Maximum Throughput Region of Multiuser Cognitive Access of Continuous Time Markovian Channels

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***Abstract*—The problem of cognitive access of multiple primary channels by multiple cognitive users is considered. The primary transmission on each channel is modeled by a continuous time Markov on-off process. Cognitive access of the primary channels is realized via channel sensing. Each cognitive user adopts a slotted transmission structure, senses one channel in each slot and makes the transmission decision based on the sensing outcome. The cognitive transmissions in each channel are subject to collision constraints that limit their interference to the primary users.**

The maximum throughput region of this multiuser cognitive network is characterized by establishing inner and outer bounds. Under tight collision constraints, the inner bound is obtained by a simple orthogonalized periodic sensing with memoryless access policy and its generalizations. The outer bound, on the other hand, is obtained by relating the sum throughput with the interference limits. It is shown that when collision constraints are tight, the outer and inner bounds match. This maximum throughput region result is further extended by a generalized periodic sensing scheme with a mechanism of timing sharing. Under general collision constraints, another outer bound is obtained via Whittle's relaxation and another inner bound obtained via Whittle's index sensing policy with memoryless access. Packet level simulations are used to validate the analytical performance prediction.

***Index Terms*—Cognitive radio networks, dynamic spectrum access, opportunistic multiaccess, constrained MDP, maximum throughput region.**

I. Introduction

In a hierarchical cognitive network [3], primary and secondary users coexist with different access priorities. The licensed primary users have high priority and transmit in their dedicated channels whenever there are packets in their queues, oblivious to the presence of cognitive users. Secondary or cognitive users, on the other hand, have low access priority and can only transmit at the times and in the channels where the primary users are currently not present. Such transmission opportunities—the so-called white space—exist in the spectrum when primary traffic is bursty. For example, experiments in Voice over IP traffic suggest the existence of significant temporal white space [4].

We consider the problem of multiuser cognitive access in a hierarchical cognitive network with N primary channels and $K \leq N$ secondary cognitive users. The cognitive users try to capture the transmission opportunities while limiting their interference to the primary users within a prescribed level. The cognitive users are capable of channel sensing and follow a slotted transmission structure. In particular, a cognitive user chooses to sense one of the primary channels in the beginning of each slot and makes the transmission decision accordingly. Each cognitive user exploits the transmission opportunities individually, *i.e.*, no communications are assumed among the cognitive users to share their observations or decisions.

For this multiuser hierarchical cognitive network, a performance measure is the throughput vector, with each component being the throughput of an individual cognitive user. Of interest is characterizing the maximum throughput region and optimal sensing and transmission policies.

To obtain a sensing and transmission policy optimal with respect to throughput region, one needs to optimize jointly the *sensing policy* that specifies which channel to sense and the *access policy* that determines whether to transmit. This joint optimization across multiple users belongs to the class of decentralized and constrained Markov decision process, and no tractable solution exists in general. However, the multiuser cognitive access problem has the special property that the evolution of the primary transmission processes is not affected by the actions of the cognitive users. This property coupled with the special structure of constraints makes it possible that simple yet optimal cognitive access policies exist. The throughput region results presented in this paper provide a theoretical limit of the quantity of service that can be delivered by the multi-channel multiuser cognitive network. The optimal cognitive access scheme presented in this paper also provides a practical access mechanism for secondary users.

A. Summary of Results

In this paper we obtain the maximum throughput region of a cognitive network with continuous time Markovian primary traffic. To this end, we first obtain two outer bounds of the maximum throughput region, one via upper bounding the sum throughput by the interference limits and the other via a relaxed problem with centralized control and no interference constraints. The first bound is tighter for more stringent collision constraints whereas the second is tighter for loose collision constraints.

Next we focus on achievable schemes that also define the inner bounds. Under tight collision constraints, we show that the multiuser policy orthogonalized periodic sensing with memoryless access, first proposed in [5], as well as its generalization that allows a cognitive user to tune the fraction

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of time spent on each channel according to the collision limits, achieves the maximum throughput region. For general collision limits, inner bound is obtained via Whittle's index sensing policy with memoryless access. This inner bound is shown to be throughput optimal under all collision limits for single user network with homogeneous channels and equal collision limits. For homogeneous primary networks with multiple cognitive users, we show that the Whittle's index sensing policy coupled with memoryless access extends the "tight collision regime" where the maximum throughput region is known through a mechanism of time sharing. These results provide so far the most complete characterization of the maximum throughput region. What left unknown is the case in which different primary users have different traffic pattern and collision constraints, and the collision constraints do not satisfy the "tight" classification.

Part of our results were presented in [1], [2], and this paper presents a more complete and significantly improved results. Specifically, the following results appear in this paper for the first time: a generalized periodic sensing with memoryless access scheme for heterogeneous channels; an index sensing with memoryless access scheme based on an application of Whittle's index policy, which has better performance when collision constraints are loose; and a new outer bound based on a centralized restless multi-armed bandit formulation using Whittle's relaxation.

B. Related Work

Single user medium access in a hierarchical cognitive network was first considered by the authors of [6], in which a slotted transmission model was adopted for both primary users and a single cognitive user. The optimal sensing policy is shown to be a myopic policy [7], [8] for homogeneous, positively correlated Markov channels. A separation principle, *i.e.*, the sensing and access policies can be designed separately without loss of optimality, is established under a per slot interference constraint in [9].

Generalizations for the multiuser cognitive network when the primary users has slotted traffic (i.i.d. over time) can be formulated as a problem of non-Bayesian multi-armed bandit with multiple players [10], [11], [12]. The case of slotted Markovian traffic with unknown parameters is considered in [13]. In addition to the modeling difference of these slotted schemes, these results do not provide characterizations of maximum rate region. Indeed, the exact characterization of the maximum throughput region for the slotted case appears to be unknown even when the traffic parameters are known. (The Whittle's index sensing policy and the Whittle's relaxation can be applied to the slotted case with known parameters to provide inner and outer bounds. However, the exact maximum throughput region seems difficult to characterize.) Another piece of work that adopts a restless Bayesian multi-armed bandit formulation in studying medium access of hierarchical cognitive network is [14], in which the sum throughput is studied with $K \leq N$ cognitive users and centralized decision. Liu and Zhao [14] explores an index policy for general restless multi-armed bandit problem proposed by Whittle in [15] on the problem with discrete primary traffic and per slot interference constraint. In this paper the Whittle's index policy is used to obtain an inner bound of the maximum throughput region for heterogeneous channels and general collision parameters.

A more realistic primary traffic model by continuous time on-off Markov processes [16], and the single user medium access of such cognitive network was first considered in [17]. Adopting a periodic sensing policy, the authors of [17] formulate a constrained Markov decision processes for the single user problem and obtain the optimal cognitive access policy. The Periodic Sensing with Memoryless Access (PS-MA) policy is proposed in [17], and the single user throughput optimality of PS-MA is established in [5]. Earlier, for the case of single primary channel, Huang, Liu, and Ding derive the structure of optimal transmission policy under a continuous time on-off channel occupancy [18]. Separation principle is established for continuous time Markov channel occupancy under a per slot interference constraint in [19].

Multiuser cognitive access for continuous time primary traffic model is considered in [20], [5]. The authors of [20] propose an ALOHA based policy, which does not require $K \leq N$ but is in general suboptimal with respect to throughput region. The multiuser OPS-MA policy is first proposed in [5] as a heuristic generalization of PS-MA to the multiuser scenario without establishing its optimality. The optimality of OPS-MA is first shown in [1].

The organization of the rest of the paper is as follows. Section II describes the model of the hierarchical cognitive network. Section III establishes two outer bounds of the maximum throughput region. Section IV and Section V develop the throughput results for single user and multiuser cognitive network, respectively. Section VI concludes the paper.

II. Network Model

A. Primary Traffic Model

We assume N parallel primary channels indexed by $i = 1, \ldots, N$ and $K \leq N$ cognitive users indexed by $k = 1, \ldots, K$. Each primary user transmits on its designated channel. The primary transmission in each channel is modeled as a continuous time Markov on-off process, with busy and idle periods exponentially distributed with mean μ_i^{-1} and λ_i^{-1}, respectively. The primary transmission processes are independent over the channels. We term the primary channels *homogeneous* if $\lambda_i = \lambda$ and $\mu_i = \mu$ for $1 \leq i \leq N$.

The generator matrix of the ith channel is given by

$$Q_i = \begin{pmatrix} -\lambda_i & \lambda_i \\ \mu_i & -\mu_i \end{pmatrix}, \quad (1)$$

and the stationary distribution for idle state is given by $v_i(0) = \mu_i/(\mu_i + \lambda_i)$. We denote the set of cognitive users by $\mathcal{K} = \{1, \ldots, K\}$.

B. Sensing and Transmission Model

The cognitive users follow a slotted sensing-before-transmission policy with slot length T in accessing the primary channels. Each cognitive user senses one out of the N channels in the beginning of each slot and makes the transmission decision. No communications among the cognitive users concerning the sensing results or transmission decisions are assumed. When a cognitive user transmits in ceratin channel in slot t, the cognitive transmission is successful if the channel is idle throughout slot t and no other cognitive users transmit in the same channel in slot t.

The cognitive access policy of each cognitive user consists of two components: sensing policy and transmission policy. The sensing policy selects a channel to sense in each slot based on the history while the transmission policy specifies the transmission probability upon idle sensing results, since busy sensing results indicate sure collisions and no successful transmission gained. Therefore we assume that the cognitive users do not transmit upon busy sensing results. The transmission probability is determined, in general, based on the history. However, in the rest of this paper we analyze cognitive access policies, for which the transmission policy only uses the current sensing result and ignores the previous history. Such transmission policy is termed Memoryless Access (MA).

C. Performance Measure and Interference Constraint

The throughput, which measures the quantity of service delivered to the cognitive users is taken to be the performance measure. Denote by $R_t^{(k)}$ the indicator that the kth cognitive user has a successful transmission in slot t. The throughput of the kth cognitive user is defined by

$$J^{(k)} = \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} R_t^{(k)}.$$

The throughput vector for the K cognitive users is given by $\mathbf{J} = (J^{(1)}, J^{(2)}, \ldots, J^{(K)})$.

Due to the low access priority, the transmissions of the cognitive users are subject to collision constraints imposed by the primary users. Specifically, the overall collision caused by the K cognitive users to the ith primary user should be capped below a collision limit γ_i. The collision for the ith primary user is the fraction of the collided time out of the total primary transmission time. Specifically, the collision for the ith primary user is defined to be the fraction of the collided slots in the slots fully or partially used by the primary user (due to the continuous time transmission process assumed for the primary users). After some manipulation it can be shown that the collision for the ith primary user defined above is

$$C_i = \kappa_i \limsup_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} 1_{\{\text{collide with PU } i \text{ in slot } t\}} \qquad (2)$$

where $\kappa_i = (1-v_i(0)\varepsilon_i)^{-1}$ is the reciprocal of the steady state probability of the ith primary user transmitting in a certain slot, $\varepsilon_i = \exp(-\lambda_i T)$ is the conditional probability of a successful transmission conditioned on an idle sensing outcome (the notation ε_i will also be used in later presentation), and $1_{\mathscr{A}}$ is the indicator function for event $\mathscr{A}$. Given collision parameter $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_N)$, collision constraint $C_i \le \gamma_i$ is imposed for each i. We speak of *equal collision constraints* if $\gamma_i = \gamma$ for $i = 1, \ldots, N$.

Our goal is to characterize the maximum throughput region of the multiuser hierarchical cognitive network. Mathematically we have the following problem formulation. The set of admissible policies $\mathbf{\Pi}(\boldsymbol{\gamma})$ is given by the set of policies that meet the interference constraints, *i.e.*, $\mathbf{\Pi}(\boldsymbol{\gamma}) = \{\pi : C_{\pi,i} \le \gamma_i, \forall i\}$. We aim to characterize the throughput region $\mathscr{J} = \bigcup_{\pi\in\mathbf{\Pi}(\gamma)} \mathbf{J}_\pi$, where $\mathbf{J}_\pi$ is the throughput vector of policy $\pi \in \mathbf{\Pi}(\boldsymbol{\gamma})$,

III. Outer Bounds of Maximum Throughput Region

A. Outer Bound $\mathscr{B}_\gamma$

The maximization of the throughput region is subject to the collision constraints imposed by the primary users. The smaller the collision parameter $\boldsymbol{\gamma}$ is, the smaller the throughput region will be. Lemma 1 formalizes this idea where outer bound $\mathscr{B}_\gamma$ is obtained by upper bounding the sum throughput of the K cognitive users with a linear combination of the collision constraints.

Lemma 1: (Outer bound $\mathscr{B}_\gamma$) The maximum throughput region $\mathscr{J}$ is outer bounded by the region

$$\mathscr{B}_\gamma = \{(y_1, \ldots, y_K) \mid \sum_{k=1}^{K} y_k \le \sum_{i=1}^{N} \varepsilon_i\phi_i\gamma_i, y_k \ge 0\} \qquad (3)$$

where

$$\phi_i \triangleq \frac{1 - v_i(0)\varepsilon_i}{1-\varepsilon_i}. \qquad (4)$$

Proof: We prove that under any admissible policy $\pi \in \mathbf{\Pi}(\boldsymbol{\gamma})$, $\boldsymbol{J}_\pi \in \mathscr{B}_\gamma$. We have assumed that the cognitive users do not transmit if their sensing result is busy. First we upper bound the total throughput of the K cognitive users by a linear combination of the allowed collisions. Specifically, by the definition of the collision for the ith primary user, we have under any admissible policy

$$\sum_{k=1}^{K} \frac{1}{1-v_i(0)\varepsilon_i} \limsup_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \le \gamma_i, \qquad (5)$$

where $C_{i,t}^{(k)}$ is the indicator of the event that *only* the kth cognitive user transmits in channel i in slot t and incurs a collision with primary user i. Note that the LHS of Eq. (5) only counts the interference to the ith primary user when a single cognitive user is transmitting while there may be other interference during the slots in which multiple cognitive users transmit in the ith primary channel.

Taking a nonnegative linear combination of the inequalities in (5), we obtain

$$\sum_{i=1}^{N} \frac{\varepsilon_i}{1-\varepsilon_i} \sum_{k=1}^{K} \limsup_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \le \sum_{i=1}^{N} \varepsilon_i\phi_i\gamma_i.$$

On the left hand side we interchange the $\limsup$ and the finite term summation over i (leading to the inequality in Eq. (6))

$$\begin{aligned} &\sum_{i=1}^{N} \frac{\varepsilon_i}{1-\varepsilon_i} \sum_{k=1}^{K} \limsup_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \\ \ge\ & \sum_{k=1}^{K} \limsup_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n}\sum_{i=1}^{N} \frac{\varepsilon_i}{1-\varepsilon_i}\mathbb{E}C_{i,t}^{(k)} \qquad (6) \\ =\ & \sum_{k=1}^{K} \limsup_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} \mathbb{E}R_t^{(k)} \qquad (7) \\ \ge\ & \sum_{k=1}^{K} J^{(k)} \end{aligned}$$

Eq. (7) follows from the fact

$$\mathbb{E}R_t^{(k)} = \sum_{i=1}^{N} \frac{\varepsilon_i}{1-\varepsilon_i}\mathbb{E}C_{i,t}^{(k)}, \tag{8}$$

which can be derived as follows. The expected number of successful transmissions cognitive user k has in slot t is

$$\mathbb{E}R_t^{(k)} = \sum_{i=1}^{N} \mathbb{P}(\mathscr{A}_i)\varepsilon_i \tag{9}$$

where $\mathscr{A}_i$ is the event that only cognitive user k transmits in channel i in slot t. On the other hand

$$\mathbb{E}C_{i,t}^{(k)} = \mathbb{P}(\mathscr{A}_i)(1-\varepsilon_i) \tag{10}$$

Substituting Eq. (10) into Eq. (9) yields Eq. (8). Therefore

$$\sum_{k=1}^{K} J^{(k)} \leq \sum_{i=1}^{N} \varepsilon_i \phi_i \gamma_i.$$

Also note that $J^{(k)} \geq 0$ for all k. This completes the proof that under any admissible policy $\pi \in \mathbf{\Pi}(\boldsymbol{\gamma})$, $\boldsymbol{J}_\pi \in \mathscr{B}_\gamma$. Therefore $\mathscr{B}_\gamma$ outer bounds $\mathscr{J}$. ■

B. Outer Bound $\mathscr{B}_K$

We obtain another outer bound $\mathscr{B}_K$ by scheduling a subset $\mathcal{K}' \subset \mathcal{K}$ of the cognitive users in a centralized manner without the collision constraints. The relaxed centralized problem falls into the category of the so called restless multi-armed bandit problem, in which a decision maker sequentially chooses k out of the pool of total N "arms" (resources evolving in a Markovian fashion) to maximize the long term average reward. In general, the decision maker should consider the state of all the N arms to make the selection decision. Whittle obtains via a Lagrangian relaxation a condition named indexability, under which he proposes a selection policy referred to as Whittle's index policy [15]. Whittle's index policy decouples the N "arm" by computing an index for each arm that only depends on its local state, and then selects the k arms with the largest indices. Meanwhile Whittle's Lagrangian relaxation produces an upper bound of the optimal value by relaxing the constraint that exactly k arm are selected each time to that on average k arm are selected in the long run.

The relaxed problem we set up for the outer bound $\mathscr{B}_K$ is formulated as follows. Consider the state of channel i at the beginning of slot t, denoted by $S_i(t)$, which takes value either busy (1) or idle (0), and evolves as a discrete time Markov chain (the skeleton of the continuous time channel state process) with transition matrix $\exp(TQ_i)$, where Q_i is the generator matrix defined in (1). At the beginning of slot t, the centralized decision maker selects $|\mathcal{K}'|$ channels and senses the channel state $S_i(t)$ for all selected channels. For each sensed channel if $S_i(t) = 0$, the decision maker assigns one cognitive user to transmit with probability 1 and obtains ε_i expected successful transmission. Let $U(t)$ denote the set of $|\mathcal{K}'|$ channels selected in slot t. The number of successful transmissions obtained in slot t is thus given by $R(t) = \sum_{i\in U(t)}(1-S_i(t))\varepsilon_i$. The performance measure of the relaxed problem is $V_\pi(k) = \mathbb{E}_\pi[\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} R(t)]$ where $k = |\mathcal{K}'|$ and π denotes the sensing policy that sequentially selects $|\mathcal{K}'|$ channels to sense in each slot.

The relaxed problem was formulated as a restless multi-armed bandit problem in [14] to study the throughput of a single cognitive user capable of sensing k channels at a time. Following Whittle's work in [15], Liu and Zhao [14] establishes the indexability condition, obtains near optimal Whittle's index sensing policy, and upper bounds the optimal value via Whittle's relaxation for this particular restless multi-armed bandit problem in medium access of hierarchical cognitive network.

We leverage the restless multi-armed bandit formulation to obtain outer bound of the maximum throughput region of the K-user cognitive network. (In Section V the restless multi-armed bandit formulation is used for an inner bound.) Write $V(k) = \sup_\pi V_\pi(k)$, the optimal value of the relaxed problem. Denote by $W(k)$ the performance of Whittle's index sensing policy and $\overline{W}(k)$ the upper bound of $V(k)$ obtained via Whittle's relaxation (see [14] for detailed computation procedures and expressions of $W(k)$ and $\overline{W}(k)$). Thus for $k = 1,\dots,K$, $W(k) \leq V(k) \leq \overline{W}(k)$. Lemma 2 provides an outer bound $\mathscr{B}_K$ of the maximum throughput region $\mathscr{J}$ using $\overline{W}(k)$, $k = 1,\dots,K$.

Lemma 2: (Outer bound $\mathscr{B}_K$) The maximum throughput region $\mathscr{J}$ is outer bounded by the region

$$\mathscr{B}_K = \{(y_1,\dots,y_K) \mid \sum_{k\in\mathcal{K}'} y_k \leq \overline{W}(|\mathcal{K}'|)\forall\mathcal{K}' \subset \mathcal{K}, y_k \geq 0\}.$$

Proof: Since the throughput region $\mathscr{J}$ is obtained under decentralized decision and interference constraints, the proof follows directly by the formulation of the relaxed problem with centralized decision and no interference constraints. ■

The outer bound $\mathscr{B}_\gamma$ is obtained via the interference constraints and increases as $\boldsymbol{\gamma}$ increases. For small $\boldsymbol{\gamma}$ $\mathscr{B}_\gamma$ is tighter than $\mathscr{B}_K$. For large $\boldsymbol{\gamma}$ the transmission probability of the cognitive users is saturated by 1 and $\mathscr{B}_K$ becomes tighter than $\mathscr{B}_\gamma$ since $\mathscr{B}_K$ exploits the limitation in the number of channels the cognitive users can sense and transmit.

IV. Maximum Throughput: Single User Network

It was established in [5] that for the single user network ($K = 1$), maximum throughput is achieved by a policy referred to as Periodic Sensing with Memoryless Access (PS-MA), when the collision constraint given by $\boldsymbol{\gamma}$ is tight. Specifically, the collision constraint given by $\boldsymbol{\gamma}$ is *tight*, if $\gamma_i \leq \gamma_i^{\text{PS}} \triangleq \frac{v_i(0)}{N\phi_i}$, for $i = 1,\dots,N$, where ϕ_i is defined in Eq. (4).

The sensing and transmission in PS-MA can be described as follows. The cognitive user senses the channels in an increasing order at the beginning of each slot, starting from an arbitrary channel. If the ith channel is sensed to be idle, the cognitive user transmits in the sensed channel with probability β_i. A sample path of the PS-MA policy is illustrated in Fig. 1(a). The transmission probability β_i is determined by the collision constraint $\boldsymbol{\gamma}$ with $\beta_i = \min\{\frac{\gamma_i N\phi_i}{v_i(0)}, 1\}$. The single user result in [5] is summarized in Proposition 1.

Proposition 1: (PS-MA, tight collision constraints [5]) Under tight collision constraints, the PS-MA policy is throughput optimal for the single user network with throughput $J_{\text{PS}} = \sum_{i=1}^N \phi_i \varepsilon_i \gamma_i$.

The appeal of memoryless probabilistic transmission extends to more general scenarios. Lemma 3 connects the performance of an arbitrary sensing policy π_S with memoryless access with $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_N)$, the expected fraction of idle sensing results under π_S, defined as

$$\theta_i \triangleq \limsup_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}), \tag{11}$$

where $\mathscr{S}_i^{(j)}$ is the event that the cognitive user senses the ith channel in slot j, $\mathscr{I}_i^{(j)}$ is the event that the ith channel is sensed to be idle in slot j, and the intersection $\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}$ is the event that the cognitive user senses the ith channel and gets an idle sensing result in slot j.

Lemma 3: (Memoryless access) Given sensing policy π_S which selects channel $i(j)$ in the jth slot, the memoryless probabilistic access can be combined with π_S with transmission probability $\beta_i = \min\{\frac{\gamma_i \phi_i}{\theta_i}, 1\}$ such that the collision constraints given by $\boldsymbol{\gamma}$ are met.

If further the limit exists

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}) = \theta_i, \tag{12}$$

then the throughput of the sensing policy π_S with memoryless access is given by $J = \sum_{i=1}^N \theta_i \varepsilon_i \beta_i$.

Proof: Verify the transmission probability β_i defined in Lemma 3 is admissible. By the definition of collision in Eq. (2), the collision for the ith primary user is

$$\begin{aligned} C_i &= \kappa_i \limsup_{n\to\infty} \frac{1}{n} \sum_{t=1}^n \mathbb{E}1_{\{\text{collide PU } i \text{ in slot } t\}} \\ &= \kappa_i \limsup_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(t)} \cap \mathscr{I}_i^{(t)})(1-\varepsilon_i)\beta_i. \end{aligned}$$

Substituting Eq. (11) into the expression of C_i yields the collision constraint

$$C_i = \frac{1}{1 - v_i(0)\varepsilon_i} \theta_i \beta_i (1-\varepsilon_i) \le \gamma_i. \tag{13}$$

Plugging the definition of ϕ_i in Eq. (4) and the transmission probability β_i chosen in Lemma 3 into Eq. (13) verifies the admissibility of the choice of β_i.

It is left to verify the throughput formula when Eq. (12) holds.

$$\begin{aligned} \sum_{i=1}^N \theta_i \varepsilon_i \beta_i &= \sum_{i=1}^N \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}) \varepsilon_i \beta_i \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}) \varepsilon_i \beta_i \\ &= \lim_{n\to\infty} \frac{1}{n} \mathbb{E} \sum_{j=1}^n R_j = J. \end{aligned}$$

■

The result for the single user network under tight collision constraint in [5] is established by showing the throughput of PS-MA matches that of a clairvoyant setting where the cognitive user senses all N channels in each slot. In this paper the upper bound $\mathscr{B}_\gamma$ and Lemma 3 enables us to extend the application of memoryless transmission to larger collision regime.

A. Extended Tight Collision Regime: Generalized PS-MA

In PS-MA policy each primary channel is sensed equally with $1/N$ fraction of time. In the situation where certain primary users can tolerate large interference while others only allow small interference, it is necessary to favor certain channels, *i.e.*, to spend larger fraction of time sensing the channels with larger tolerance. Therefore we add the fraction of time the cognitive user spends sensing channel i, denoted by τ_i, into the design and introduce the Generalized PS-MA (GPS-MA) policy. The channel sensing in GPS-MA is still periodic, with τ_i fraction of time in the ith channel. The transmission is still memoryless, *i.e.*, if the ith channel is sensed to be idle, the cognitive user transmits in the sensed channel with probability β_i. A sample path of the GPS-MA policy is illustrated in Fig. 1(b), where the cognitive user stays in Channel 1 for 3 slots, Channel 2 for 1 slot, and then Channel 3 for 2 slot, in a period of 6 slots. The performance of GPS-MA is characterized in Theorem 1.

Theorem 1: (GPS-MA extended tight collision constraints) In the extended tight collision regime defined by

$$\sum_{i=1}^N \frac{\gamma_i}{\gamma_i^{\text{PS}}} \le N, \tag{14}$$

the GPS-MA policy with the time fraction parameter

$$\tau_i = \frac{\gamma_i}{N \gamma_i^{\text{PS}}}. \tag{15}$$

and the transmission probability β_i defined in Lemma 3 is throughput optimal for the single user network.

It can be verified that the tight collision regime for PS-MA is a subset of the extended tight collision regime for GPS-MA. PS-MA requires the collision constraints on all channels to be small while GPS-MA just needs the weighted sum of the collision constraints to be small.

Proof: The time fraction defined in Eq. (15) is admissible since $\sum_{i=1}^N \tau_i \le 1$ due to Eq. (14).

In order to apply Lemma 3, we compute θ_i according to the structure of GPS-MA

$$\begin{aligned} \theta_i &= \limsup_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}) \\ &= \limsup_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \mathbb{P}(\mathscr{S}_i^{(j)}) \mathbb{P}(\mathscr{I}_i^{(j)}) \qquad (16) \\ &= \lim_{n\to\infty} \frac{1}{n} \sum_{j=1, i(j)=i}^n \mathbb{P}(\mathscr{I}_i^{(j)}) \qquad (17) \\ &= \tau_i \lim_{n\to\infty} \frac{1}{n\tau_i} \sum_{j=1, i(j)=i}^n \mathbb{P}(\mathscr{I}_i^{(j)}) \\ &= \tau_i v_i(0), \end{aligned}$$

where Eq. (16) is due to the fact that the event $\mathscr{S}_i^{(j)}$ is deterministic under GPS-MA (deterministic channel sensing) and therefore independent of the event $\mathscr{I}_i^{(j)}$, and the limit in Eq. (17) exists because the limiting distribution of the channel state exists.

By Lemma 3 $\beta_i = \min\{\frac{\gamma_i\phi_i}{\tau_i v_i(0)}, 1\}$. Under extended tight collision constraints Eq. (15) implies $\beta_i = \frac{\gamma_i\phi_i}{\tau_i v_i(0)}$. Lemma 3 and the fact that the limit in Eq. (12) exists imply that the throughput of GPS-MA under the extended tight collision constraints is

$$J_{\text{GPS}} = \sum_{i=1}^{N} \theta_i \varepsilon_i \beta_i = \sum_{i=1}^{N} \phi_i \varepsilon_i \gamma_i. \qquad (18)$$

Combining the outer bound $\mathscr{B}_\gamma$ and Eq. (18) completes the proof. ■

B. Index Sensing with Memoryless Access Policy

In this Section we combine the Whittle's index sensing policy with memoryless access (IS-MA) to give lower bound of the maximum throughput. In the single user network, the outer bounds $\mathscr{B}_\gamma$ and $\mathscr{B}_K$ reduce to the upper bound $\min\{\overline{W}(1), \sum_{i=1}^{N} \varepsilon_i\phi_i\gamma_i\}$.

Similar result to Theorem 1 holds for IS-MA, *i.e.*, IS-MA achieves the maximum throughput when $\boldsymbol{\gamma}$ is small. However, it is difficult to compute θ_i for Whittle's index sensing policy and therefore difficult to characterize the corresponding optimal regime for IS-MA. When $\boldsymbol{\gamma}$ is sufficiently large, $\overline{W}(1) = \min\{\overline{W}(1), \sum_{i=1}^{N} \varepsilon_i\phi_i\gamma_i\}$ and the maximum throughput is bounded from below by $W(1)$, the throughput of IS-MA with transmission probability 1 and bounded from above by $\overline{W}(1)$. It was observed in [14] via simulation that the gap between $W(1)$ and $\overline{W}(1)$ is small.

In the special case where the primary channels are homogeneous, it is shown in [14] that $\overline{W}(1) = W(1)$ and the Whittle's index sensing policy coincides with a sensing policy proposed earlier by Zhao, Krishnamachari, and Liu in [7] for homogeneous primary traffic in discrete time with per slot collision constraints. IS-MA policy for homogeneous channels is described below.

When the channels are homogeneous, the cognitive user first fixes an ordered list of the N channels and the transmission probability β_i. To start, the cognitive user senses the first channel in the list. If idle, the cognitive user transmits with the corresponding β. The cognitive user then keeps sensing the first channel on the list until the first busy sensing result, after which the cognitive user switches to the next channel in the list and transmits probabilistically. The cognitive user moves down along the list as described above until reaching the last channel. After the first busy sensing result from the last channel, the cognitive user goes back to the first channel again. Equivalently, the cognitive user stays in the same channel and transmits probabilistically, if the channel is sensed to be idle, and moves down along the ordered list otherwise. With homogeneous channels the IS-MA policy induces a $N \times 2^N$ state Markov chain having state space $\{1, \ldots, N\} \times \{0, 1\}^N$ with state vector $(I, \boldsymbol{X})$, where I indicates the current channel index and $\boldsymbol{X}$ indicates the state of the N channels. Specifically, if the state vector for slot t $(I_t, \boldsymbol{X}_t)$ is known, then the sensing result of the cognitive user in slot t is $\boldsymbol{X}_t(I_t)$. Following the sensing policy of IS-MA, the channel index for slot $t+1$ can be determined according to $\boldsymbol{X}_t(I_t)$. Therefore the distribution of $(I_{t+1}, \boldsymbol{X}_{t+1})$ only depends on $(I_t, \boldsymbol{X}_t)$ and is conditionally independent of the previous history. A sample path of IS-MA is illustrated in Fig. 1(c).

Denote by $\omega(i, \boldsymbol{x})$ the stationary distribution of the Markov chain induced by IS-MA where i is the channel index the cognitive user currently senses and $\boldsymbol{x}$ is the current state for the N channels. Let $\gamma_i^{\text{IS}} \triangleq \frac{1}{\phi_i}\sum_{x_i=0} \omega(i, \boldsymbol{x})$, where x_i is the ith component of vector $\boldsymbol{x}$. Due to homogeneity $\gamma_i^{\text{IS}} = \gamma^{\text{IS}}$. Theorem 2 shows that the single user IS-MA policy matches the outer bound $\mathscr{B}_\gamma$ for larger $\boldsymbol{\gamma}$ compared with PS-MA for homogeneous channels.

Theorem 2: (IS-MA, homogeneous channels) For homogeneous channels, IS-MA is throughput optimal when $\gamma_i \le \gamma^{\text{IS}}$. Also it holds that $\gamma^{\text{IS}} > \gamma^{\text{PS}}$.

The fact that $\gamma^{\text{IS}} > \gamma^{\text{PS}}$ implies that the "optimal" regime (matching $\mathscr{B}_\gamma$) of IS-MA is larger than that of PS-MA for homogeneous channels.

Proof: The finite state Markov chain structure induced by IS-MA guarantees the existence of the limit in Eq. (12), which is computed below

$$\begin{aligned} \theta_i &= \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \mathbb{P}(\mathscr{S}_i^{(j)} \cap \mathscr{I}_i^{(j)}) \\ &= \omega(\mathscr{S}_i \cap \mathscr{I}_i) = \sum_{x_i=0} \omega(i, \mathbf{x}) \end{aligned}$$

where $\omega(\cdot)$ is the stationary distribution of the induced Markov chain. The intersection $\mathscr{S}_i \cap \mathscr{I}_i$ is the event that the cognitive user senses the ith primary channel and gets an idle sensing result, *i.e.*, $x_i = 0$.

Therefore by Lemma 3 and the definition of γ^{IS} the transmission probability β_i is given by $\beta_i = \min\{\frac{\gamma_i}{\gamma^{\text{IS}}}, 1\}$. The minimum is assumed by the first term when $\gamma_i \le \gamma^{\text{IS}}$. Therefore the throughput of IS-MA is given by

$$J_{\text{IS}} = \sum_{i=1}^{N} \theta_i \exp(-\lambda T)\beta_i = \phi \exp(-\lambda T) \sum_{i=1}^{N} \gamma_i \qquad (19)$$

when $\gamma_i \le \gamma^{\text{IS}}$. Therefore combining the outer bound $\mathscr{B}_\gamma$ and Eq. (19), the IS-MA policy is throughput optimal for the single user network when $\gamma_i \le \gamma^{\text{IS}}$.

It is left to prove that for homogeneous channels, $\gamma^{\text{IS}} > \gamma^{\text{PS}}$, or equivalently,

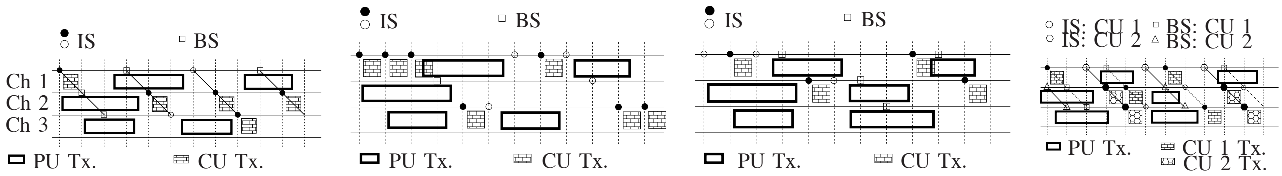
$$\sum_{x_i=0} \omega(i, \boldsymbol{x}) = \omega(\mathscr{S}_i \cap \mathscr{I}_i) > \frac{v(0)}{N}. \qquad (20)$$

Lower bounds on $\omega(\mathscr{S}_i \cap \mathscr{I}_i)$ is established for homogeneous channels in [7]. Some algebra shows that the lower bound there is greater than $\frac{v(0)}{N}$. ■

For homogeneous channels with equal collision constraints we establish the single user throughput optimality of IS-MA in the following corollary.

Corollary 1: For homogeneous channels with equal collision constraints $\gamma_i = \gamma$, IS-MA is throughput optimal in the set of all admissible policies $\boldsymbol{\Pi}(\boldsymbol{\gamma})$ for all $\gamma \in [0, 1]$.

Proof: For the regime $\gamma \le \gamma^{\text{IS}}$, Theorem 2 gives the throughput optimality.



(a) PS-MA. (b) GPS-MA. (c) IS-MA. (d) OPS-MA.

Fig. 1. Illustration of access policies. Filled(Open) circle: cognitive user decides to(not to) transmit. IS(BS): idle(busy) sensing.

For the regime $\gamma > \gamma^{\mathrm{IS}}$, the cognitive user transmits with probability 1 on each channel since $\beta_i = \min\{\frac{\gamma_i}{\gamma^{\mathrm{IS}}}, 1\}$. The optimality of IS-MA reduces to the known result obtained in [7], [8] if we can establish that the underlying Markov chain for the channel state process is positive correlated.

The transition matrix of the Markov chain is given by

$$e^{TQ} = \frac{1}{\lambda+\mu}\begin{pmatrix} \mu + \lambda e^{-(\lambda+\mu)T} & \lambda - \lambda e^{-(\lambda+\mu)T} \\ \mu - \mu e^{-(\lambda+\mu)T} & \lambda + \mu e^{-(\lambda+\mu)T} \end{pmatrix}.$$

The underlying Markov chain is positive correlated since $\mathbb{P}(\text{idle} \mid \text{idle}) \geq \mathbb{P}(\text{idle} \mid \text{busy})$. ■

Remark In this section the acknowledgement of the cognitive transmission is not included in the sensing and transmission decision. With the acknowledgement taken into consideration, the optimality of GPS-MA in the extended tight collision regime is not affected. However, the IS-MA policy for homogeneous channels needs the following modification. The cognitive user will stay in the same channel and transmits probabilistically, if the channel is sensed to be idle and the acknowledgement is success, and moves down along the ordered list if the channel is sensed to be busy or the acknowledgement is failure. Similar result to Corollary 1 holds for the problem with the acknowledgement in consideration.

C. Simulation Results: Single User Case

We show the simulation results for cognitive network with two primary channels and a single cognitive user. The slot length in the simulation is taken to be $T = 0.25$ms. The channel parameters we use are as follows.

1) For heterogeneous channels, $\mu = [1/1, 1/1.43]\mathrm{ms}^{-1}$, $\lambda = [1/4.2, 1/3.23]\mathrm{ms}^{-1}$.
2) For homogeneous channels, $\mu = 1/2\mathrm{ms}^{-1}$, $\lambda = 1/3\mathrm{ms}^{-1}$.

Fig. 3(a) depicts the throughput versus the collision constraint parameter γ_1 for Channel 1, while fixing the collision constraint parameter for Channel 2 $\gamma_2 = 0.02$, for PS-MA and GPS-MA. The plot shows that the throughput of PS-MA matches the outer bound $\mathscr{B}_\gamma$ until a certain point whereas the throughput of GPS-MA matches the upper bound further than PS-MA by spending large fraction of time sensing Channel 1. Fig. 3(b) depicts the throughput versus the collision constraint parameter γ, for PS-MA and IS-MA with homogeneous channels. It can be observed from the plot that, for homogeneous channels, the throughput of both PS-MA and IS-MA match the outer bound until certain breakpoint points and the breakpoint of IS-MA is larger than that of PS-MA, *i.e.*, $\gamma^{\mathrm{PS}} < \gamma^{\mathrm{IS}}$. This validates that the throughput of IS-MA is superior to that of PS-MA for homogeneous channels.

V. Maximum Throughput Region: Multiuser Network

A. Tight and Extended Tight Collision Constraints

For the multiuser network, referred to as the Orthogonalized Periodic Sensing with Memoryless Access, OPS-MA implements PS-MA for each cognitive user in an orthogonal manner. The K cognitive users periodically sense the channels using different sensing phases. See Fig. 1(d) for an illustration. Since in the network there are fewer cognitive users than primary channels ($K \leq N$), the K cognitive users can be fit in K orthogonal sensing phases and precludes collisions among the cognitive users.

Upon idle sensing results on the ith channel, the kth cognitive user transmits with probability $\beta_i^{(k)}$. OPS-MA splits the collision constraints among the cognitive users. Specifically, let $\gamma_i^{(k)} = \gamma_i \alpha_i^{(k)}$ where $\gamma_i^{(k)}$ is the equivalent collision limit for cognitive user k and $\alpha_i^{(k)}$ is such that $\alpha_i^{(k)} \geq 0$, and $\sum_{k=1}^{K} \alpha_i^{(k)} \leq 1$. Different splitting coefficients $\boldsymbol{\alpha}$ yield different points in the throughput region $\mathscr{J}$. Theorem 3 states that this collision-split sharing achieves the maximum throughput region under tight collision constraints.

Theorem 3: (OPS-MA, tight collision constraints) Under tight collision constraints, *i.e.*, $\gamma_i \leq \gamma_i^{\mathrm{PS}}$ for all i, the maximum throughput region is given by $\mathscr{J} = \mathscr{B}_\gamma$, and achieved by OPS-MA with all possible $\boldsymbol{\alpha}$ and $\beta_i^{(k)}$ determined by $\gamma_i^{(k)}$ with the procedure in Lemma 3.

Remark In implementation of OPS-MA, a cognitive user can first flip a coin for transmission decision and then sense the channel only if it decides to transmit. This modification still achieves the maximum throughput region with the gain of less channel sensing energy.[1]

Proof: First determine the transmission probabilities $\beta_i^{(k)}$'s for given $\boldsymbol{\gamma}$. Following similar arguments to that of Lemma 3, the collision constraint can be written in detail as

$$\frac{v_i(0)(1-\varepsilon_i)}{N(1-v_i(0)\varepsilon_i)}\beta_i^{(k)} \leq \alpha_i^{(k)}\gamma_i. \tag{21}$$

where the right hand side gives the collision allowed for the kth cognitive user.

Write $\beta_i = \gamma_i N\phi_i / v_i(0)$ and choose the transmission probability $\beta_i^{(k)} = \alpha_i^{(k)}\beta_i$ where the $\alpha_i^{(k)}$'s can be interpreted as back-off coefficients to guarantee the collision constraints to be met. Each cognitive user transmits less to accommodate other cognitive users.

Then we show that OPS-MA achieves the region $\mathscr{B}_\gamma$. First fix a set of $\alpha_i^{(k)}$'s and analyze the throughput vector $\mathbf{J}$ achieved by OPS-MA with parameter $\alpha_i^{(k)}$.

[1]We would like to thank an anonymous reviewer for this suggestion.

The throughput of the kth cognitive user under OPS-MA is

$$\begin{aligned} J^{(k)} &= \liminf_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{j=1}^{n} R_j^{(k)} \\ &= \liminf_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{N}\mathbb{P}(\mathscr{S}_{k,i}^{(j)}\cap\mathscr{I}_{k,i}^{(j)})\varepsilon_i\beta_i\alpha_i^{(k)} \\ &= \sum_{i=1}^{N}\liminf_{n\to\infty}\frac{1}{n}\sum_{j=1,i_k(j)=i}^{n}\mathbb{P}(\mathscr{I}_{k,i}^{(j)})\varepsilon_i\beta_i\alpha_i^{(k)} \\ &= \frac{1}{N}\sum_{i=1}^{N} v_i(0)\varepsilon_i\beta_i\alpha_i^{(k)} \quad (22)\\ &= \sum_{i=1}^{N}\phi_i\varepsilon_i\gamma_i\alpha_i^{(k)}. \quad (23)\end{aligned}$$

where $i_k(j)$ denotes the channel index the kth cognitive user is at in slot j, Eq. (22) follows from the structure of OPS-MA (each cognitive user senses each channel once in every N time slots) and Eq. (23) follows from the tight collision assumption.

Eq. (23) gives a linear relation between the throughput vector $\mathbf{J} = (J^{(1)}, J^{(2)}, \ldots, J^{(K)})$ and OPS-MA parameter $\boldsymbol{\alpha}$. Then we apply Farkas lemma to prove that for any $(y_1, \ldots, y_K)$ in $\mathscr{B}_\gamma$ there exist $\alpha_i^{(k)} \geq 0$ such that $\sum_{k=1}^{K}\alpha_i^{(k)} \leq 1$ and $y_k = \sum_{i=1}^{N}\varepsilon_i\phi_i\gamma_i\alpha_i^{(k)}$. We omit the detail of the application of Farkas lemma. Combining the outer bound $\mathscr{B}_\gamma$ and the achievability proves Theorem 3. ∎

The throughput region under tight collision constraints is a polytope. Specifically, it is the convex hull of the origin and the K points corresponding to exclusively serving one single cognitive user. A point in the positive orthant is in the throughput region if and only if the total throughput of the K cognitive users is below the upper bound $\sum_{i=1}^{N}\varepsilon_i\phi_i\gamma_i$.

Remark The orthogonalization of the cognitive users is not trivial and we have addressed the procedure of orthogonalization in another paper [21] for the case $K \leq N$. On the other hand, if there are more cognitive users than primary users ($K > N$), the orthogonalization becomes difficult and the $K > N$ case requires fundamentally different schemes from the techniques in this paper to deal with the contention among cognitive users.

Theorem 3 is valid under tight collision constraints. Corollary 2 treats extended tight collision constraints via time sharing of GPS-MA.

Corollary 2: For multiuser network under the extended tight collision constraints $\sum_{i=1}^{N}\frac{\gamma_i}{\gamma_i^{\rm PS}} \leq N$, time sharing of the K single user GPS-MA policies, each exclusively serving one cognitive user gives the maximum throughput region $\mathscr{J} = \mathscr{B}_\gamma$.

B. IS-MA and inner bound $\underline{\mathscr{B}}$

Similar to the single user case, $\mathscr{B}_K$ and $\mathscr{B}_\gamma$ serve as the outer bounds and time sharing of single user IS-MA matches $\mathscr{B}_\gamma$ when $\boldsymbol{\gamma}$ is small. (The characterization of the tight collision regime is difficult.) For general $\boldsymbol{\gamma}$ we use single user IS-MA with channel partition for inner bound $\underline{\mathscr{B}}$.

The procedure of determining the inner bound $\underline{\mathscr{B}}$ is described as follows. First define the set of all possible partitions of the primary channels

$$\mathcal{S} = \{(\mathcal{A}_1, \ldots, \mathcal{A}_K) \mid \cup_{j=1}^{K}\mathcal{A}_j = \mathcal{N}, \mathcal{A}_i\cap\mathcal{A}_j = \emptyset, i \neq j\}$$

where $\mathcal{N}$ is the set of all primary channels and $\mathcal{A}_j$ is a subset of $\mathcal{N}$. The K subsets of primary channels $\mathcal{A}_1, \ldots, \mathcal{A}_K$ (may be empty set) are assigned to K cognitive users and the cognitive user with $\mathcal{A}_j$ assigned follows single user IS-MA policy with transmission probability 1, restricted to $\mathcal{A}_j$. If $\mathcal{A}_j = \emptyset$, then the corresponding cognitive user has no sensing or transmission action. Therefore a specific partition $(\mathcal{A}_1, \ldots, \mathcal{A}_K)$ defines a multiuser cognitive access policy and precludes collisions among the cognitive users.

For each multiuser cognitive access policy π defined by certain partition $(\mathcal{A}_1, \ldots, \mathcal{A}_K) \in \mathcal{S}$, there is a matrix $\mathbf{M}_\pi$ associated with it where

$$\mathbf{M}_\pi = \begin{pmatrix} M_{\pi,1}^{(1)} & \ldots & M_{\pi,1}^{(K)} \\ \vdots & \ddots & \vdots \\ M_{\pi,N}^{(1)} & \ldots & M_{\pi,N}^{(K)} \end{pmatrix},$$

$$M_{\pi,i}^{(k)} = \lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\mathbb{P}(\mathscr{S}_{k,i}^{(j)}\cap\mathscr{I}_{k,i}^{(j)}). \quad (24)$$

The limit in Eq. (24) can be shown to exist by the fact that under Whittle's index sensing policy, $\boldsymbol{\Omega}(t) = (\Omega_1(t), \ldots, \Omega_N(t))$ evolves as a Markov process where $\Omega_i(t)$ denotes the conditional probability channel i is idle in the beginning of slot t, and the fact that $\mathbb{P}(\mathscr{S}_{k,i}^{(j)}\cap\mathscr{I}_{k,i}^{(j)})$ can be written as a function of $\boldsymbol{\Omega}(t)$.

Then we define the throughput region $\underline{\mathscr{B}}$ achievable by the subset of admissible policies that can be generated by using time sharing of the cognitive access policies defined by partitions as well as adjusting the transmission probability on the primary channels.

Define

$$\begin{aligned}\mathscr{A} \;=\; &\{\boldsymbol{A} = (\boldsymbol{\alpha}_\pi)_{\pi\in\mathcal{S}} \mid \forall\pi\in\mathcal{S}, \boldsymbol{\alpha}_\pi = (\alpha_{\pi,1}, \ldots, \alpha_{\pi,N}), \\ &\sum_{\pi\in\mathcal{S}}\max_{1\leq i\leq N}\alpha_{\pi,i} \leq 1, \alpha_{\pi,i} \geq 0\}.\end{aligned}$$

An element $\boldsymbol{A}$ of $\mathscr{A}$ specifies for each $\pi \in \mathcal{S}$ an $\boldsymbol{\alpha}_\pi = (\alpha_{\pi,1}, \ldots, \alpha_{\pi,N})$. If $\sum_{\pi\in\mathcal{S}}\max_{1\leq i\leq N}\alpha_{\pi,i} \leq 1$ and $\alpha_{\pi,i} \geq 0$ hold, then we take the time sharing of policies in $\mathcal{S}$ with time fraction $\max_{1\leq i\leq N}\alpha_{\pi,i}$ for policy π and tune the transmission probability in channel i of policy π from 1 to $\frac{\alpha_{\pi,i}}{\max_{1\leq i\leq N}\alpha_{\pi,i}}$.

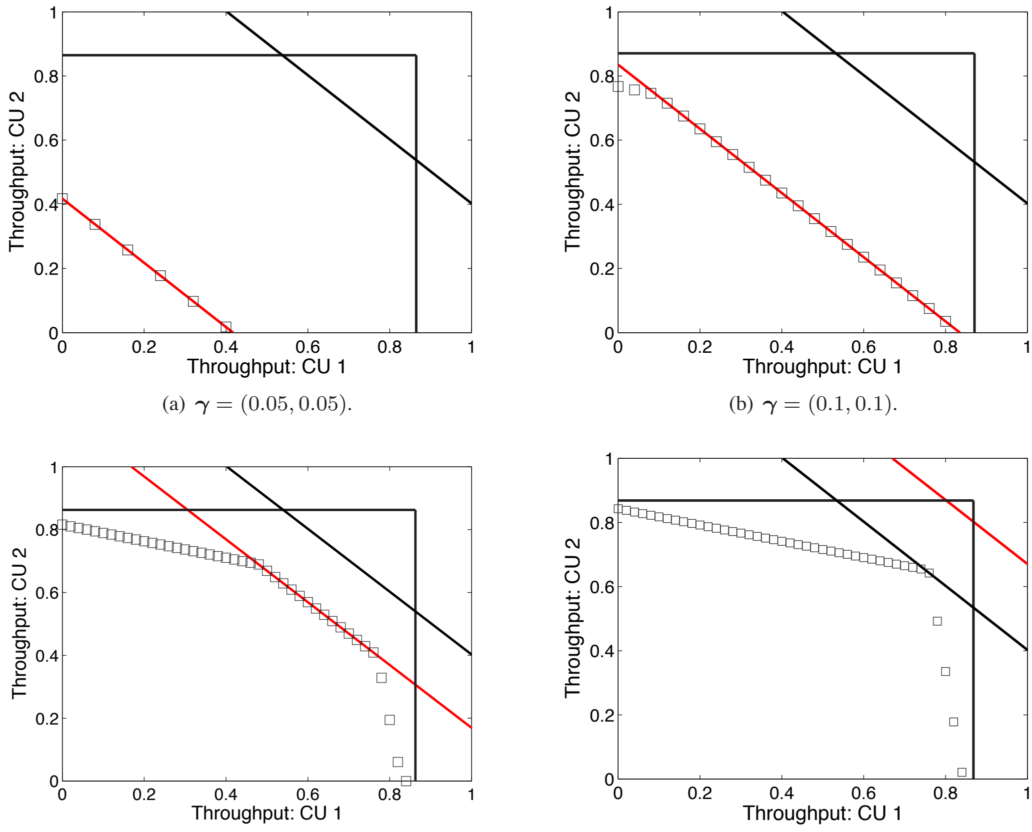
Given an element $\boldsymbol{A}$ of $\mathscr{A}$, the throughput vector $\mathbf{J}_{\boldsymbol{A}} = (J^{(1)}, \ldots, J^{(K)})$ and the collision vector $\mathbf{C}_{\boldsymbol{A}} = \{C_1, \ldots, C_N\}$ of the time sharing scheme can be associated with the matrix $\mathbf{M}_\pi$ using the relations below.

$$J_{\boldsymbol{A}}^{(k)} = \sum_{\pi\in\mathcal{S}}\sum_{i=1}^{N}\varepsilon_i M_{\pi,i}^{(k)}\alpha_{\pi,i}.$$

$$C_{\boldsymbol{A},i} = \sum_{\pi\in\mathcal{S}}\frac{1-\varepsilon_i}{1-v_i(0)\varepsilon_i}\sum_{k=1}^{K}M_{\pi,i}^{(k)}\alpha_{\pi,i}.$$

We are ready to define the inner bound $\underline{\mathscr{B}}$

$$\underline{\mathscr{B}} = \{\mathbf{J}_{\boldsymbol{A}} \mid \mathbf{C}_{\boldsymbol{A}} \leq \boldsymbol{\gamma}, \boldsymbol{A}\in\mathscr{A}\}.$$



(a) $\boldsymbol{\gamma} = (0.05, 0.05)$.

(b) $\boldsymbol{\gamma} = (0.1, 0.1)$.

(c) $\boldsymbol{\gamma} = (0.14, 0.14)$.

(d) $\boldsymbol{\gamma} = (0.2, 0.2)$.

Fig. 2. Inner bound $\underline{\mathscr{B}}$.

We take the cognitive access policies defined by the partitions as building block and obtain the inner bound $\underline{\mathscr{B}}$. The inner bound uses both the collision parameter $\boldsymbol{\gamma}$ as well as the fact that only K cognitive users are present in the system. $\underline{\mathscr{B}}$ is a polytope, however the structure of $\underline{\mathscr{B}}$ is complicated in general. One simple case is when the collision parameter $\boldsymbol{\gamma}$ satisfies

$$\gamma_i \leq C_{\pi_j,i}, \text{ for } 1 \leq i \leq N$$

where the policy π_j is such that $\mathcal{A}_j = \mathcal{N}$ (the IS-MA policy with all channels dedicated to cognitive user j with transmission probability 1). In this case the maximum throughput region can be achieved by time sharing of single user IS-MA. (This is the counterpart of Theorem 1 for IS-MA and in this case the throughput is $\underline{\mathscr{B}} = \mathscr{B}_\gamma$.)

Another simple case is when there are two homogeneous channels with very loose collision constraints and two cognitive users. There are four partitions of primary channels, namely $(\{1,2\},\emptyset)$, $(\{1\},\{2\})$, $(\{2\},\{1\})$ and $(\emptyset,\{1,2\})$. Due to homogeneity, $(\{1\},\{2\})$ and $(\{2\},\{1\})$ gives identical matrix $\mathbf{M}$. Therefore we obtain three throughput vectors corresponding to the cognitive access policy with partition $(\{1,2\},\emptyset)$, $(\{1\},\{2\})$ or $(\{2\},\{1\})$, and $(\emptyset,\{1,2\})$. $\underline{\mathscr{B}}$ is given by the convex hull of the origin and the three points corresponding to the three throughput vectors.

C. Simulation Results: Inner bound $\underline{\mathscr{B}}$

We show the simulation for the inner bound $\underline{\mathscr{B}}$ for two primary channels and two cognitive users ($N = K = 2$). The channel parameters are identical to the heterogeneous single user network. In Fig. 2 the blue lines correspond to the constraints in $\mathscr{B}_K$ with $|\mathcal{K}'| = 1$ while the black line corresponds to the constraints in $\mathscr{B}_K$ with $|\mathcal{K}'| = 2$. The red line corresponds to $\mathscr{B}_\gamma$. We observe from Fig. 2 that $\mathscr{B}_K$ stay constant while $\mathscr{B}_\gamma$ increases as $\boldsymbol{\gamma}$ increases.

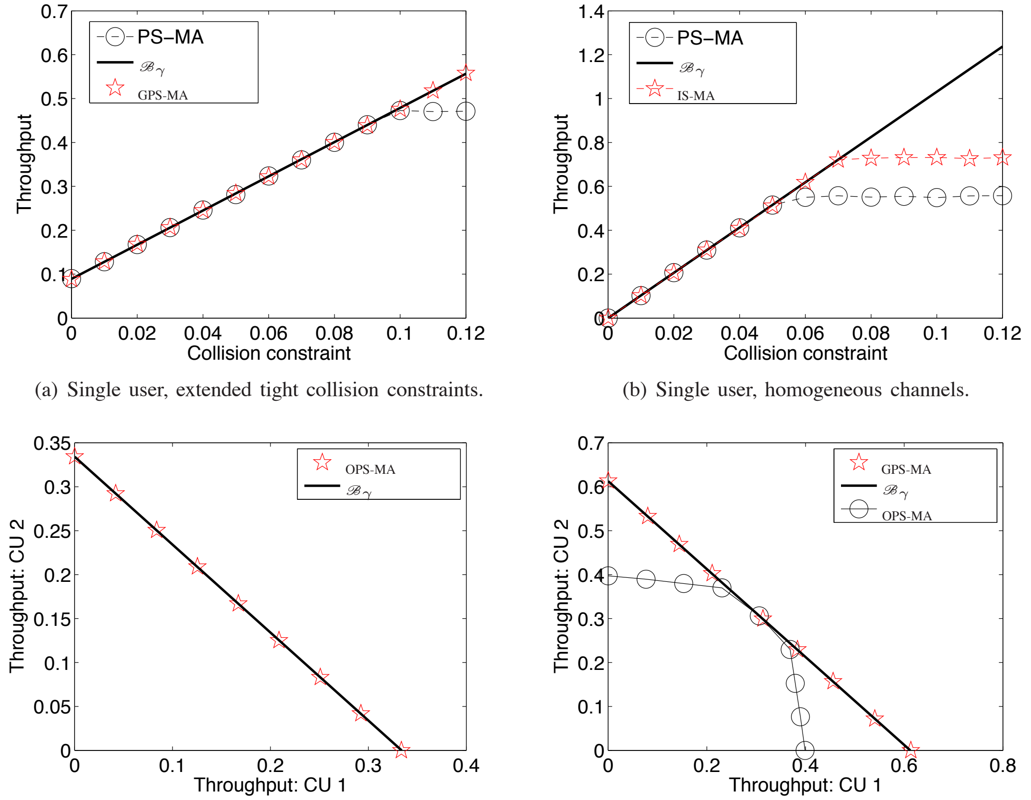
Fig. 2(a) depicts the scenario of tight collision constraint for IS-MA. Time sharing of IS-MA achieves the outer bound $\mathscr{B}_\gamma$. As $\boldsymbol{\gamma}$ increases, the matched part between the inner bound $\underline{\mathscr{B}}$ and $\mathscr{B}_\gamma$ decreases and $\underline{\mathscr{B}}$ is confined by $\mathscr{B}_K$. We also observe that there is a gap between $\underline{\mathscr{B}}$ and $\mathscr{B}_K$, which is due to the fact that $\underline{\mathscr{B}}$ is achieved by decentralized decision of the two cognitive users while $\mathscr{B}_K$ relies on centralized decision.

D. Simulation Results: Multiuser Network

In this subsection we simulate a multiuser network with two primary channels and two cognitive users ($N = K = 2$). The channel parameters are identical to the single user network.

1) For heterogeneous channels, The collision constraint parameter for tight collision regime is $\gamma = [0.04, 0.04]$ and is $\gamma = [0.02, 0.12]$ for extended tight collision regime.
2) For homogeneous channels, we use four different collision constraint parameters $\gamma = [0.02, 0.02]$, $\gamma = [0.07, 0.07]$, $\gamma = [0.09, 0.09]$ and $\gamma = [0.2, 0.2]$ for four different regimes, $\gamma \leq \gamma^{\mathrm{PS}}$, $\gamma^{\mathrm{PS}} \leq \gamma \leq \gamma^{\mathrm{IS}}$, $\gamma^{\mathrm{IS}} \leq \gamma \leq N\gamma^{\mathrm{PS}}$ and $\gamma \geq N\gamma^{\mathrm{PS}}$, separately.

Fig. 3(c) depicts the throughput regions of OPS-MA under tight collision constraints, which matches $\mathscr{B}_\gamma$, and validates



(a) Single user, extended tight collision constraints.

(b) Single user, homogeneous channels.

(c) Multiple users, tight collision constraints.

(d) Multiple users, extended tight collision constraints.

Fig. 3. Throughput region simulation.

the optimality of OPS-MA under tight collision constraints. Fig. 3(d) depicts the throughput regions of OPS-MA and GPS-MA under the collision constraint $\gamma = [0.02, 0.12]$. The throughput region of GPS-MA matches the outer bound $\mathscr{B}_\gamma$ while OPS-MA does not suffice under extended tight collision constraints.

Fig. 4 depicts the throughput region achieved by time sharing of IS-MA and OPS-MA. The four throughput regions correspond to four different collision parameter γ. We can observe that the simulation results validates the inner bound $\underline{\mathscr{B}}$ illustrated in Section V-B.

VI. Conclusions

The problem of multiuser cognitive access in a hierarchical cognitive radio network with $K \leq N$ cognitive users and N primary channels is considered. The maximum throughput region is characterized under different scenarios. Under tight collision constraints the maximum throughput region is achieved by the OPS-MA policy. This optimality result is extended to more general collision parameters by adjusting the fraction of time spent in each channel. An inner bound is proposed via IS-MA. For homogeneous channels the IS-MA policy is shown to be throughput optimal for single user cognitive network. The throughput region result for GPS-MA and IS-MA employs time sharing of different cognitive schemes, which requires synchronization in the design of realistic cognitive network. There are several interesting features that we do not consider in this paper, including the scenario with more cognitive users than primary users ($K > N$), spatial reuse, and the effect of primary retransmission due to the collisions caused by cognitive users.

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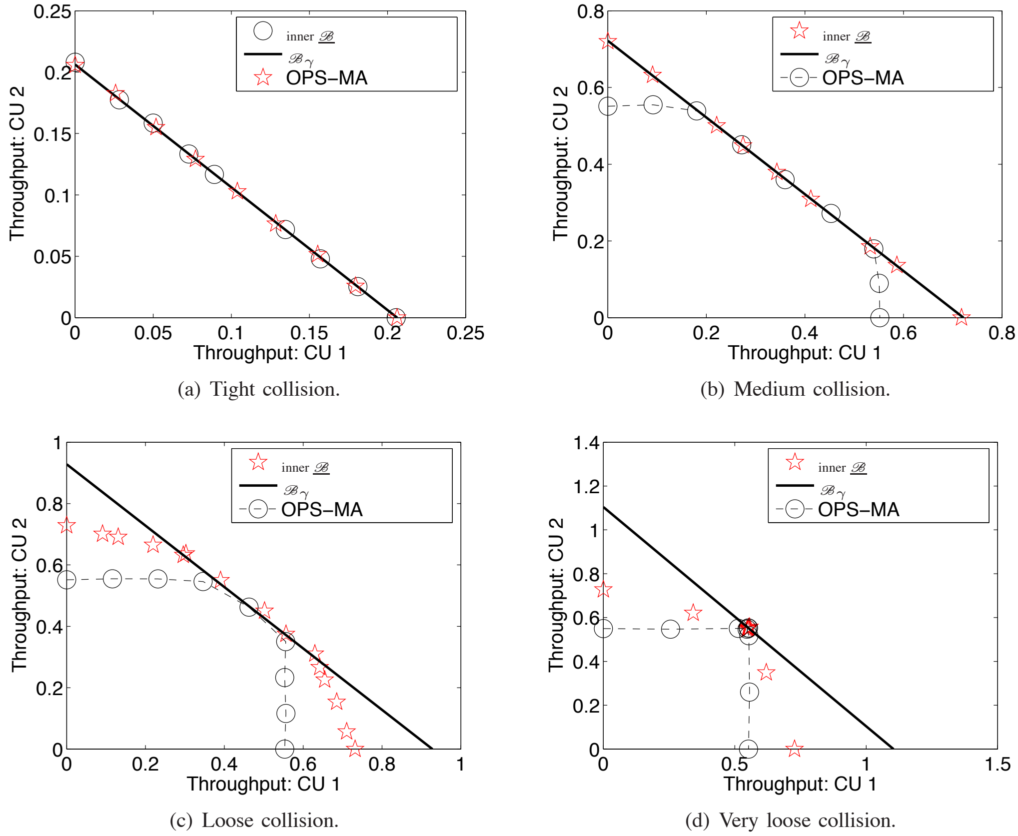

(a) Tight collision.
(b) Medium collision.
(c) Loose collision.
(d) Very loose collision.

Fig. 4. Throughput region, homogeneous channels.

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