.11ac Task Group (TGac) was established to develop a WLAN standard that supports a maximum aggregate throughput of at least 1 Gbps [10]. In order to realize the targeted data rates, TGac recognizes the need for higher data multiplexing efficiency and has identified MPR techniques like OFDMA and spatial division multiple access (SDMA), also known as MU multiple-input-multiple-output (MU-MIMO), as candidate technologies [10]. TGac now has converged on adapting MU-MIMO as one of the core technologies of the 802.11ac PHY layer [11].

B. Main Contributions of This Paper

This paper is an extension of the seminal multiaccess theory on MPR channels set forth in [1] and [2] to embody another essential MAC protocol — CSMA. We find that CSMA will always provide a higher throughput than S-ALOHA when closed-loop control is used, but this margin diminishes when the ability to resolve simultaneous transmissions is strong. Nonetheless, for systems that are evolving from SU to MU, CSMA can furnish significantly better performance than S-ALOHA via more efficient utilization of MPR capacity. This is of practical significance because the emerging generation of MPR-enabled WLANs likely will operate in this regime. We illustrate the applicability of our theoretical results by showing the effective usage of channel resources for MPR and the advantages that MU-MIMO can offer to WLANs. In particular, by simulating early 802.11ac specifications, we show that its SU-oriented MAC layer can under-utilize the capacity offered by an MPR-enabled MU-oriented PHY layer.

C. Other Related Work

In 2009, Zhang *et al*. reported in [12] their analysis of MPR in WLANs, employing 802.11g as an example. While their conclusions corroborate some themes in the theory we discuss here, their main results for CSMA are not shown analytically but via numerical results. And although they looked at S-ALOHA too, its intricate relationship with CSMA is not extensively investigated. Their entire focus is also only on one specific type of MPR channel. Moreover, their discussion for designing WLAN with MPR is on the limiting case when number of users is infinite, which differs from ours on the finite user case as reflected in practice. According to [13], their research in [12] was conducted unbeknownst of our earlier work.

Other researchers also have published results on this topic that enrich the theory and advance our understanding of it, like [14] and [15]. Particularly, Gau [14] provided another treatment of this topic by applying the traditionally used but possibly unrealistic framework wherein the overall *offered* traffic is Poisson; we discuss in III-D how this provides an alternative perspective of the theory in this paper.

II. The Network Model

A. Topology, Timing Relations and the CSMA Protocol

We consider a network with an unbounded number of stations (STAs) contending to transmit data packets to a central base station. Such a network model corresponds to the uplink of the *infrastructure mode* in 802.11 WLANs, wherein the base station is referred to as access point (AP) [6].

We assume each STA can perform *carrier sensing*, namely to detect whether the channel is currently idle or busy (i.e., that there is at least one other STA transmitting), and we assume the time required to do so is negligible (i.e., zero detection time).[1] As illustrated in Fig. 1, we employ a slotted-time system in which transmissions may begin only at the start of a slot and that every STA is synchronized to the slots.[2] Slot duration is designated to be at least the maximum signal propagation time of $t_{\text{prop}} = d_{max}/c$ seconds, where d_{max} is the maximum separation distance between the STAs and c is the signal's speed of propagation through the network's PHY medium.[3] Such a setup ensures that, after a transmission stops, every STA will find the channel to be clear for transmission after one slot's time; thus, each transmission must be preceded by an idle slot. We also assume packets are of constant length lasting T seconds. As in [5] and [17, §4.4], without loss of generality, we choose $T = 1$, which is equivalent to expressing time in units of T. We also express the slot duration in terms of this normalized time unit as $\tau = t_{\text{prop}}/T$, $0 < \tau < \infty$. Consequently, a packet will last for $1/\tau$ slots, where we have assumed as in [5] that $1/\tau$ is an integer.[4] In our model,

[1] Note that in 802.11 WLANs [6] a STA has to detect that the channel has become busy within 4 μs, which is also less than half of the slot duration.

[2] We will forgo analyzing the non-slotted versions as their performances are inferior to those of slotted versions [16] [5].

[3] Eg., c is the speed of light for the air interface in wireless networks.

[4] We can achieve this integral assumption practically by designing one's slot duration to fit this requirement.

each STA immediately becomes aware of its transmission outcomes,[5] either successful or unsuccessful, without expending extra cost; in this regard, acknowledgements introduce only fixed overheads and hence can be neglected from the model without affecting comparative analyses.

Fig. 1 illustrates a typical realization of channel activities with the CSMA protocol. It is not difficult to see that CSMA's channel activities can be modelled by a renewal process. Namely, the process regenerates itself after either an idle slot or an idle slot followed by transmission attempts that last for a packet's duration. We will refer to either of these events as a *transmission period* (TP).

Our analysis focuses on CSMA with *immediate first transmission* (IFT) admission control. Under this policy, when a packet arrives at an inactive STA and the channel is sensed idle, then the STA will attempt to transmit the packet at the start of the slot immediately following the packet's arrival. If the channel is sensed busy, then the STA will defer its attempt until there is idleness, and thereupon decides randomly whether to "backoff" or transmit. This random backoff is modelled as an independent sampling from a geometric distribution with parameter p, $0 \leq p \leq 1$; in other words, each STA attempts transmission with probability p or backs off with probability $1-p$.[6] The process is repeated if the channel is sensed busy after a backoff or that the packet transmission is not successfully received.

Note that CSMA with IFT is exactly the slotted *non-persistent CSMA* protocol introduced by Kleinrock and Tobagi in [5]. This variant of CSMA is, moreover, the primary MAC protocol practiced in 802.11 WLANs, called *distributed coordinated function* (DCF) [6].

B. Network Traffic and Throughput Characteristics

Each STA in our infinite population network model can have up to one packet to be transmitted at any time, be it a newly arrived packet or a so-called *backlogged* packet that needs to be retransmitted. Packet arrivals for non-backlogged STAs are assumed to be independent and identically distributed from slot to slot. Let $\hat{A}_t$ denote the number of new packets that arrived during the idle slot of TP t, $t \geq 0$. Assume $\hat{A}_t$ has probability distribution $P[\hat{A}_t = n] = \hat{\lambda}_n$, for $n \geq 0$, such that the mean arrival rate per slot is

$$\hat{\lambda} = \sum_{n=1}^{\infty} n\hat{\lambda}_n = \tau\lambda, \tag{1}$$

where λ is the mean arrival rate per normalized time unit (i.e., per packet duration) and is finite. Described more precisely, our packet arrival model is a point process over the real time line, with mean measure equal to the unit rate λ scaled by the Lebesgue measure. Note that a homogeneous Poisson point process of intensity λ satisfies condition (1).

[5]This assumption reflects 802.11 WLANs wherein the STA that has successfully received a data packet has to transmit an acknowledgement packet immediately after a fixed short duration; otherwise, receipt of the data packet is considered unsuccessful.

[6]Note that the exponential backoff process used in 802.11 can be equivalently modelled by such a sampling from a geometric distribution with an appropriate p [18].

When the arrival statistics are described in this manner, λ is equivalent to the mean arrival rate considered in many well known results, such as [1] and [2] for S-ALOHA with MPR or [5] and [17, §4.4] for CSMA on the collision channel. In other words, our current setup facilitates direct and meaningful comparison of our results with those in classical multiple access theory. In particular, the maximum λ for which packets can be successfully transmitted by a multiaccess protocol on a channel with asymptotically finite average delay is the channel's maximum achievable stable throughput with that protocol [17, §4.2.3].

C. The MPR Channel Model and Examples

We employ the *symmetric MPR channel model* of [1], with which the successful reception probabilities depend only on the number of packets transmitted in the slot. For the infinite population scenario, given that n packets are transmitted, for $1 \leq n \leq \infty$, $0 \leq k \leq n$, let

$$C_{n,k} = P[k \text{ packets are correctly received} |\ n \text{ are transmitted}].$$

Clearly, the symmetric MPR channel model is a generalized formulation and embodies as a special case the classical collision channel, which has $C_{1,0} = 0$ and $C_{n,0} = 1$ for all $n > 1$.

We denote the expected number of packets correctly received from a transmission set of n STAs by

$$\mathcal{C}_n \triangleq \sum_{k=1}^{n} kC_{n,k},$$

and assume that its limit $\mathcal{C} = \lim_{n\to\infty} \mathcal{C}_n$ exists, which is usually the case in practice [1]. For instance, it is natural to expect $\mathcal{C} = 0$ because practical PHY layers have finite resources and hence cannot support an unbounded number of simultaneous transmissions. We define *channel capacity* to be

$$\mathbb{C} \triangleq \sup_n \mathcal{C}_n,$$

namely the largest expected successes with simultaneous transmissions on a channel.

In addition, we also assume that $\mathcal{C}_n$ is concave in n, a property possessed by reasonable MPR channels. To see why, say we have $n_1 < n_2 < \arg\sup_n \mathcal{C}_n$. Then it is unreasonable to expect $\mathcal{C}_{n_1} > \mathcal{C}_{n_2} < \mathbb{C}$ because, if the channel is still capable of supporting more successful receptions even when greater than n_2 STAs transmit, then we should have $\mathcal{C}_{n_1} \leq \mathcal{C}_{n_2}$. Similarly, say $\arg\sup_n \mathcal{C}_n < n_3 < n_4$, then we should have $\mathcal{C}_{n_3} \geq \mathcal{C}_{n_4}$, because the channel can only yield fewer successes with an increasing number of transmitting STAs if there already are greater than $\arg\sup_n \mathcal{C}_n$ of them. It is possible for $\sup_n \mathcal{C}_n$ to be achieved at more than one n, but with said assumption this must occur over one and only one consecutive set of n's.

In sections below we discuss two specific MPR channels we consider exemplary and refer the reader to [1] and [2] for additional ones. In the *q-frequencies frequency-hopping channel*, each STA chooses with equal probability one of q frequencies on which to transmit. If more than one STAs choose the same frequency, then their transmissions will fail.

Such a channel has $\mathcal{C}_n = n(1-1/q)^{n-1}$ [1]. We will have an equivalent channel model if orthogonal multiuser codes are used instead of the q frequencies, i.e., STAs select one of q codes to encode their transmissions, failure occurring whenever two or more STAs select the same code. Another example we consider is the N-*user channel*, which assumes failed reception when the transmission set is strictly greater than N. The expected success of this channel is simply $\mathcal{C}_n = n$ for $n \leq N$ and $\mathcal{C}_n = 0$ for $n > N$. Note that the $\mathcal{C}_n$'s of both channels are concave in n.

A meaningful performance measure of a multiaccess protocol is how well it utilizes a channel's MPR capacity; accordingly we define *efficiency* to be the protocol's maximum stable throughput divided by $\mathbb{C}$.

III. Capacity of CSMA on MPR Channels

A. Ergodicity Region of CSMA

There have been many results on the stability and dynamic control of CSMA, but they pertain to the collision channel [17, §4.4] [19] [20]. Although they used different definitions of stochastic stability and attacked the problem with unique approaches, they all concluded that CSMA with the collision channel is inherently unstable, which is expected from its traffic load-throughput curve (cf. [17, §4.2] and [5]). We employ instead the analytic framework used for S-ALOHA in [1] and [2]:[7] A system is defined to be stable if the discrete-time Markov chain $\{X_t\}_{t\geq 0}$, whose state is the number of backlogged packets in the system at the beginning of the tth TP, is ergodic and unstable otherwise [17, §3A.5].

We give the state transition probabilities of $\{X_t\}$ in Appendix A. We can see from them that $\{X_t\}$ will be irreducible and aperiodic as long as it satisfies the sufficient condition that $0 < \hat{\lambda}_0 < 1$, which we assume holds as it is true for all reasonable scenarios.

To obtain our first main result, we examine the expected drift of the Markov chain $\{X_t\}$ at state $n, n \geq 0$, which is given by

$$d_n = E[X_{t+1} - X_t | X_t = n] = E[A_t - \Sigma_t | X_t = n],$$

where A_t and Σ_t are respectively the no. of new packets that arrived and the no. of successful transmissions during TP t.

Observe that the expected number of arrivals during a TP depends on whether there are arrivals during its idle slot and whether any of the backlogged STAs decide to transmit in the slot after it. Due to IFT, if there is at least one arrival in the idle slot, then the TP will consist of this idle slot followed by a transmission, regardless of how many backlogged STAs also decide to transmit during this time too. And of course, this situation also occurs even if the idle slot has no packet arrivals but at least one of the n backlogged STAs transmits, which occurs with probability $1-(1-p)^n$. Since the expected number of arrivals during a slot and during a packet transmission are respectively $\tau\lambda$ and λ, we have

$$\begin{aligned} E[A_t | X_t = n] &= \tau\lambda + (1-\hat{\lambda}_0)\lambda + \hat{\lambda}_0[1-(1-p)^n]\lambda \\ &= \lambda[1+\tau-\hat{\lambda}_0(1-p)^n]. \end{aligned}$$

[7]In homage to Ghez, Verdú and Schwartz, in adapting their analytic framework of [1] and [2], we have kept many of their notations unchanged or as similar as possible.

To find Σ_t, observe that the STAs contributing to channel contention in the current TP are only those with packets that arrived during the idle slot and those that are retransmissions from STAs backlogged at the start of this TP. As in the derivation in [1], let R_t be the number of transmissions from backlogged STAs in TP t. Then

$$P[\Sigma_t = k | X_t = n, \hat{A}_t = i, R_t = j] = C_{i+j,k},$$

for $i \geq 0$, $0 \leq j \leq n, 0 \leq k \leq i+j$. With the convention that $C_{0,0} = 0$, we have

$$E[\Sigma_t | X_t = n, \hat{A}_t = i, R_t = j] = \mathcal{C}_{i+j}$$

and hence get

$$E[\Sigma_t | X_t = n] = \sum_{i=0}^{\infty} \hat{\lambda}_i \sum_{j=0}^{n} B_n(j)\mathcal{C}_{i+j},$$

where we have let $B_n(j) = \binom{n}{j} p^j (1-p)^{n-j}$. Therefore,

$$d_n = \lambda(1+\tau) - \lambda\hat{\lambda}_0(1-p)^n - \sum_{i=0}^{\infty} \hat{\lambda}_i \sum_{j=0}^{n} B_n(j)\mathcal{C}_{i+j}. \quad (2)$$

Then by applying a result from [1], we arrive at the following result on the ergodicity region of CSMA.

Theorem 1. *A CSMA system is stable for all arrival distributions such that* $\lambda < \frac{1}{1+\tau}\,\mathcal{C}$ *and is unstable for* $\lambda > \frac{1}{1+\tau}\,\mathcal{C}$, *where* $\mathcal{C} = \lim_{n\to\infty} \mathcal{C}_n$. *(This also holds if* $\mathcal{C}$ *is infinite: if* $\lim_{n\to\infty} \mathcal{C}_n = +\infty$, *then the system is always stable.)*

Proof: From (2) we can see that $-2\lambda - n < d_n < \lambda(1+\tau)$, so $|d_n|$ is finite for $n < \infty$. Then we can use Lemma 1 of [1], which states that

$$\lim_{n\to\infty} \sum_{i=0}^{\infty} \hat{\lambda}_i \sum_{j=0}^{n} B_n(j)\mathcal{C}_{i+j} = \mathcal{C},$$

and see that the limit of equation (2) as n goes to infinity is

$$\lim_{n\to\infty} d_n = \lambda(1+\tau) - \mathcal{C}.$$

Then our result on the stable region follows by Pakes' Lemma (Theorem 2 of [21]).

To obtain our result on the unstable region, we will verify that Kaplan's condition [22] is satisfied provided that $\mathcal{C}_n < L$, $n \geq 1$, for some $L \in (0,\infty)$. According to [23], this is equivalent to showing the downward part of the drift,

$$D(i) = -\sum_{k=1}^{i} kP_{i,i-k}, \quad (3)$$

is bounded below, as shown in Appendix B. □

Because the ergodicity region in Theorem 1 is achieved with an arbitrary transmission probability p, $\frac{1}{1+\tau}\mathcal{C}$ is the maximum stable throughput that CSMA can achieve with open-loop control. Denoting $\eta_{c,o}$ to be CSMA's open-loop throughput, then Theorem 1 states that $\eta_{c,o} = \frac{1}{1+\tau}\lim_{n\to\infty}\mathcal{C}_n = \frac{1}{1+\tau}\mathcal{C}$.

Another implication of Theorem 1 is that, since $\mathcal{C} = 0$ for the collision channel, the corresponding open-loop throughput is then $\eta_{c,o} = 0$, which agrees with the known results that CSMA over the collision channel is inherently unstable without resort to a control protocol. It is also revealing to

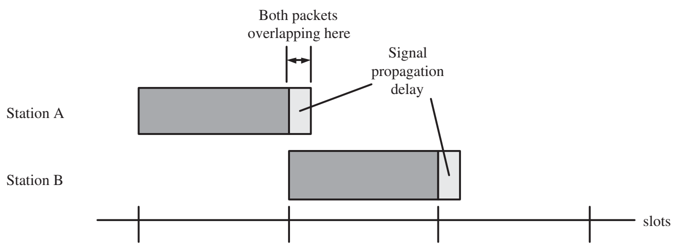

Fig. 2. Unfavorable interference can occur in S-ALOHA if signal propagation delay is not properly accounted for. Thus, in practice, S-ALOHA's slot size has to be at least $1+\tau$ time units. Note that, when compared with Fig. 1, the only difference in the channel activities is that CSMA's idle slots are shorter.

compare $\eta_{c,o}$ with the open-loop throughput for S-ALOHA given in [1], which we will denote by $\eta_{a,o}$. Although [1] shows that stability for uncontrolled S-ALOHA is achieved as long as $\lambda < \mathcal{C}$, this does not mean S-ALOHA actually outperforms CSMA by a factor of $1 + \tau$. This is because the network model assumed in [1] hasn't accounted for signal propagation delay. Certainly, if one were to implement S-ALOHA in practice, as illustrated in Fig. 2, propagation delay will destroy the synchronous assumption and detrimental interferences can occur between packets. Thus, as Roberts originally proposed [16], S-ALOHA's slot size should have at least $1 + \tau$ time units. By applying the same derivations set forth in [1], one can extend their results to show that S-ALOHA with a slot size of $1+\tau$ actually has $\eta_{a,o} = \frac{1}{1+\tau}\mathcal{C}$. This means S-ALOHA and CSMA perform identically with open-loop control. In other words, Theorem 1 has the important implication that CSMA's extra capability to sense the channel affords no additional advantage for random multiple access when no control protocol is used; we'll see below how this is different when a control protocol is employed.

B. Maximum Decentralized Stable Throughput of CSMA

Similar to the decentralized control strategies analyzed in [2] and the references therein, we consider strategies in which STAs adjust their retransmission probability according to the feedback information they can obtain from the channel. These schemes can be characterized in such a form:

$$p_t = F(S_t) \quad \text{and} \quad S_{t+1} = G(S_t, Z_t), \tag{4}$$

where p_t is the retransmission probability to be used in TP t, S_t is an estimate of the backlogged STAs X_t at the beginning of TP t, and Z_t is the feedback information at the end of TP t.[8] To proceed with the analysis, we represent this system by a discrete-time homogeneous Markov chain $\{X_t, S_t\}_{t\geq 0}$, the state of which at time t is the pair $\{X_t, S_t\}$. This Markov chain also will be irreducible and aperiodic if the same sufficient condition previously stated for $\{X_t\}$ is satisfied, which we henceforth assume to be the case.

Let $\eta_{c,c}$ be the maximum stable throughput achievable by CSMA with decentralized closed-loop control. Toward finding $\eta_{c,c}$, we will first study the case when each STA knows X_t at the beginning of TP t and determine the optimal control function $F^*(X_t)$ for it. To do that, we proceed by analyzing the Markov chain $\{X_t\}$ in Section III-A but with $p = p_t = F(X_t)$. This will then reveal the largest ergodicity region with decentralized control (4), and thus the highest possible achievable stable throughput (cf.[2]).

[8]See [2] for discussion on implementing such a closed-loop control.

Theorem 2. *There exists a retransmission probability p_n^* that minimizes the expected drift, d_n. With that p_n^* the CSMA system is stable for $\lambda < \eta_{c,c}$ and unstable for $\lambda > \eta_{c,c}$, where*

$$\eta_{c,c} = \sup\left\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} e^{-x}\left(\hat{\lambda}_0\lambda + \sum_{n=0}^{\infty}\frac{x^n}{n!}\sum_{j=0}^{\infty}\hat{\lambda}_j\mathcal{C}_{n+j}\right)\right\}. \tag{5}$$

The largest stable achievable throughput by any decentralized control algorithm of the form (4) *for CSMA is $\eta_{c,c}$ of* (5)*.*

Proof: We can write the expected drift equation (2) as $d_n(p) = \lambda(1+\tau) - Y_n(p)$, where $Y_n(p) = \lambda\hat{\lambda}_0(1-p)^n + \sum_{i=0}^{\infty}\hat{\lambda}_i\sum_{j=0}^{n}\binom{n}{j}p^j(1-p)^{n-j}\mathcal{C}_{i+j}$. Since $Y_n(p)$ is a polynomial on $[0,1]$, it attains a global maximum and there exists a $p_n^* = \arg\max_{p\in[0,1]} Y_n(p) = \arg\min_{p\in[0,1]} d_n(p)$; in other words, there exists a retransmission probability p_n^* that minimizes the drift d_n at state n.

Following the same steps used in the proof of Theorem 1 of [2], we can show that $Y_n(x/n)$ converges uniformly to $Y(x)$ for $x \geq 0$, where

$$Y(x) = e^{-x}\left(\hat{\lambda}_0\lambda + \sum_{n=0}^{\infty}\frac{x^n}{n!}\sum_{j=0}^{\infty}\hat{\lambda}_j\mathcal{C}_{n+j}\right). \tag{6}$$

So it follows that

$$\lim_{n\to\infty} Y_n(p_n^*) = \lim_{n\to\infty}\sup_{x\geq 0} Y_n(x/n) = \sup_{x\geq 0} Y(x).$$

The stable region specified by (5) follows by applying Pakes' Lemma [21, Theorem 2].

To obtain the unstable region, we can proceed as we did in Theorem 1 but with $p_t = F(X_t)$ substituted for p. Since $p_t \in [0,1]$, the result obtained there holds with said p_t.

To prove the second part of the theorem, we need to show that the $\{X_t, S_t\}$ is non-ergodic with any control when $\lambda > \eta_{c,c}$. The proof of this is essentially the same as that for Theorem 2 of [2], which gives the analogous result for S-ALOHA. (Cf. [24] for such details.) □

Though $\eta_{c,c}$ of (5) is derived by assuming the STAs have perfect knowledge of X_t, $\eta_{c,c}$ can also be achieved with a control with partial state information. In fact, by directly applying the derivation in [2, §III] to our CSMA framework here, we can show the same control they described for S-ALOHA — a control that computes the backlogged estimate S_t from feedback information Z_t, where $Z_t = 0$ when TP t is empty (contains no transmission) and $Z_t = 1$ otherwise — can also achieve $\eta_{c,c}$.

Because $Y(x)$ is expressed in terms of the arrival probabilities $\{\hat{\lambda}_n\}_{n\geq 0}$, the expression for $\eta_{c,c}$ given by (5) is an implicit equation of λ. Thus, unless the entire packet arrival probability distribution is specified, it is unclear whether there exist solutions to (5) and we cannot draw more meaningful general conclusions from it. It turns out, however, that a Poisson distribution will give a solution to (5), i.e., that the arrival per slot is Poisson distributed with parameter $\tau\lambda$. Since

Poisson arrivals are also a fitting — if not the de facto — statistical model for packet generation in networks of large numbers of STAs, we assume henceforth that the packet arrival per slot has such a probability distribution. Then,

$$\begin{aligned} Y(x) &= e^{-(x+\tau\lambda)}\left(\lambda+\sum_{n=0}^{\infty}\frac{x^n}{n!}\sum_{j=0}^{\infty}\mathcal{C}_{n+j}\frac{(\tau\lambda)^j}{j!}\right) \\ &= e^{-(x+\tau\lambda)}\left(\lambda+\sum_{n=1}^{\infty}\mathcal{C}_n\frac{(x+\tau\lambda)^n}{n!}\right), \end{aligned} \tag{7}$$

and hence,

$$\eta_{c,c} = \sup\left\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} e^{-(x+\tau\lambda)}\left(\lambda+\sum_{n=1}^{\infty}\mathcal{C}_n\frac{(x+\tau\lambda)^n}{n!}\right)\right\}, \tag{8}$$

which readily makes way for further analysis.

For example, under the collision channel model, $Y(x) = e^{-(x+\tau\lambda)}[x+\lambda(1+\tau)]$. And $\sup_{x\geq 0} Y(x)$ is then attained with $x = 1-\lambda(1+\tau)$, so (8) reduces to the transcendental equation $\eta_{c,c} = \sup\left\{\lambda : \lambda < \frac{1}{1+\tau}e^{\lambda-1}\right\}$. Solving numerically for $\tau = 0.01$, we find that $\eta_{c,c} = 0.865$ packets per unit time (packets/T), which is equal to the corresponding value calculated from Kleinrock and Tobagi's throughput expression for slotted non-persistent CSMA, i.e., equation (8) of [5]. (Recall that the non-persistent CSMA is equivalent to CSMA with IFT.) They also reported that as $\tau \to 0$, CSMA could approach perfect (collision) channel utilization, or 100% efficiency, which can be shown here as well. While usage of (8) is probably limited to numerical solutions, an equivalent closed form expression can be obtained for Poisson arrivals:

Theorem 3. For Poisson distributed packet arrival, the maximum achievable stable throughput of CSMA, $\eta_{c,c}$, is equivalently given by both

$$\eta_{c,c} = \sup\left\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} e^{-x}\left(\lambda+\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}\right)\right\} \tag{9}$$

and

$$\eta_{c,c} = \sup_{x\geq 0}\frac{1}{1+\tau-e^{-x}}e^{-x}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}. \tag{10}$$

If $\eta_{c,c} > \frac{1}{1+\tau}\mathcal{C} = \frac{1}{1+\tau}\lim_{n\to\infty}\mathcal{C}_n$, then there exists a constant $A > 0$ such that the control $p_t = A/X_t$ for $X_t > A$ yields the optimal throughput $\eta_{c,c}$.

Proof: Observe that by solving directly for λ from the inequality in (9), which is an operation that is independent of x, we can obtain the equivalent closed form expression for $\eta_{c,c}$ of (10). So, to prove the first part of the theorem it remains only to show that (9) and (8) are equivalent.

Let $y(x) = e^{-x}(\lambda+\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!})$. Then $y(x+\tau\lambda) = Y(x)$; i.e., $Y(x)$ of (7) is a translation of $y(x)$ by $-\tau\lambda$. (Recall that $\tau \in (0,\infty)$ and $\tau\lambda \geq 0$.) To obtain (9), we need to show that $\sup_{x\geq 0} Y(x) = \sup_{x\geq 0} y(x)$ holds within the solution space of the inequality $\lambda < \frac{1}{1+\tau}\sup_{x\geq 0} Y(x)$, and thus this solution space must then be identical to that of $\lambda < \frac{1}{1+\tau}\sup_{x\geq 0} y(x)$, with which (9) will follow by the definition of $\eta_{c,c}$ in (8).

Consider first the case in which $y(x)$ does not achieve its supremum. Then together with Property 5 of [2], we can also see that $\sup_{x\geq 0} y(x) = \lim_{x\to\infty} y(x) = \mathcal{C}$. Since $Y(x) = y(x+\tau\lambda)$, it follows that $Y(x)$ will also not achieve its supremum, and so for all $\tau \in (0,\infty)$ and $\tau\lambda \geq 0$, $\sup_{x\geq 0} y(x+\tau\lambda) = \lim_{x\to\infty} y(x+\tau\lambda) = \mathcal{C}$. Then for all $\tau \in (0,\infty)$ and $\tau\lambda \geq 0$, $\sup_{x\geq 0} y(x) = \sup_{x\geq 0} Y(x)$. Therefore, by the definition of $\eta_{c,c}$ in (8), we have $\eta_{c,c} = \sup\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} y(x)\}$, as given by (9).

Now consider the case in which $\sup_{x\geq 0} y(x) = y(x_o)$ at some finite $x_0 \geq 0$. Then for all $\lambda \in [0, x_0/\tau]$,

$$\sup_{x\geq 0} Y(x) = Y(x_o - \tau\lambda) = y(x_0) = \sup_{x\geq 0} y(x). \tag{11}$$

Also, for all $x \geq 0$, $y(x) \leq e^{-x}\lambda + x$. So, for all $\lambda \in [0, x_0/\tau]$,

$$\begin{aligned} \tfrac{1}{1+\tau}\sup_{x\geq 0} Y(x) &= \tfrac{1}{1+\tau}\sup_{x\geq 0} y(x) \\ &\leq \tfrac{1}{1+\tau}\left(e^{-x_0}\lambda + x_0\right) \\ &\leq \tfrac{1}{1+\tau}\left(e^{-x_0}+\tau\right)\tfrac{x_0}{\tau} \\ &\leq \tfrac{x_0}{\tau}. \end{aligned}$$

It follows that for all $\lambda \in [0, \frac{1}{1+\tau}\sup_{x\geq 0} Y(x)]$, the identity of (11) holds, which implies that the solution spaces of $\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} Y(x)\}$ and $\{\lambda : \lambda < \frac{1}{1+\tau}\sup_{x\geq 0} y(x)\}$ are identical. Therefore, by the definition of $\eta_{c,c}$ in (8), $\eta_{c,c}$ can be expressed equivalently by (9) for this case as well.

The second part of this theorem can be proved by directly applying the steps for the analogous result for S-ALOHA given in the proof of Theorem 2 in [2]. □

By Theorem 3, if we have $\eta_{c,c} = \frac{1}{1+\tau}\mathcal{C}$, which occurs when $\mathcal{C} = \lim_{n\to\infty}\mathcal{C}_n = \sup_{n\geq 1}\mathcal{C}_n$, then open-loop control can already attain the maximum stable throughput, i.e., $\eta_{c,c} = \eta_{c,o}$; otherwise, closed-loop control with $p_t = A/X_t$ should be used to achieve the optimal throughput $\eta_{c,c} > \eta_{c,o}$ (cf. [2]).

C. CSMA's Throughput in Relation with S-ALOHA's

We can straightforwardly adapt the derivation in [2] to obtain S-ALOHA's optimal closed-loop throughput for a slot size of $1+\tau$ time units. Denoting said throughput by $\eta_{a,c}$, such a derivation will yield

$$\eta_{a,c} = \sup_{x\geq 0}\frac{1}{1+\tau}e^{-x}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}. \tag{12}$$

Comparing (10) and (12), we see the only difference between closed-loop CSMA and S-ALOHA comes down to just a single $-e^{-x}$ term in the denominator. Since $\frac{1}{1+\tau} \in (0,1)$, whenever $\sup_{0\geq 0}\frac{e^{-x}}{1+\tau-e^{-x}}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}$ is achieved by an $x \in [0,\infty)$, due to said $-e^{-x}$ term, CSMA's maximum throughput will always be higher than that of S-ALOHA. But as it turns out, this advantage will diminish as the channel supports more MPR. Though, before we can formally describe this outcome, we need to define what is meant by supporting more MPR. Since a larger expected success for a transmission set corresponds to a better MPR capability for that set, accordingly, we can assert the following definition.

Definition 1 (MPR Strength): Consider two channels given respectively by their expected transmission successes $\{\mathcal{C}_n^{(1)}\}$ and $\{\mathcal{C}_n^{(2)}\}$, $n \geq 1$. We say that the channel with $\{\mathcal{C}_n^{(2)}\}$ has a *stronger MPR strength* than the channel with $\{\mathcal{C}_n^{(1)}\}$ if there

TABLE I
THROUGHPUTS AND EFFICIENCIES OF S-ALOHA AND CSMA FOR q-ORTHOGONAL-CODES CHANNEL WITH PROPAGATION DELAY $\tau = 0.01$

No. of codes (q)	$\mathbb{C}$	$\eta_{c,c}$	$\eta_{a,c}$	$\frac{\eta_{c,c}}{\mathbb{C}}$	$\frac{\eta_{a,c}}{\mathbb{C}}$	x_c	x_a
1	1.0000	0.8655	0.3642	0.8655	0.3642	0.1345	1.0000
2	1.0000	0.9652	0.7285	0.9652	0.7285	0.4865	2.0000
3	1.3333	1.1752	1.0927	0.8814	0.8195	2.1706	3.0000
4	1.6875	1.4895	1.4569	0.8826	0.8634	3.5994	4.0000
5	2.0480	1.8346	1.8212	0.8958	0.8893	4.8034	5.0000
10	3.8742	3.6425	3.6424	0.9402	0.9402	9.9955	10.0000

exists a non-empty set $\mathcal{N} = \{n : \mathcal{C}_n^{(2)} > \mathcal{C}_n^{(1)}\}$ and that $\mathcal{C}_n^{(2)} = \mathcal{C}_n^{(1)}\ \forall n \notin \mathcal{N}$.

Since $\{\mathcal{C}_n\}$ directly determines the value of the summation in (10) and (12), Definition 1 gives a sufficient condition for attaining greater throughput; i.e., $\eta_{c,c}$ and $\eta_{a,c}$ are increasing functions of MPR strength. Moreover, we can show a stronger MPR strength also leads to a greater x that achieves the supremum in (10). For brevity, let $\Upsilon(x,\{\mathcal{C}_n\}) = \frac{e^{-x}}{1+\tau-e^{-x}}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}$, where $\{\mathcal{C}_n\}$ denotes a set of $\mathcal{C}_n, n \geq 1$.

Lemma 1. *For all expected transmission success $\{\mathcal{C}_n^{(1)}\}$ with $x_1 = \arg\sup_{x\geq 0}\Upsilon(x,\{\mathcal{C}_n^{(1)}\}) < \infty$, there exists a $\{\mathcal{C}_n^{(2)}\}$ with $x_2 = \arg\sup_{x\geq 0}\Upsilon(x,\{\mathcal{C}_n^{(2)}\}) < \infty$ such that $\{\mathcal{C}_n^{(2)}\}$ has stronger MPR strength than $\{\mathcal{C}_n^{(1)}\}$ and $x_2 > x_1$.*

The proof is given in Appendix C. Note that an analogue to Lemma 1 for S-ALOHA can be stated by substituting $\Upsilon(x,\{\mathcal{C}_n\})$ with $\Gamma(x,\{\mathcal{C}_n\}) = \frac{e^{-x}}{1+\tau}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}$.[9] Moreover, it can be shown that $\arg\sup_{x\geq 0}\Upsilon(x,\{\mathcal{C}_n\}) \leq \arg\sup_{x\geq 0}\Gamma(x,\{\mathcal{C}_n\})$, with equality if and only if $\mathcal{C}_n = 0$ for all n. The relationship between CSMA and S-ALOHA is further revealed by employing Lemma 1:

Theorem 4. *For Poisson distributed packet arrival, as the channel's MPR strength becomes stronger, the maximum stable throughput achievable by CSMA, $\eta_{c,c}$, approaches the maximum stable throughput achievable by S-ALOHA, $\eta_{a,c}$. In the limit as MPR strength becomes stronger, $\eta_{c,c} = \eta_{a,c}$ and the two protocols both have efficiency of 1.*

Proof: If the MPR channel's expected transmission success $\{\mathcal{C}_n\}$ has limit $\mathcal{C} = \sup_{n\geq 1}\mathcal{C}_n$, then we know from the proof of our Theorem 3 and Property 5 of [2], respectively, that the best that CSMA and S-ALOHA can achieve are their open-loop throughputs, which are the same. Therefore, $\eta_{c,c} = \eta_{a,c} = \eta_{a,o} = \frac{1}{1+\tau}\mathcal{C}$ to start with and the proof is already complete for this case.

So let us consider now only channels with $\mathcal{C} < \sup_{n\geq 1}\mathcal{C}_n$, namely those with a $\{\mathcal{C}_n\}$ such that $\Upsilon(x,\{\mathcal{C}_n\})$ achieves its supremum at some $x \in [0,\infty)$. Consider a sequence of expected transmission successes with increasing MPR strength $\{\{\mathcal{C}_n^{(i)}\}\}_{i\geq 1} = \{\{\mathcal{C}_n^{(1)}\},\{\mathcal{C}_n^{(2)}\},\{\mathcal{C}_n^{(3)}\},\ldots\}$. Let $x_i = \arg\sup_{x\geq 0}\Upsilon(x,\{\mathcal{C}_n^{(i)}\})$. Denote $\eta_{c,c}^{(i)}$ and $\eta_{a,c}^{(i)}$ to be the respective maximum stable throughput of CSMA and S-ALOHA for channel $\{\mathcal{C}_n^{(i)}\}$.

[9] This is because, apparent from Appendix C, $d\Gamma(x_0,\{\mathcal{C}_n\})/dx = -\Gamma(x,\{\mathcal{C}_n\})+C(x,\{\mathcal{C}_n\})$, so $\Gamma(x,\{\mathcal{C}_n\})$ has identities similar to (16) and (17). Note that both $d\Upsilon(x,\{\mathcal{C}_n\})/dx$ and $d\Gamma(x,\{\mathcal{C}_n\})/dx$ are linear ODEs.

TABLE II
THROUGHPUTS AND EFFICIENCIES OF S-ALOHA AND CSMA FOR THE N-USER CHANNEL WITH PROPAGATION DELAY $\tau = 0.01$

$\mathbb{C} = N$	$\eta_{c,c}$	$\eta_{a,c}$	$\frac{\eta_{c,c}}{\mathbb{C}}$	$\frac{\eta_{a,c}}{\mathbb{C}}$	x_c	x_a
1	0.8655	0.3642	0.8655	0.3642	0.1345	1.0000
2	1.1541	0.8316	0.5770	0.4158	0.8097	1.6180
3	1.5570	1.3575	0.5190	0.4525	1.7735	2.2695
4	2.0455	1.9231	0.5114	0.4808	2.6496	2.9452
5	2.5916	2.5184	0.5183	0.5037	3.4654	3.6395
10	5.7775	5.7737	0.5778	0.5774	7.2872	7.2970

From (10) and (12) we can see an upper bound: $\eta_{c,c}^{(i)} \leq \frac{1}{1+\tau-e^{-x_i}}\sup_{x\geq 0}e^{-x}\sum_{n=1}^{\infty}\mathcal{C}_n^{(i)}\frac{x^n}{n!} = \frac{1+\tau}{1+\tau-e^{-x_i}}\eta_{a,c}^{(i)}$. Because $e^{-x} > 0$ for all $x \in [0,\infty)$, we also have a lower bound of $\eta_{c,c}^{(i)} \geq \eta_{a,c}^{(i)}$, with equality if and only if $i = 1$ and $\mathcal{C}_n^{(1)} = 0$ for all n. Stated equivalently, if we let $\gamma_i = \frac{1+\tau}{1+\tau-e^{-x_i}}$, then

$$\eta_{a,c}^{(i)} < \eta_{c,c}^{(i)} \leq \gamma_i\eta_{a,c}^{(i)} \quad \text{for all } i > 1. \tag{13}$$

By our definition of stronger MPR strength, given any $\{\mathcal{C}_n^{(i)}\}$ in our sequence $\{\{\mathcal{C}_n^{(i)}\}\}_{i\geq 1}$, there must exist a $\{\mathcal{C}_n^{(j)}\}$, $j \geq i$, in $\{\{\mathcal{C}_n^{(i)}\}\}_{i\geq 1}$ with $\mathcal{C}_n^{(j)} = \max\{\mathcal{C}_n^{(j)}\} = n$ for all $n \leq \lfloor x_i + 1\rfloor$, where $\lfloor x_i + 1\rfloor$ denotes the largest integer less than or equal to $x_i + 1$. Thereafter in the sequence, $\{\mathcal{C}_n^{(j+1)}\}$ must satisfy the construction method given in the proof of Lemma 1, as if $\{\mathcal{C}_n^{(j+1)}\}$ is constructed from $\{\mathcal{C}_n^{(i)}\}$ with that method. So, by Lemma 1 we can construct from $\{x_i\}_{i\geq 1}$ a subsequence $\{\hat{x}_k\}_{k\geq 1}$, with $\hat{x}_{k+1} > \hat{x}_k$, and by our definition of higher MPR strength there does not exist a subsequence in $\{x_i\}_{i\geq 1}$ that is constant or strictly decreasing. Then because $\frac{1+\tau}{1+\tau-e^{-\hat{x}_{k+1}}} < \frac{1+\tau}{1+\tau-e^{-\hat{x}_k}}$ for all k, we have $\lim_{i\to\infty}\gamma_i = 1^+$, i.e., that $\{\gamma_i\}_{i\geq 1}$ is approaching 1 from above. Therefore, along with (13), as the MPR strength becomes stronger with each i, $\eta_{c,c}^{(i)}$ is approaching $\eta_{a,c}^{(i)}$ from above.

Finally, in the limit as MPR strength increases, we have the ideal MPR channel that has $\mathcal{C}_n = n$ for all n and channel capacity $\mathbb{C} = \infty$. Accordingly we then have $\mathcal{C} = \infty$, with which $\eta_{c,c} = \eta_{a,c} = \eta_{a,o} = \infty$ and efficiency is $\eta_{c,c}/\mathbb{C} = 1$. □

Theorem 4 essentially shows that as the MPR strength becomes stronger, we can perform equally well without carrier sensing by simply accessing the channel with random scheduling — i.e., closed-loop S-ALOHA. And then when the MPR strength is sufficiently strong, in lieu of scheduling, STAs could even transmit at any slot and achieve the same throughput performance by relying solely on the PHY layer as the means for separating the STAs' transmissions. This conclusion regarding when MAC-layer scheduling becomes unnecessary by virtue of the PHY layer's MPR strength has been pointed

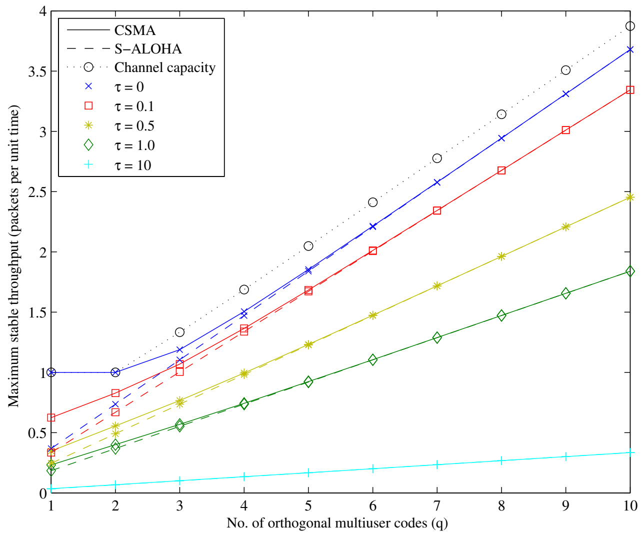

Fig. 3. Maximum stable throughput of CSMA and S-ALOHA for the q-orthogonal codes MPR channel with various propagation delays (τ). The channel's MPR capacity is displayed for comparison. As in Figs. 4–6, note also the overall throughput-lowering effect of τ.

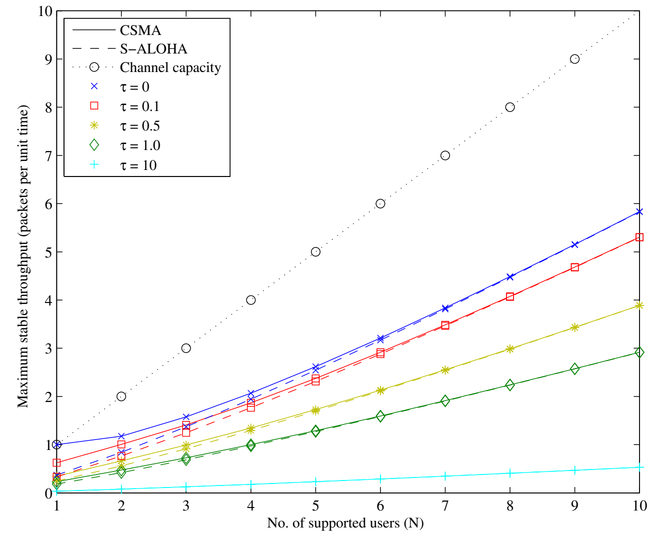

Fig. 5. Maximum stable throughput of CSMA and S-ALOHA for the N-user MPR channel with various propagation delays (τ). The channel's MPR capacity is also displayed for comparison.

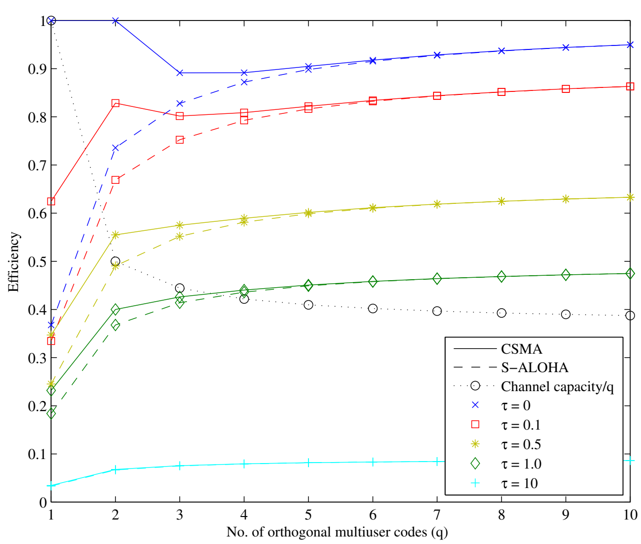

Fig. 4. Efficiency of CSMA and S-ALOHA for the q-orthogonal codes MPR channel with various propagation delays (τ). The channel's MPR capacity normalized by q is also displayed for comparison.

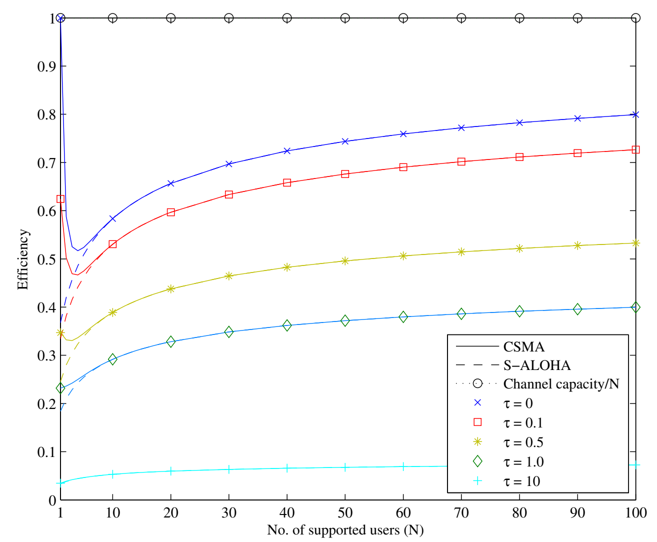

Fig. 6. Efficiency of CSMA and S-ALOHA for the N-user MPR channel with various propagation delays (τ). The channel's MPR capacity normalized by N is also displayed for comparison. Note that the horizontal scale here is different from that of Fig. 5 in order to show the relative growth of the efficiency curves.

out [25]. An additional insight herein and implicit in [2] is the juncture that occurs, i.e., when closed-loop control is first rendered unnecessary and open-loop control alone suffices, is when the MPR channel has $\mathcal{C} = \limsup_{n\geq 1} \mathcal{C}_n = \mathbb{C}$ (cf. [24]).

To illustrate the results in our discussion thus far, we plot $\eta_{c,c}$, $\eta_{a,c}$ and their efficiencies in Figs. 3–6 for the q-orthogonal-codes and N-user MPR channels. These values and the x's that attain them are also given in Tables I–II, where $x_c = \arg\sup_{x\geq 0} \Upsilon(x, \{\mathcal{C}_n\})$ and $x_a = \arg\sup_{x\geq 0} \Gamma(x, \{\mathcal{C}_n\})$.

Observe that the N-user channel has $\mathcal{N} = \{N\}$ at each increment in channel capacity; so, its MPR strength is becoming stronger. The same is true for the q-orthogonal-codes channel, which has $\mathcal{N} = \{n : n \geq 1\}$ at each q because $\mathcal{C}_n^{(q+1)} = n(1-\frac{1}{q+1})^{n-1} > n(1-\frac{1}{q})^{n-1} = \mathcal{C}_n^{(q)}$, $\forall n \geq 1$. But while their associated throughputs are increasing with channel capacity, they also settle into linear growths in Figs. 3 and 5. Actually, this is expected for the q-orthogonal-codes channel, since $\eta_{a,c} = \frac{1}{1+\tau} q e^{-1}$ [2] and so its growth has a constant slope of $\frac{1}{1+\tau} e^{-1}$. And, an approximately linear growth for the N-user channel has been predicted by [26], wherein throughputs of random access protocols for said channel are bounded between $N - \sqrt{N \ln N}$ and $N - \sqrt{N}$ when N is large. Because $\sqrt{N \ln N}$ and $\sqrt{N}$ are $o(N)$ when N is large, this channel's $\eta_{a,c}$ will eventually be growing approximately linearly at a small positive slope.

These throughputs' linear asymptotic growth trends are as well reflected in Figs. 4 and 6, where the associated efficiencies become nearly constant with MPR strength. This too is expected for the q-orthogonal-codes channel, since $(1 - \frac{1}{q}) \to e^{-1/q}$ at large q, it can be easily shown that

$\mathbb{C} \to qe^{-1}$ at large q (as confirmed by the normalized capacity in Fig. 4); thus, its efficiency evolves into $\sim \frac{1}{1+\tau}$ at large q. As for the N-user MPR channel, said trend is due to the fact that $\eta_{a,c}/N$ is bounded between $1 - \sqrt{(\ln N)/N}$ and $1 - \sqrt{1/N}$ at large N, where both increase at a slow rate.

Interestingly, while S-ALOHA's efficiency always rises from below, CSMA's efficiency may initially reach a local maximum — even as high as 1 when τ is small for some channels — before tapering off to approach S-ALOHA's. This perhaps unintuitive effect can be shown from (15). For instance, as long as $\sup_{n\geq 1} \mathcal{C}_n \leq \mathcal{C}_1$, efficiency can be maintained at or increase toward 1 (cf. Fig. 4). And even after said local maximum, CSMA's efficiency (and throughput) can still be considerably higher than S-ALOHA's. This indicates CSMA can be much more efficient than S-ALOHA at utilizing the MPR capability in the region where the SU channel is just beginning to evolve into a MU one. We discuss in Section IV how this region is an important one to consider because resources for enabling more MPR can be scarce in practice.

D. Connection to Results from Classical Multiaccess Theory

An alternative interpretation is that $\frac{1}{\tau}\sup_{x\geq 0} e^{-x}\sum_{n=1}^{\infty} \mathcal{C}_n \frac{x^n}{n!}$ is equivalent to $\frac{1}{\tau}\sup_{X\sim Poiss(x),x\geq 0} E[\mathcal{C}_X]$, i.e., the supremum of the expectation of C_X over all X, where X is a Poisson distributed random variable with mean x [2]. Because each TP for S-ALOHA is $1 + \tau$, be it idle or not, (12) can be explained by said interpretation via renewal theory arguments.[10] This applies to CSMA too: Due to the benefits of channel sensing, its difference from S-ALOHA is that an idle slot lasts only τ time units, so the TP for CSMA has expected value of $(1 + \tau)P[X > 0] + \tau P[X = 0] = 1 + \tau - e^{-x}$, which is reflected in (10).

Another perspective on CSMA's diminishing throughput gain also follows. As the MPR strength becomes stronger, STAs can then transmit at any slot and have a lower chance of collision; thus, there are fewer idle slots. This is confirmed by x_c and x_a both increasing with MPR strength, i.e., $P[X = 0]$ is decreasing. When channel activities are viewed solely as a renewal process, the only difference between CSMA and S-ALOHA is that the former has shorter idle slots, and so, if idle slots are becoming fewer, then the TPs for both protocols must be approaching the same duration; ergo, the same throughput.

Finally, note that X, the number of transmission attempts in each slot, is actually the overall *offered* network traffic, i.e., the aggregate of both new and backlogged traffic. That's why our $\eta_{c,c}$ value for the collision channel is the same as that from Kleinrock and Tobagi [5] despite the fact that their derivation, like many multiaccess research of that era, hinges on the flawed assumption [2] [17, §4.2.2] that the network's offered traffic is approximately Poisson distributed. Note that this is also true for the CSMA with MPR throughputs calculated from Gau's equation (5) of [14], which actually has the same Poisson traffic assumption.

[10] By renewal theory arguments, throughput (as measured in units of packets per unit time) is the expected duration of successful attempts divided by the expected duration of the transmission period (i.e., renewal cycle), and multiplied by the expected number of packets in each successful attempt.

IV. Applying MPR in Practice and Performance of 802.11 WLANs with MU-MIMO

A. Insights for Practical Designs of MPR-enabled Networks

Perhaps not emphasized enough in MPR multiaccess research is the unstated assumption that even as more simultaneously transmitted packets can be successfully received, the packet's data rate has to be maintained at some minimum level in order to achieve actual increase in network throughput. In other words, we cannot examine solely the improvement on multiaccess throughput expressed in packets per unit time; we need to also consider whether and how much the packet's data rate has been sacrificed by the diverting of channel resources to support MU capability. Simply put, if we denote r_1 and η_1 to be the respective data rate (in bits per packet) and multiaccess throughput (in packets per unit time) for a PHY layer, and r_2 and η_2 for those of another PHY layer with stronger MPR strength, then, even though $\eta_2 > \eta_1$, we still need

$$\eta_2/\eta_1 > r_1/r_2 \tag{14}$$

in order for the latter PHY layer to deliver a higher network throughput of $\eta_2 r_2$, in bits per unit time.

The basis for MU capability is to exploit resources in the channel that provide orthogonal properties to support MPR. Ultimately, this is equivalent to finding what is known as *degrees of freedom* (DoF) from the channel to multiplex data streams [3, §1]. The design criteria that we describe here is: A resource that gives rise to new DoF should be used to increase MU capability when (14) can be satisfied; otherwise, the network throughput can be made higher by applying the resource to instead increase the data rate of each packet.

B. WLAN Design Constraints and Advantages of MU-MIMO

MIMO signaling with multiple antennas exploits the spatial diversity "resource" in multipaths of a wireless channel to generate DoF that can be used for boosting a point-to-point link's spectral efficiency or robustness. Because of independent fading statistics among users' propagation paths, MU diversity also exists as a resource for creating DoF to multiplex MU data streams [27]. The challenge in designing MIMO systems centers on balancing the trade-offs among said various gains with the available DoF.

While MIMO spatial multiplexing of additional data streams increases data rates linearly, the SNR required to robustly receive them also grows noticeably. In fact, a currently impractical level of SNR is required to receive just a few spatial streams signaled at high orders of modulation and coding rates [28, §5.3]. Contrastingly, indoor measurements show MU-MIMO can achieve considerably higher spectral efficiencies than SU-MIMO for the same SNR and number of receive antennas per user [29]–[30]. This implies that, as SNR and number of receive antennas are typical WLAN design constraints, the extra DoF obtained from adding transmit antennas should be applied on MU multiplexing rather than on raising the level of a SU's spatial multiplexing toward levels that the system cannot sustain robustly. And since a PHY layer that uses these new DoF to improve its SU data rate cannot practically do so below a certain SNR, when this PHY layer is compared to another that instead diverts these DoF to increase

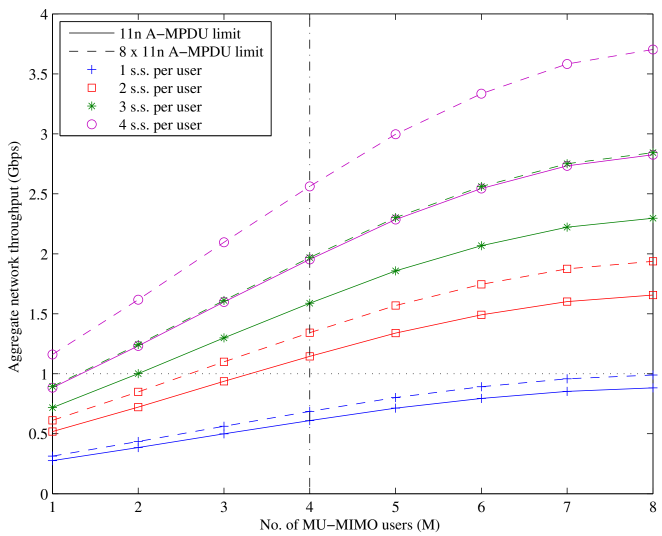

Fig. 7. Aggregate network throughput of the proposed 802.11ac WLAN for 1–4 spatial streams (s.s.) per user. As in Fig. 8, the A-MPDU size limits considered are 65 KB and 520 KB, which are, respectively, the maximum defined in 802.11n and eight times of that; note that larger sizes do not show changes to the trends. 802.11ac proposes to support up to four MU-MIMO users, where each user can have up to 4 s.s. for a system maximum of 8 s.s. 802.11ac has a maximum aggregate throughput of at least 1 Gbps.

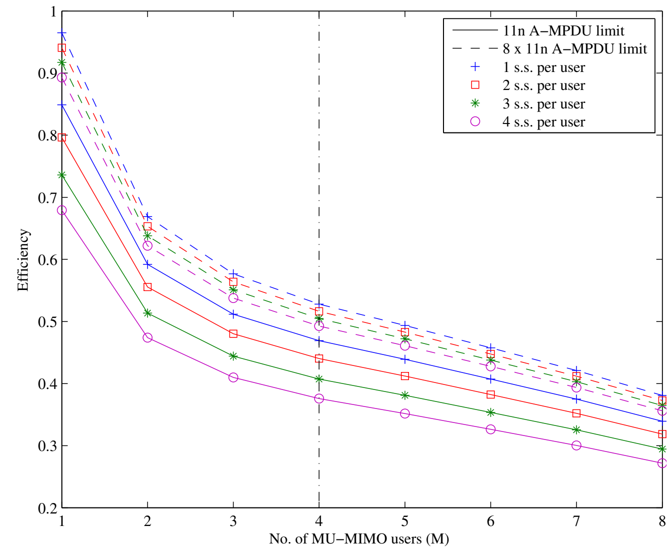

Fig. 8. Efficiency of the proposed 802.11ac WLAN in the infrastructure mode. The decrease in efficiency with M indicates the current 802.11 SU-oriented MAC cannot fully utilize the PHY layer's MU capacity.

TABLE III
A SELECTION OF DATA RATES FOR AN 80-MHz 802.11AC OFDM SYMBOL WITH SHORT GUARD INTERVAL (400 NS) [11]

No. of spatial streams	Modulation	Code rate	Spectral efficiency (Mb/s/Hz)	Data rate (Mbps)
1	BPSK	1/2	0.40625	32.5
1	64-QAM	5/6	4.0625	325.0
2	64-QAM	5/6	8.125	650.0
3	64-QAM	5/6	12.1875	975.0
4	64-QAM	5/6	16.25	1300.0

the system's MU capacity, we can also expect (14) to likely be satisfied. Thus, MU diversity can be an effective resource for realizing throughput gains from MPR in current WLANs.

Lastly, we note that finding new channel resources to enhance MPR can become progressively difficult because of practical issues. For instance, the number of antennas is a potential constraint due to space limitations and costs. Also, the maximum transmit power allowable by regulatory rules is a constraint on the power density per spatial stream, limiting the achievable level of MU multiplexing. Consequently, although the throughput gain with the next generation of WLANs promises to be rewarding, the increment in MPR capacity will only be moderate, not dramatic.

C. Performance Results and Enhancements for MU-based 802.11ac WLANs

To see the benefits from applying MPR to CSMA wireless networks in practice, we have simulated the performance of WLANs based on the initial draft text (version D0.1) [11] proposed for the 802.11ac standard.[11] While this standardization effort is still ongoing and industry dynamics are difficult to predict, our results provide the best means for speculating at present about how the finalized specification may perform.

The plan for 802.11ac to perform MU-MIMO is to extend the SU-MIMO framework of 802.11n. As a result, the fundamental components in the 802.11n PHY layer are adopted. Table III lists the 80-MHz data rates from [11] that we simulated. Details of our simulation framework are explained in Table IV. Of note is that 802.11ac has adapted the same MAC backoff policy as 802.11n. And since this policy's parameters are originally designed for a SU PHY layer, the simulation results represent those of a SU-oriented MAC layer.

In Figs. 7-8 we plot the network throughput and efficiency for the described framework. Fig. 7 shows that MU-MIMO can offer significant increase in network throughput. However, these throughputs exhibit a ceiling, and hence the corresponding efficiencies in Fig. 8 also show a gradual decrease with M. To remedy this effect, we extend the maximum A-MPDU size to 8 times that of the 802.11n limits. This would mitigate the degradation on the ratio of preamble overhead to payload duration as the packet's data rate increases. But while this method improves performances, which are also plotted in Figs. 7-8, the downward trend in efficiencies persists. Evidently such performances are those of a MAC layer that is not optimized for its underlying PHY layers; otherwise, as discussed in Section III, the throughputs will unboundedly scale with MU capacity. Note that we have found the general trends displayed of the plots in Figs. 7-8 remain unchanged when the number of clients in the network is varied.

These results clearly indicate that the current SU-oriented MAC layer in 802.11 cannot fully utilize the PHY layer's MU capacity. As expected from what we pointed out in Section III, when the MU capacity increases, the number of idle slots has to correspondingly decrease to achieve the optimal throughput. Thus, the fundamentals for 802.11ac can be enhanced with an appropriate cross-layer design approach, in particular, by applying the theory of CSMA with MPR that we discussed.

[11]The particular parameters we took from D0.1 have not been changed in D4.0, the latest and nearly final draft version at this paper's press time (Nov. 2012).

TABLE IV
DETAILS OF OUR 802.11AC WLAN SIMULATION FRAMEWORK

Simulation parameter	***Value***	***Remarks***
Operating mode	Infrastructure mode	Downlink and uplink transmissions cannot concurrently coexist; i.e., when STA(s) transmit together with the AP, none of their frames are correctly received.
No. of AP	1	AP transmissions to the clients are called downlink [6].
No. of clients	29	Client transmissions to the AP are called uplink [6].
No. of simultaneous transmissions	Downlink: M Uplink: 1 to M	M is the maximum no. of simultaneous transmissions allowable by the MU-MIMO PHY layer. Note that, unlike in our simulations, TGac has ultimately decided to define only downlink MU-MIMO in the current 802.11ac standard, despite early proposals to also adapt uplink MU-MIMO and leaving this for potential consideration in future amendments.
Channel width and PHY data rates	See Table III	Only those signaled by 64-QAM at rate 5/6 are used here. Each client's link has the same data rate. SNR is assumed sufficient enough for a negligible PER.
PHY header duration	16 μs	This is based on the header structure proposed in [11], i.e., the SIG fields.
No. of LTFs in preamble	Depends on no. of spatial streams, per [11]	This is necessary for receivers to estimate the MIMO channel properly (cf. 802.11n [6]). These Long Training Fields (LTFs) are those proposed in [11].
MAC backoff policy	As in 802.11n	802.11ac has adapted essentially the same backoff policy as in 802.11n. Note that these parameters were designed for a SU-oriented PHY.
RTS-CTS frame exchange	Enforced for all TXOPs	Transmitted with 1 spatial stream by BPSK at rate 1/2 (cf. Table III), as in practice to maximize effect against hidden nodes.
TXOP duration	3.008 ms	This is the maximum specified for OFDM-based PHY layers by 802.11 [6].
Frame aggregation	Enforced for all data frames	Each TXOP is packed with as many A-MPDUs as possible. In 802.11 parlance, an aggregated data frame is known as aggregated MPDU (A-MPDU).
MPDUs per A-MPDU	64	This is the maximum specified by 802.11 [6].
A-MPDU size	65 535 bytes	This is the maximum specified by 802.11 [6]; 802.11ac is proposing a maximum 16 times of that.
Acknowledgement (ACK) policy	Implicit and Compressed BlockACK	We assume the M clients ACK simultaneously via uplink MU-MIMO. Although currently in 802.11ac each client's ACK is sent individually to the AP, our results will still show the general expected performance trend. Implicit and Compressed BlockACK minimizes ACK overheads.
CSI feedback or channel sounding	Not included	Obtaining channel state information (CSI), which is essential for MU-MIMO, can incur small overheads. Since our goal is on showing the general expected performance, we have assumed accurate CSI is known at the transmitter.
Network traffic model	STAs always saturated with packets to send	This forces the system to operate in a critical region where it is on the verge of drifting into instability, i.e, throughput approaching zero and delay approaching infinity, allowing us to examine a protocol's fundamental performance.
No. of iterations in simulation	100 000	Running more, e.g. 1 000 000, did not exhibit any significant differences in the scale of our plots.

APPENDIX A: STATE TRANSITION PROBABILITIES OF MARKOV CHAIN $\{X_t\}$

Let $P_{i,k}$ be the conditional probability that, if j packets are backlogged at the start of the current TP, then k packets will be backlogged at the start of the next TP. With Λ_k denoting the probability there are exactly k new arrivals during the $1/\tau$ slots when there are transmissions (e.g., $\Lambda_0 = \hat{\lambda}_0^{1/\tau}$), we have:

- $P_{0,0} = \hat{\lambda}_0 + \sum_{n=1}^{\infty} \hat{\lambda}_n C_{n,n} \Lambda_0$
- $P_{0,k} = \sum_{n=1}^{\infty} \hat{\lambda}_n \sum_{s=\max(0,n-k)}^{n} C_{n,s} \Lambda_{k-(n-s)}$, for $k \geq 1$;
- $P_{i,i-k} = \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{j=k}^{i} B_i(j) \sum_{s=k}^{j} C_{n+j,n+s} \Lambda_{s-k}$, for $i \geq 1$ and $1 \leq k < i$;
- $P_{i,i} = \hat{\lambda}_0 B_i(0) + \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{j=0}^{i} B_i(j) \sum_{s=0}^{j} C_{n+j,n+s} \Lambda_s$, for $i \geq 1$; and
- $P_{i,i+k} = \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{j=k}^{i} B_i(j) \sum_{s=\max(0,n-k)}^{n+j} C_{n+j,s} \Lambda_{k-(n-s)}$, for $i \geq 1$ and $k \geq 1$.

APPENDIX B: PROOF OF (3) IS BOUNDED BELOW

$$
\begin{aligned}
D(i) &= -\sum_{k=1}^{i} k P_{i,i-k} \\
&= -\sum_{k=1}^{i} k \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{j=k}^{i} B_i(j) \sum_{s=k}^{j} C_{n+j,n+s} \Lambda_{s-k} \\
&= -\sum_{j=1}^{i} B_i(j) \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{s=1}^{j} \sum_{k=1}^{s} k C_{n+j,n+s} \Lambda_{s-k} \\
&> -\sum_{j=1}^{i} B_i(j) \sum_{n=0}^{\infty} \hat{\lambda}_n \sum_{s=1}^{j} s C_{n+j,n+s} \sum_{k=1}^{s} \Lambda_{s-k} \\
&> -\sum_{j=1}^{i} B_i(j) \sum_{n=0}^{\infty} \hat{\lambda}_n \mathcal{C}_{n+j} \\
&> -L,
\end{aligned}
$$

where $P_{i,i-k}$ and definition of Λ_k are given in Appendix A.

Appendix C: Proof of Lemma 1

Proof: We first establish some properties to be used in the proof. Consider an $x_0 = \arg\sup_{x\geq 0} \Upsilon(x, \{\mathcal{C}_n\})$ for some general $\{\mathcal{C}_n\}$ where $x_0 < \infty$. The concavity assumption we have on expected transmission successes $\{\mathcal{C}_n\}$ implies that $\Upsilon(x, \{\mathcal{C}_n\})$ is concave down, and thus x_0 is where the global maximum occurs. Naturally, $\frac{d\Upsilon(x,\{\mathcal{C}_n\})}{dx}\big|_{x=x_0} = 0$, which after some simplification of terms can be written as

$$\frac{d\Upsilon(x_0,\{\mathcal{C}_n\})}{dx} = -\frac{e^{-x}}{1+\tau-e^{-x}}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^n}{n!}+\frac{e^{-x}}{1+\tau}\sum_{n=1}^{\infty}\mathcal{C}_n\frac{x^{n-1}}{(n-1)!} = 0. \quad (15)$$

Interestingly, the first term of $\frac{d\Upsilon(x,\{\mathcal{C}_n\})}{dx}$ in (15) is exactly the expression for $-\Upsilon(x, \{\mathcal{C}_n\})$. Thus, if we denote $C(x, \{\mathcal{C}_n\})$ to be the second term of $\frac{d\Upsilon(x,\{\mathcal{C}_n\})}{dx}$ in (15), we will have

$$\sup_{x\geq 0}\Upsilon(x, \{\mathcal{C}_n\}) = \Upsilon(x_0, \{\mathcal{C}_n\}) = C(x_0, \{\mathcal{C}_n\}). \quad (16)$$

Since x_0 is the global maximum, $\frac{d\Upsilon(x,\{\mathcal{C}_n\})}{dx} > 0$ for $x < x_0$, so it follows from (15) that

$$\Upsilon(x, \{\mathcal{C}_n\}) < C(x, \{\mathcal{C}_n\}) \quad \text{for } x < x_0. \quad (17)$$

Similarly, because $\frac{d\Upsilon(x,\{\mathcal{C}_n\})}{dx} < 0$ for $x > x_0$, $\Upsilon(x, \{\mathcal{C}_n\}) > C(x, \{\mathcal{C}_n\})$ for $x > x_0$.

We now construct an expected transmission success $\{\mathcal{C}_n^{(2)}\}$ that will complete the proof. Consider a $\{\mathcal{C}_n^{(2)}\}$ with $\mathcal{C}_n^{(2)} > \mathcal{C}_n^{(1)}$ for all $n \in \mathcal{N}$ and $\mathcal{C}_n^{(2)} = \mathcal{C}_n^{(1)}$ for all $n \notin \mathcal{N}$, where $\mathcal{N} \in \{n : n > x_1 + 1\}$ and $\mathcal{N} \neq \emptyset$. In other words, $\{\mathcal{C}_n^{(2)}\}$ contains one or more $\mathcal{C}_n^{(2)}$ that are greater than $\mathcal{C}_n^{(1)}$ at some (integer) indices $n > x_1 + 1$, while the rest of $\{\mathcal{C}_n^{(2)}\}$ is the same as $\{\mathcal{C}_n^{(1)}\}$. Further construct $\{\mathcal{C}_n^{(2)}\}$ such that $\lim_{n\to\infty}\mathcal{C}_n^{(2)} < \sup_{n\geq 1}\{\mathcal{C}_n^{(2)}\}$. Then we know from the proof of Theorem 3 that $\Upsilon(x, \{\mathcal{C}_n^{(2)}\})$ will achieve its supremum at some finite x_2. From the construction of $\{\mathcal{C}_n^{(2)}\}$ we can readily see that it has stronger MPR strength than $\{\mathcal{C}_n^{(1)}\}$. So, what remains to be shown is that $x_2 > x_1$ with this $\{\mathcal{C}_n^{(2)}\}$.

Obviously if $x_1 = 0$, which happens when $\mathcal{C}_n^{(1)} = 0$ for all n, then we have $x_2 > x_1$ already because by our construction of $\{\mathcal{C}_n^{(2)}\}$ we can only have $x_2 > 0$. Thus, we need consider only those $\{\mathcal{C}_n^{(1)}\}$'s that give $x_1 \in (0, \infty)$. Let us compare the values of $\Upsilon(x, \{\mathcal{C}_n^{(2)}\})$ and $C(x, \{\mathcal{C}_n^{(2)}\})$ when evaluated at $x = x_1$. Let $\{\delta_n\} = \{\mathcal{C}_n^{(2)} - \mathcal{C}_n^{(1)}\}$. Then $\Upsilon(x, \{\mathcal{C}_n^{(2)}\}) = \Upsilon(x, \{\mathcal{C}_n^{(1)}\}) + \frac{e^{-x}}{1+\tau-e^{-x}}\sum_{n\in\mathcal{N}}\delta_n\frac{x^n}{n!}$ and $C(x, \{\mathcal{C}_n^{(2)}\}) = C(x, \{\mathcal{C}_n^{(1)}\}) + \frac{e^{-x}}{1+\tau}\sum_{n\in\mathcal{N}}\delta_n\frac{x^{n-1}}{(n-1)!}$, where in both expressions all the δ_n terms are greater than zero. Because the supremum of $\Upsilon(x, \{\mathcal{C}_n^{(1)}\})$ is attained at x_1, from (16) we know that $\Upsilon(x_1, \{\mathcal{C}_n^{(1)}\}) = C(x_1, \{\mathcal{C}_n^{(1)}\})$. Observe that for all $x \in (0, \infty)$, since $\frac{x^n}{n!} < \frac{x}{1+x}\frac{x^{n-1}}{(n-1)!}$ when $n > x+1$ and $\frac{x}{1+x} < \frac{1+\tau-e^{-x}}{1+\tau}$ for any $\tau > 0$, it will also be true that $\frac{1}{1+\tau-e^{-x}}\frac{x^n}{n!} < \frac{1}{1+\tau}\frac{x^{n-1}}{(n-1)!}$ when $n > x+1$ and for any $\tau > 0$. Thus we have,

$$\begin{aligned}&\Upsilon(x_1, \{\mathcal{C}_n^{(1)}\}) + \frac{e^{-x_1}}{1+\tau-e^{-x_1}}\sum_{n\in\mathcal{N}}\delta_n\frac{x_1^n}{n!}\\ <\ &C(x_1, \{\mathcal{C}_n^{(1)}\}) + \frac{e^{-x_1}}{1+\tau}\sum_{n\in\mathcal{N}}\delta_n\frac{x_1^{n-1}}{(n-1)!},\end{aligned}$$

i.e., $\Upsilon(x_1, \{\mathcal{C}_n^{(2)}\}) < C(x_1, \{\mathcal{C}_n^{(2)}\})$. Finally, given this fact and that the supremum of $\Upsilon(x, \{\mathcal{C}_n^{(2)}\})$ is attained at x_2, it follows from (17) that we must have $x_1 < x_2$. □

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Disclaimer

The opinions in this paper represent only the personal views of the authors, not necessarily those of their employers.

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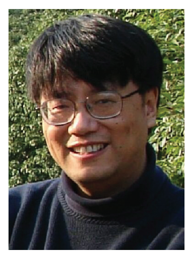

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