Low-Complexity Distributed Spectrum Sharing Among Multiple Cognitive Users

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***Abstract*—The problem of sharing multiple primary channels among multiple cognitive users is considered. The occupancy of each primary channel is modeled by a continuous time on-off process with exponentially distributed idle (off) period and arbitrarily distributed busy (on) period. Each cognitive user follows a slotted sensing-before-transmission access protocol, with the capability of sensing one primary channel in each slot. To limit the interference to the primary users, the transmissions of cognitive users on each channel are subject to a prescribed collision constraint.**

In the absence of sensing error, it is shown that a distributed spectrum sharing scheme with low complexity achieves the throughput region under tight collision constraints. Cognitive access with sensing error is also investigated and optimal transmission policy is obtained for the orthogonalized periodic sensing. Packet level simulations are conducted to validate the performance of the spectrum sharing scheme under various channel models as well as perfect and imperfect channel sensing.

***Index terms*—Cognitive radio networks, dynamic spectrum access, opportunistic multiaccess, constrained POMDP, sensing error.**

I. Introduction

In a hierarchical cognitive network [1], primary users are licensed to communicate over designated channels. As owners of the spectrum, they transmit whenever they have packets in their queues. Secondary or cognitive users, on the other hand, have lower access priority. They should not interfere with primary users' communications; therefore can only transmit in channels and at times when primary users are idle. Such transmission opportunities, also referred to as the "white space" in the channel-time domain, do exist when traffic of primary users are bursty. For example, it has been demonstrated experimentally that Voice over IP traffic has over 90% idle time that could have been exploited by secondary users [2].

Hierarchical cognitive networks have potential applications in network centric military operations. For example, when low priority sensors (secondary users) are deployed along with assets with high priority communication needs, cognitive access by the secondary users is one way to share a common set of channels without prearranged or rigid allocation of network resources.

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We consider the problem of distributed spectrum sharing among K secondary cognitive users who exploit transmission opportunities individually in an N channel ($N \geq K$) wireless network. The cognitive users must discover the transmission opportunities and limit their interference within a prescribed level. The cognitive access of the primary channels is realized via channel sensing. We assume that each cognitive user can only sense one channel at a time, and their sensing may not be reliable. Each cognitive user makes its own decision on which channel to sense and transmit without coordination through a central controller or communications among themselves.

We focus in this paper on the maximum throughput region achievable by the cognitive users. Devising an optimal sensing and access strategy requires a judicious choice of a channel to sense and a policy to transmit, taking into account that sensing outcomes may not be reliable. The problem in general falls in the category of dynamic programming that, in general, does not have tractable solutions. The problem considered in this paper, however, has the special property that the actions of secondary users do not affect the underlying dynamics of primary channels, which makes it possible that simple yet optimal access policies exist. The throughput region provides a theoretical limit of the service that can be delivered by the multi-channel multiuser cognitive access network. Our analysis of the throughput region also provides insights into the effect of unreliable channel sensing.

A. Summary of results

In this paper we obtain the throughput region of the cognitive access network where the primary traffic is modeled by a continuous time on-off process. We relax the Markovian assumption on the primary traffic and assume that the busy period may be arbitrarily distributed whereas the idle period has an exponential distribution. We show in Section IV that under tight collision constraints, a simple policy Orthogonalized Periodic Sensing with Memoryless Access (OPS-MA), first proposed in [3], achieves the maximum throughput region of the multiuser cognitive network. The regime of tight collision constraints is of particular interest in practice. This result generalizes that in [4] where the multiuser throughput region is obtained for Markovian primary traffic. The relaxation of the exponential assumption of the busy period distribution does not follow directly from the approach in [3].

Next we analyze the effect of channel sensing error. We show that, for Markovian primary traffic, if the cognitive users employ Orthogonalized Periodic Sensing, the structure of the optimal transmission policy is a threshold policy. See Theorem 2 in Section V. This result indicates that Memoryless Access is no longer optimal if sensing is not perfect.

B. Related work

One of the earliest approach to medium access in a hierarchical cognitive network is given in [5], [6] based on a slotted transmission model for both primary users and a single cognitive user. The primary traffic is a two state Markov chain. The optimal sensing policy is shown to be a myopic policy [7], [8] for independent, identically distributed, and positively correlated Markov chains. Sensing error is considered in [9] where the authors establish a separation principle, *i.e.*, the sensing and transmission policies can be designed separately without loss of optimality.

Generalizations to the continuous time model for primary traffic are in [10], [11], which corresponds to the exponential/exponential traffic model in this paper. Restricting to the periodic sensing policy, the authors of [10] formulate a constrained Markov decision process for the single user problem and obtain the optimal cognitive access policy. The PS-MA policy is proposed in [10] and independently in [12]. It is only recent that the optimality of PS-MA is established in [13], [3] for the single (cognitive) user access. Earlier, for the case of single primary channel, Huang, Liu, and Ding derive the structure of optimal transmission policy assuming general channel occupancy [14], [15], [16]. The optimal transmission policy there is related to the policy we obtain for multichannel multiuser cognitive access with sensing error. In particular, both transmission policies uses reward-budget ratio as a measure of efficient use of the collision budget. Other related work assuming multiple channels with continuous time channel model can be found in [17], [18], [19].

The first multiuser cognitive access policy for multichannel continuous time primary traffic appears to be [20] where the authors propose an ALOHA based policy. The resulting policy does not require $K \leq N$ but is in general suboptimal. The multiuser OPS-MA policy is first proposed in [3] as a heuristic generalization to the multiuser scenario without establishing its optimality. The optimality of OPS-MA is shown in [4] for Markovian channel occupancy. We relax the Markovian assumption used in [3], [4] in this paper.

II. Network Model

A. Traffic models

We assume N parallel primary channels indexed by $i = 1, \ldots, N$ and $K \leq N$ cognitive users indexed by $k = 1, \ldots, K$. Each primary user transmits on its designated channel. Two occupancy models for primary traffic are considered: (i) the general/exponential occupancy; (ii) exponential/exponential (Markovian) occupancy. By general/exponential occupancy we mean that the busy period length is arbitrarily distributed with mean μ_i^{-1} while the idle period length is exponentially distributed with mean λ_i^{-1}. In both channel occupancy models each primary transmission process is independent of the transmission processes of other primary users.

In the Markovian occupancy model the generator matrix of the ith channel is given by

$$Q_i = \begin{pmatrix} -\lambda_i & \lambda_i \\ \mu_i & -\mu_i \end{pmatrix}, \tag{1}$$

and the stationary distribution for idle and busy states are given by $v_i(0) = \mu_i/(\mu_i + \lambda_i)$ and $v_i(1) = \lambda_i/(\mu_i + \lambda_i)$, where 0 and 1 denote the idle and busy states, respectively. In the general/exponential occupancy model the steady state distribution of idle and busy states are given by $v_i(0)$ and $v_i(1)$, respectively.

B. Sensing and transmission models

The cognitive users adopt a slotted sensing-before-transmission policy with slot length T in accessing the primary channels. Each cognitive user senses one out of the N channels in each slot and makes the transmission decision. For the channel sensing model we look into both noiseless and noisy sensing for the cognitive users. In the noisy sensing scenario, the sensing error is characterized by two parameters, probability of false alarm $P_f = P(\text{claim idle} \mid \text{busy})$ and probability of miss detection $P_m = P(\text{claim busy} \mid \text{idle})$. No communications among the cognitive users to share observations or decisions are assumed. When a cognitive user accesses a ceratin channel in slot t, the cognitive user collects unit reward if the channel is idle throughout slot t and no other cognitive users access the same channel.

C. Performance measure and constraints

The performance measure used in this paper is throughput, which measures the quantity of service delivered to the cognitive users. Denote by $R_t^{(k)}$ the reward that the kth cognitive user collects in slot t. The throughput of the kth cognitive user is defined by the infinite horizon average reward, *i.e.*,

$$J^{(k)} = \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \sum_{t=1}^{n} R_t^{(k)}.$$

The throughput vector for the K cognitive users is given by $\mathbf{J} = (J^{(1)}, J^{(2)}, \ldots, J^{(K)})$.

Due to the lack of access priority, the transmissions of the cognitive users are subject to collision constraints imposed by the primary users. Specifically, the overall collision caused by the K cognitive users to the ith primary channel should be limited below a collision constraint parameter γ_i. The collision for the ith primary user is the fraction of the collided time out of the total primary transmission time. Specifically, the collision for the ith primary user is defined to be the fraction of the collided slots in the slots fully or partially used by the primary user (due to the continuous time transmission process assumed for the primary users). We use the infinite horizon average collision scaled by the reciprocal of the steady state probability of the ith primary user transmitting in a certain slot, as given below, for the overall collision on channel i.

$$C_i = \frac{1}{1 - v_i(0)e^{-\lambda_i T}} \lim_{n\to\infty} \frac{\mathbb{E}\left(\sum_{t=1}^{n} 1_{\{\text{collide PU } i \text{ in slot } t\}}\right)}{n}$$

where $1_{\mathscr{A}}$ is the indicator function for event $\mathscr{A}$. Given $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_N)$, we impose collision constraints $C_i \le \gamma_i$.

III. OPS-MA: A Cognitive Access Policy

We present here a simple multiaccess policy for cognitive users. Referred to as the Orthogonalized Periodic Sensing with Memoryless Access, OPS-MA is later shown to be optimal under tight collision constraints (to be defined later).

OPS-MA has two components: sensing policy and transmission policy. The sensing policy is orthogonal and periodic. Specifically, each cognitive user senses the primary channels in an increasing order at the beginning of each slot. The K cognitive users sense the channels using different sensing phases. See Fig. 1 for an illustration. Since there are fewer cognitive users than primary channels in the network ($K \le N$), the K cognitive users can be fit in K orthogonal sensing phases and precludes collisions between cognitive users. The collision suffered by a primary user is the sum of collisions caused by each individual cognitive user under OPS. For each cognitive user, OPS induces N independent Markov chains, each with state space $\{0(\text{idle}), 1(\text{busy})\}$ and transition matrix $\exp(NTQ_i)$ in the Markovian occupancy model, where Q_i is the generator matrix of the ith channel.

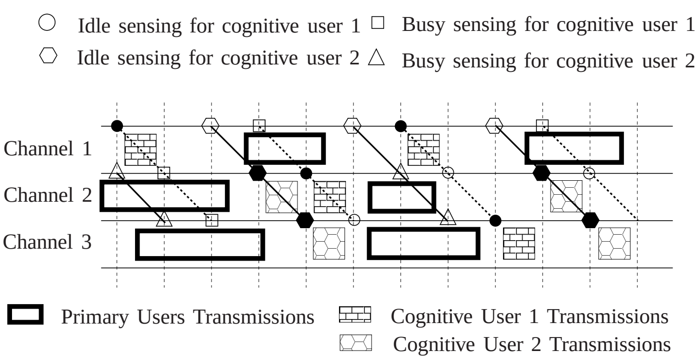


Fig. 1. Illustration of OPS-MA policy. Open circle: cognitive users decide not to transmit. Filled circle: cognitive users decide to transmit.

The transmission policy of OPS-MA is memoryless and can be described as follows, assuming perfect channel sensing. If the ith channel is sensed to be idle by the kth cognitive user, the kth cognitive user transmits in the sensed channel with fixed probability $\beta_i^{(k)}$, regardless of the previous history. The $\beta_i^{(k)}$'s are the design variables for the transmission policy.

To determine the transmission probability $\beta_i^{(k)}$'s for given collision parameter $\boldsymbol{\gamma}$, it can be shown following [13] that we need

$$\frac{v_i(0)(1-\exp(-\lambda_i T))}{N(1 - v_i(0)\exp(-\lambda_i T))} \sum_{k=1}^{K} \beta_i^{(k)} \le \gamma_i \quad \forall\ i, \tag{2}$$

where the left hand side gives the total collision caused by the K cognitive users on the ith channel. Equivalently,

$$\sum_{k=1}^{K} \beta_i^{(k)} \le \frac{\gamma_i N(1 - v_i(0)\exp(-\lambda_i T))}{v_i(0)(1-\exp(-\lambda_i T))} \triangleq \beta_i. \tag{3}$$

By Eq. (3) if the kth cognitive user increases its transmission probability $\beta_i^{(k)}$, then the other cognitive users need to decrease their transmission probabilities on the ith primary channel accordingly to limit the total collision. Therefore there is a tradeoff between the throughput for different cognitive users. OPS-MA addresses the tradeoff in the following manner. To specify $\beta_i(k)$, let $\beta_i^{(k)} = \beta_i \alpha_i^{(k)}$ where $\alpha_i^{(k)} \ge 0$ for all i and k, and $\sum_{k=1}^{K} \alpha_i^{(k)} \le 1$ for all i. The $\alpha_i^{(k)}$'s are backoff coefficients to guarantee the collision constraints to be met. Each cognitive user transmits less to accommodate other cognitive users. Different α in OPS-MA would yield different points in the throughput region, corresponding to different spectrum sharing among cognitive users. We remark that OPS-MA requires the knowledge of available sensing phases. Such information may be available through a basestation broadcast or on line learning.

IV. Throughput Region: Noiseless Sensing

It is established in [3] that for the single user network ($K = 1$), optimal throughput is achieved by PS-MA (single user OPS-MA) under tight collision constraints under Markovian occupancy model with perfect channel sensing, by showing the throughput of PS-MA matches the optimal throughput of a clairvoyant setting where the single cognitive user senses all N channels in each slot. The proof in [3] is long and relies on the Markovian occupancy model. In this paper we obtain an upper bound in a simpler way and show the optimality of multiuser OPS-MA for the general/exponential occupancy model. That is, the optimality of OPS-MA under tight collision constraints only requires the memoryless property of the idle period distribution, as shown in the following theorem.

Theorem 1. *(OPS-MA general/exponential occupancy) Given collision parameter $\boldsymbol{\gamma} = (\gamma_1, \cdots, \gamma_N)$, the collision constraints are defined to be tight if $\gamma_i \le \overline{\gamma}_i$ for $1 \le i \le N$, where*

$$\overline{\gamma}_i = \frac{v_i(0)(1-\exp(-\lambda_i T))}{N(1 - v_i(0)\exp(-\lambda_i T))}. \tag{4}$$

Under tight collision constraints, the throughput region for perfect channel sensing is given by

$$\mathscr{I}_{\boldsymbol{\gamma}} = \{(y_1, \ldots, y_K) \mid \sum_{k=1}^{K} y_k \le \sum_{i=1}^{N} e^{-\lambda_i T} \phi_i \gamma_i, y_k \ge 0\}, \tag{5}$$

where

$$\phi_i = \frac{1 - v_i(0)\exp(-\lambda_i T)}{1 - \exp(-\lambda_i T)} \tag{6}$$

Proof: See Appendix. ■

The throughput region under tight collision constraints is a polytope. It is the convex hull of the origin and the K points corresponding to exclusively serving one cognitive user. By

Theorem 1, a point in the positive orthant is in the throughput region (5) if and only if the total throughput of the K cognitive users is below the upper bound $\sum_{i=1}^{N} \exp(-\lambda_i T)\phi_i\gamma_i$. This upper bound is a linear combination of the collision constraint parameters and comes from the exponential assumption of the idle period distribution. Specifically, if the total throughput of the K cognitive users from channel i is S_i, then the collision caused on channel i is at least $\frac{S_i(1-\exp(-\lambda_i T))}{\exp(-\lambda_i T)(1-v_i(0)\exp(-\lambda_i T))}$.

To see this, conditioned on an idle sensing result by a certain cognitive user and that the cognitive user transmits with probability β, on one hand, if no collision among cognitive users is involved, then on average $\beta\exp(-\lambda_i T)$ reward will be accrued, on the other hand, if collision among cognitive users happens, then 0 reward will be accrued. In both cases $\frac{\beta(1-\exp(-\lambda_i T))}{1-v_i(0)\exp(-\lambda_i T)}$ amount of collision is incurred since the primary user returns within time T with probability $1-\exp(-\lambda_i T)$.

This derivation of the upper bound is simpler than the derivation from the clairvoyant setting and does not require the Markov property of the channel state process.

The corresponding reward-collision ratio $\frac{\exp(-\lambda_i T)(1-v_i(0)\exp(-\lambda_i T))}{1-\exp(-\lambda_i T)}$ can be viewed as the coefficient formalizing the idea that the stricter the collision parameters are, the smaller the throughput will be. The reward-collision ratio also provides intuition in imperfect channel sensing scenario in Section V.

V. OPS with Noisy Sensing

In this section we incorporate the sensing error into the network model and design optimal transmission policy for OPS. We focus on the Markovian occupancy model since the lack of Markov property makes the analysis with sensing error difficult.

A. Hidden Markov chains and belief vectors

We mention in Section III that under Orthogonalized Periodic Sensing with perfect sensing, the observed channel state for the ith channel evolves as a Markov chain with transition matrix $P_i = \exp(NTQ_i)$. Now if the channel sensing is imperfect, the sensing outcome for each channel becomes a hidden Markov chain. In order for each cognitive user to transmit optimally, a belief for each primary channel needs to be maintained and updated. Specifically, the belief $\Omega_{i,t}$ is the probability for the ith channel being idle in slot t conditioned on the history. It can be shown that $\Omega_{i,t}$ is a sufficient statistic for optimal decision [21]. Thus there exists a stationary randomized policy mapping the belief $\Omega_{i,t}$ to a probability distribution on the action space. For a cognitive user on channel i, only two actions are available: to transmit or not to transmit. Therefore we need mappings $f_{k,i} : [0,1] \to [0,1]$ from the belief for channel i maintained by cognitive user k to the transmission probability.

The cognitive users update their believes for channel i upon receiving noisy sensing results, according to the Bayes rule

$$\omega_{i,t+1} = \begin{cases} \frac{\overline{\omega}(1-P_m)}{\overline{\omega}(1-P_m)+(1-\overline{\omega})P_f} & \text{observe idle in slot } t \\ \frac{\overline{\omega}P_m}{\overline{\omega}P_m+(1-\overline{\omega})(1-P_f)} & \text{observe busy in slot } t \end{cases}$$

where $\overline{\omega} = (1-\omega_{i,t})P_{i,10} + \omega_{i,t}P_{i,00}$, and $P_{i,10}$, $P_{i,00}$ are the transition probabilities from busy to idle and from idle to idle, respectively. We use Ω to denote belief as a random variable and use ω to denote the realization of Ω. The belief $\Omega_{i,t}^{(k)}$ evolves as a Markov chain on state space $[0,1]$.

B. Optimal transmission for OPS

For each cognitive user, OPS induces N independent Markov chains for primary channels. Therefore we can focus on a particular channel i in the design of mapping $f_{k,i}$. Also OPS precludes the collision among the cognitive users. Therefore the spectrum sharing can be achieved by splitting the collision budget γ_i among K cognitive users. We thus restrict to the design of mapping $f_{k,i}$ for particular cognitive user k and primary channel i with collision constraint for cognitive user k on channel i given by $C_{i,t}^{(k)} \leq \gamma_i^{(k)}$ without loss of generality. We drop the subscript k and i in later development. Theorem 2 characterizes the threshold structure of the optimal transmission.

Theorem 2. *(Threshold optimal transmission policy) The optimal transmission policy for Orthogonalized Periodic Sensing is given by a randomized policy f^* where*

$$f^*(\omega) = \begin{cases} 1 & \omega > \tau^* \\ \eta^* & \omega = \tau^* \\ 0 & \omega < \tau^* \end{cases} \tag{7}$$

where threshold τ^* and randomization probability $\eta^* \in (0,1)$ are chosen to satisfy the collision constraint

$$C^* = Q(\{\Omega > \tau^*\}) + \eta^* Q(\{\Omega = \tau^*\}) = \gamma \tag{8}$$

where $Q(\cdot)$ is the long term average distribution of the belief.

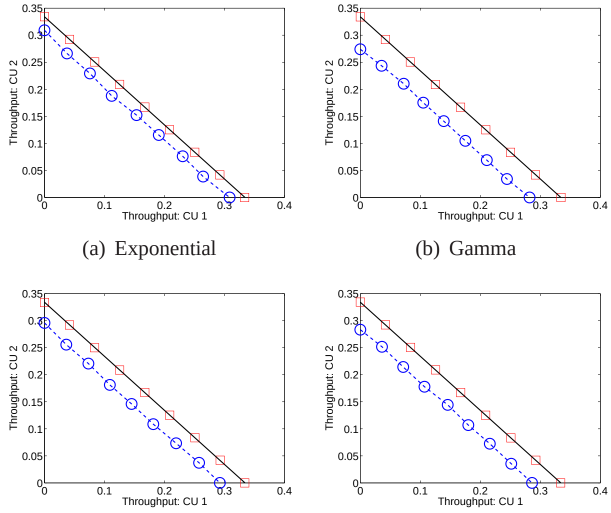
Proof: See Appendix. ∎

The transmission policy in Theorem 2 is a threshold policy on ω. As in Section IV we can define the reward-collision ratio function $g(\omega)$, and it can be shown that $g(\omega)$ is monotone increasing in ω. Therefore larger belief ω indicates more efficient use of the allowed collision, which is also intuitive.

VI. Simulation Results

In this section we show the simulation results for throughput regions for the cognitive network with two primary channels and two cognitive users. The channel parameters we use are as follows. $\mu = [1/1, 1/1.43]\text{ms}^{-1}$, $\lambda = [1/4.2, 1/3.23]\text{ms}^{-1}$, slot length $T = 0.25$ms and collision constraint parameters $\gamma = [0.04, 0.04]$. The horizon is taken to be 1500ms. In the imperfect sensing case, we take $P_f = P_m = 0.08$.

Fig. 2 depicts the throughput regions obtained by packet level simulation. We simulate exponential, Gamma, Weibull and Pareto distributions for the busy period and for each distribution we show the theoretically obtained, simulated with perfect sensing and simulated with imperfect sensing throughput regions. It can be seen from Fig. 2 that the throughput region obtained by simulation with perfect sensing matches the theoretical results, validating that OPS-MA achieves the throughput region with perfect sensing for general busy period



(a) Exponential (b) Gamma (c) Weibull (d) Pareto

Fig. 2. Throughput region. Solid line: theoretical. Square: perfect sensing. Circle and dashed line: imperfect sensing.

distribution. Also we can see that under OPS with imperfect sensing, the transmission policy we design yields performance close to the upper bound given by perfect sensing setup.

VII. Conclusions

The problem of spectrum sharing in multiuser cognitive access network with $K \leq N$ cognitive users sharing N primary channels with tight collision constraints is considered. We characterize the throughput region under tight collision constraints for general busy period distribution and show that the throughput region is achieved by a low-complexity distributed spectrum sharing scheme. We also incorporate sensing error into the analysis and characterize the optimal transmission policy to be a threshold policy with respect to the channel belief.

There are several future directions that we wish to pursue, such as generalizations on the idle period distribution and the impact of sensing error to other performance measure, *e.g.*, effective bandwidth.

Appendix: Proofs

A. Proof of Theorem 1

Upper bound: We can assume that cognitive users do not transmit if their sensing result is busy since the channel sensing results are perfect. Under tight collision constraints, the achievable total throughput of the K cognitive users is bounded below by a linear combination of the allowed collisions. Specifically, the collision constraint for the ith channel is given by

$$\sum_{k=1}^{K} \frac{1}{1 - v_i(0)\exp(-\lambda_i T)} \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \leq \gamma_i,$$

and we make linear combination and achieve

$$\begin{aligned} &\sum_{i=1}^{N} \frac{\exp(-\lambda_i T)}{1-\exp(-\lambda_i T)} \sum_{k=1}^{K} \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \\ \leq\;& \sum_{i=1}^{N} \exp(-\lambda_i T)\phi_i\gamma_i. \end{aligned}$$

On the left hand side we interchange the limit and the sum

$$\begin{aligned} &\sum_{i=1}^{N} \frac{\exp(-\lambda_i T)}{1-\exp(-\lambda_i T)} \sum_{k=1}^{K} \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{t=1}^{n} C_{i,t}^{(k)} \\ =\;& \sum_{k=1}^{K} \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n}\sum_{i=1}^{N} \frac{\exp(-\lambda_i T)}{1-\exp(-\lambda_i T)} \mathbb{E}C_{i,t}^{(k)} \\ =\;& \sum_{k=1}^{K} \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mathbb{E}R_t^{(k)}. \end{aligned}$$

Therefore

$$\sum_{k=1}^{K} J^{(k)} \leq \sum_{i=1}^{N} \exp(-\lambda_i T)\phi_i\gamma_i.$$

Also note that $J^{(k)} \geq 0$. Thus the region (5) is an upper bound of the throughput region.

Achievability: We show OPS-MA achieves the region (5). According to OPS-MA the kth cognitive user transmits with probability $\beta_i^{(k)} = \beta_i\alpha_i^{(k)}$ on the ith channel upon idle sensing result where the expression of β_i is given in Eq. (3), $\alpha_i^{(k)} \geq 0$ for all i and k, and $\sum_{k=1}^{K} \alpha_i^{(k)} \leq 1$ for all i.

Fix a set of $\alpha_i^{(k)}$'s. We analyze the throughput vector $\mathbf{J}$ achieved by OPS-MA with parameter $\alpha_i^{(k)}$.

Since the Markov property of the channel occupancy does not hold, we view the channel occupancy process as an alternating renewal process with two type of periods, idle and busy. Define $P_i(t) = P(\text{channel } i \text{ is idle at time } t)$. Alternating renewal theorem states that

$$\lim_{t\to\infty} P_i(t) = \frac{\mathbb{E}I}{\mathbb{E}I + \mathbb{E}B} = \frac{\lambda_i^{-1}}{\lambda_i^{-1} + \mu_i^{-1}} = v_i(0). \tag{9}$$

The throughput of the kth cognitive user under OPS-MA is

$$\begin{aligned} J^{(k)} &= \lim_{n\to\infty} \frac{1}{n}\mathbb{E}\sum_{j=1}^{n} R_j^{(k)} \\ &= \lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n} P_{i(jT)}(jT) e^{-\lambda_{i(jT)}T} \beta_{i(jT)}\alpha_{i(jT)}^{(k)} \\ &= \sum_{i=1}^{N} \lim_{n\to\infty} \frac{1}{n} \sum_{j=1, i(jT)=i}^{n} P_i(jT)e^{-\lambda_i T}\beta_i\alpha_i^{(k)} \\ &= \sum_{i=1}^{N} \lim_{n\to\infty} \frac{1}{n} \sum_{j=1, j\equiv i \bmod N}^{n} P_i(jT)e^{-\lambda_i T}\beta_i\alpha_i^{(k)} \end{aligned}$$

where $i(\tau)$ denotes the channel index the kth cognitive user is on at time τ. By Eq. (9) we have

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1, j\equiv i \bmod N}^{n} P_i(jT) = \frac{v_i(0)}{N}. \tag{10}$$

Therefore

$$J^{(k)} = \frac{1}{N}\sum_{i=1}^{N} v_i(0)e^{-\lambda_i T}\beta_i\alpha_i^{(k)} = \sum_{i=1}^{N}\phi_i \exp(-\lambda_i T)\gamma_i\alpha_i^{(k)}. \quad (11)$$

Eq. (11) gives a linear relation between the throughput vector $\mathbf{J} = (J^{(1)}, J^{(2)}, \ldots, J^{(K)})$ and OPS-MA parameter $\alpha_i^{(k)}$. Then apply Farkas lemma to prove that for any $(y_1, \ldots, y_K)$ in region (5) there exist $\alpha_i^{(k)} \geq 0$ such that $\sum_{k=1}^{K}\alpha_i^{(k)} \leq 1$ for all i and $y_k = \sum_{i=1}^{N}\exp(-\lambda_i T)\phi_i\gamma_i\alpha_i^{(k)}$. We omit the detail of the application of Farkas lemma. Combining the upper bound and the achievability proves Theorem 1.

B. Proof of Theorem 2

By [22, p. 199], under regularity condition,

$$\lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}P^{(m)}(\omega_0, E) = Q(\omega_0, E). \quad (12)$$

where $P^{(m)}(\omega_0, E) = P(\Omega_m \in E \mid \omega_0)$ is the m step transition probability. Since in this specific belief Markov chain there is one ergodic set, $Q(\omega_0, E) = Q(E)$. Denote by J the throughput of an arbitrary policy f and we have

$$\begin{aligned} J &= \lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}\mathbb{E}_m(\Omega f(\Omega)\exp(-\lambda T) \mid \omega_0) \\ &= \exp(-\lambda T)\lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}\int \omega f(\omega)dP^{(m)} \\ &= \int \omega\exp(-\lambda T)f(\omega)dQ. \end{aligned}$$

With similarly argument,

$$C = \int (1-\omega\exp(-\lambda T))f(\omega)dQ. \quad (13)$$

Define $g(\omega) = \frac{\omega\exp(-\lambda T)}{(1-\omega\exp(-\lambda T))}$. Note this is the reward-budget ratio when the belief of channel being idle is ω. Divide $[0,1]$ into three disjoint sets: $\{\omega : g(\omega) > \tau\}$, $\{\omega : g(\omega) = \tau\}$ and $\{\omega : g(\omega) < \tau\}$. Since $f(\cdot)$ gives the transmission probability, $0 \leq f(\omega) \leq 1$. Therefore

$$\begin{aligned} f^*(\omega) = 1 \geq f(\omega), &\qquad \text{if } g(\omega) > \tau \\ f^*(\omega) = 0 \leq f(\omega), &\qquad \text{if } g(\omega) < \tau. \end{aligned}$$

Treat the three sets defined above separately, we can show that

$$(f^*(\omega) - f(\omega))g(\omega) \geq (f^*(\omega) - f(\omega))\tau. \quad (14)$$

Denote the collision by C for f and expand $J^* - J$. We have

$$\begin{aligned} & J^* - J \\ = & \int (f^*(\omega) - f(\omega))\omega\exp(-\lambda T)dQ \\ = & \int (f^*(\omega) - f(\omega))g(\omega)(1-\omega\exp(-\lambda T))dQ \\ \geq & \int \tau(f^*(\omega) - f(\omega))(1-\omega\exp(-\lambda T))dQ \quad (15) \\ = & \ \tau(C^* - C) \geq 0 \quad (16) \end{aligned}$$

where Eq. (15) follows from Eq. (14) and Eq. (16) follows from the fact that $C \leq \gamma$ since f is admissible and $C^* = \gamma$ in Eq. (8). τ^* in Eq. (7) (threshold on ω) can be obtained from τ in Eq. (14) (threshold on $g(\omega)$) since $g(\cdot)$ is monotone. This completes the proof of the optimality of f^*.

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