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Channel Estimation and Equalization With Block Interleavers*

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Abstract — **The problem of channel estimation and equalization is considered for the block interleaved transmission over diversity channels. Effects on channel estimation and equalization from the amount of training symbols, the depth of the interleavers and the size of data packets are investigated by evaluating the Cramér-Rao Bound (CRB). The simulation results show that the block interleaved transmission may offer considerable gain in estimation and equalization performance.**

I. Introduction

Reliable communications over time varying wireless channels require the exploitation of channel diversities. A well known technique is the automatic repeat request (ARQ) with (temporal) diversity combining where the receiver requests retransmissions whenever detection fails, and signals from multiple transmissions are combined to improve detection performance [7, 6, 4, 1]. In this case, source signals through different diversity channels are highly correlated, and the exploitation of this correlation is crucial in diversity combining schemes.

In this paper, we consider the problem of channel estimation and equalization in a block interleaved transmission scheme where symbols are interleaved and transmitted through separate diversity channels. Although interleaving is widely used in wireless systems such as EDGE [3], few results have been reported concerning the effect of interleaving on the channel estimation and equalization. Of particular interest are issues relating to the effects of interleaving on the Cramér-Rao Bound(CRB) [5] of channel estimators, equalizer and the suitable channel estimation algorithms for interleaved transmission.

This paper is organized as follows. In Section II presents the block-interleaved transmission model. The CRB for the semi-blind channel estimation is presented in Section III and the CRB for the equalizer estimators is obtained in Section IV. A maximum likelihood channel estimator is derived in Section **??** followed by simulations in Section V.

Notation used in this paper are mostly standard. Upper- and lower-case bold letters denote matrices and vectors, respectively. $(\cdot)^*$ denotes conjugate, and $(\cdot)^H$ denotes hermitian transpose. For a matrix $\mathbf{X}$, $\mathcal{P}_{\mathbf{X}}^{\perp}$ is the orthogonal projection onto nullspace of $\mathbf{X}$.

II. The Model

We consider the diversity transmission scheme illustrated in Figure 1 where the source sequence $\{s_k\}$ of length M_s is interleaved and transmitted through K vector finite impulse response (FIR) channels $\{\mathbf{h}_i(z)\}_{i=1}^K$ which may or may not be the same, and then pass through K vector linear equalizers $\{\mathbf{f}_i(z)\}_{i=1}^K$. In ARQ schemes, $\mathbf{h}_i(z)$ corresponds to the the channel for the ith re-transmission. For time varying fading, there are substantial differences among these diversity channels. If the fading is relatively time invariant, on the other hand, these channels are highly correlated. The role of interleavers is to provide diversities in the presence of fading.

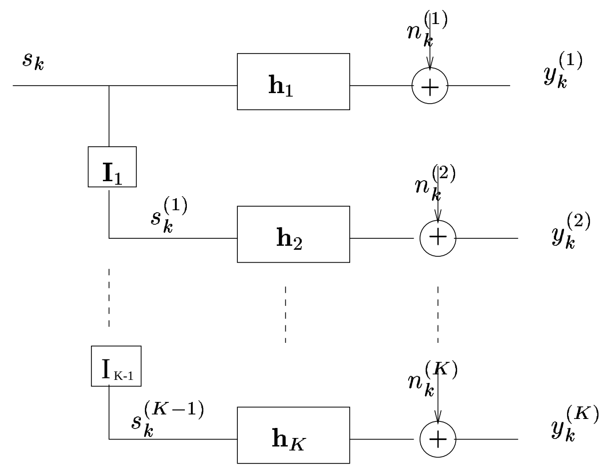

Figure 1: A diversity transmission with block interleaving.

We assume that each packet consists of M_s symbols with M_p pilot symbols placed consecutively in the middle. The $m \times n$ block interleaver for the ith channel, modeled as an permutation matrix $\mathbf{P}_i$, interleaves only the unknown data symbols as illustrated in Fig. 2.

For the ith diversity channel, the system equation for the received data through the ith channel is given by:

$$\mathbf{y}^{(i)} \quad = \quad \mathbf{H}(\mathbf{h}_i)\mathbf{P}_i\mathbf{s} + \mathbf{n}_i = \mathcal{H}(\mathbf{h}_i, \mathbf{P}_i)\mathbf{s} + \mathbf{n}_i, \qquad (1)$$

where $\mathbf{H}(\mathbf{h}_i)$ is the $(M_s - L) \times M_s$ Toeplitz matrix filled with ith channel coefficients $\mathbf{h}_i$, matrix $\mathbf{P}_i$ is the permutation matrix for the ith channel, vector $\mathbf{s} = \left[s_{M_s-1}^H, \cdots, s_0^H\right]^H$ the source sequence, $\mathbf{y}^{(i)} = \left[y_{M_s-L-1}^{(i)H}, \cdots, y_0^{(i)H}\right]^H$ the received data sequence from ith channel; $\mathbf{n}_i$ is an additive independent white Gaussian noise with zero-mean and variance σ_n^2. Stacking equations for all K channels and using the commutativity of convolutions, we have

$$\mathbf{y} \quad = \quad \mathcal{H}(\mathbf{h}, \mathbf{P})\mathbf{s} + \mathbf{n} = \mathcal{S}(\mathbf{P})\mathbf{h} + \mathbf{n}, \qquad (2)$$

where

$$\mathbf{h} = [\mathbf{h}_1^H, \cdots, \mathbf{h}_K^H]^H, \quad \mathbf{P} = \{\mathbf{P}_1, \cdots, \mathbf{P}_{K-1}\}, \qquad (3)$$

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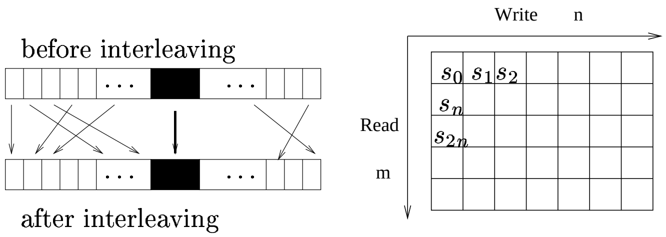

Figure 2: The block interleaving of the sequence

and the combined channel matrix $\mathcal{H}(\mathbf{h}, \mathbf{P})$ has the form

$$\mathcal{H}(\mathbf{h}, \mathbf{P}) = \begin{bmatrix} \mathcal{H}(\mathbf{h}_1) \\ \vdots \\ \mathcal{H}(\mathbf{h}_K, \mathbf{P}_K) \end{bmatrix}, \tag{4}$$

$\mathcal{S}(\mathbf{P})$ is a block diagonal matrix with ith diagonal block, denoted as $\mathcal{S}(\mathbf{P}_i)$, as a hankel matrix made of interleaved symbols for the ith channel input.

III. The CRB for Block Interleaved Channel Estimation

A. *The Cramér-Rao Bound*

Channel estimation under the above model is semi-blind because the data packet contains some pilot symbols. As a measure of performance, the CRB provides the lower bound on the error covariance of all the unbiased estimators.

Rewrite the system equation(2) as

$$\mathbf{y} = \mathcal{H}_d(\mathbf{h}, \mathbf{P}_d)\mathbf{s}_d + \mathcal{H}_p(\mathbf{h})\mathbf{s}_p + \mathbf{n}, \tag{5}$$

where $\mathbf{s}_d$ contains only the data symbols and $\mathbf{s}_p$ the pilot symbols. Their corresponding channel matrices are $\mathcal{H}_d(\mathbf{h}, \mathbf{P}_d)$ and $\mathcal{H}_p(\mathbf{h})$, defined as

$$\mathcal{H}_d(\mathbf{h}, \mathbf{P}_d) = \begin{bmatrix} \mathbf{H}_d(\mathbf{h}_1) \\ \mathbf{H}_d(\mathbf{h}_2)\mathbf{P}_{d_1} \\ \vdots \\ \mathbf{H}_d(\mathbf{h}_K)\mathbf{P}_{d_{K-1}} \end{bmatrix} \quad \mathcal{H}_p(\mathbf{h}) = \begin{bmatrix} \mathbf{H}_p(\mathbf{h}_1) \\ \mathbf{H}_p(\mathbf{h}_2) \\ \vdots \\ \mathbf{H}_p(\mathbf{h}_K) \end{bmatrix}, \tag{6}$$

where $\mathbf{H}_p(\mathbf{h}_i)$ is obtained by deleting the columns of $\mathbf{H}(\mathbf{h}_i)$ corresponding to the data. Similarly, deleting columns corresponding to the pilots from $\mathbf{H}(\mathbf{h}_i)$ gives $\mathbf{H}_d(\mathbf{h}_i)$. Matrix $\mathbf{P}_{d_i}$ is obtained from deleting columns and rows of $\mathbf{P}_i$ corresponding to the pilots.

Let $\boldsymbol{\theta} = \begin{bmatrix} \mathbf{s}_d \\ \mathbf{h} \end{bmatrix}$ be the parameter vector to be estimated. The complex Fisher Information Matrix (FIM) is

$$\mathbf{J}_{\theta\theta} = E\left(\frac{\partial \ln f(\mathbf{y}; \boldsymbol{\theta})}{\partial \boldsymbol{\theta}^*}\right)\left(\frac{\partial \ln f(Y; \boldsymbol{\theta})}{\partial \boldsymbol{\theta}^*}\right)^H, \tag{7}$$

where $f(\mathbf{y}; \boldsymbol{\theta})$ is the likelihood function of $\boldsymbol{\theta}$ given observation $\mathbf{y}$. It is shown in [?, 2] that when $\mathbf{J}_{\theta\theta^*} = 0$, the real CRB is completely determined by $\mathbf{J}_{\theta\theta}$, and the corresponding complex CRB is the inverse of $\mathbf{J}_{\theta\theta}$. In our case, since the symbols are deterministic and noise is complex circular Gaussian, $\mathbf{J}_{\theta\theta^*} = 0$ is satisfied. Thus we use complex CRB as the equivalent performance measure.

As in [?], the complex FIM for the channel model in (5) is

$$\mathbf{J}(\mathbf{h}, \mathbf{s}_d, \mathbf{P}) = \frac{1}{\sigma_n^2} \begin{bmatrix} \mathcal{H}_d^H(\mathbf{h}, \mathbf{P}_d)\mathcal{H}_d(\mathbf{h}, \mathbf{P}_d) & \mathcal{H}_d^H(\mathbf{h}, \mathbf{P}_d)\mathcal{S}(\mathbf{P}) \\ \mathcal{S}^H(\mathbf{P})\mathcal{H}_d(\mathbf{h}, \mathbf{P}_d) & \mathcal{S}^H(\mathbf{P})\mathcal{S}(\mathbf{P}) \end{bmatrix} \tag{8}$$

and the CRB for the channel coefficients, defined as $\mathbf{\Lambda}_h(\mathbf{h}, \mathbf{s}_d, \mathbf{P})$, can be expressed as

$$\mathbf{\Lambda}_\mathbf{h}(\mathbf{h}, \mathbf{s}_d, \mathbf{P}) = \sigma_n^2[\mathcal{S}^H(\mathbf{P})P^{\perp}_{\mathcal{H}_d(\mathbf{h}, \mathbf{P}_d)}\mathcal{S}(\mathbf{P})]^{-1}. \tag{9}$$

B. *Discussions*

From (9), we observe that different choices of $\mathbf{P}$ affect the value of $\mathbf{\Lambda}_\mathbf{h}(\mathbf{h}, \mathbf{s}_d, \mathbf{P})$. We have observed that, in general, the presence of interleaver does not guarantee the reduction CRB for a particular channel coefficient. If the lower bound $C_\mathbf{h}(\mathbf{h}, \mathbf{s}_d, \mathbf{P}) = tr(\mathbf{\Lambda}_h(\mathbf{h}, \mathbf{s}_d, \mathbf{P}))$ on the the mean square error (MSE) $E(||\hat{\mathbf{h}} - \mathbf{h}||^2)$ is used, however, the presence of interleaver lowers the CRB in simulations. Furthermore, the best interleaver is given by one that the original data symbols are maximally dispersed. Specifically, let $\{\tilde{s}_k\}_{k=0}^{M_s-1}$ be the interleaved sequence from the original $\{s_i\}_{i=0}^{M_s-1}$. Mapping the index of the new sequence $\{\tilde{s}_k\}$ from that of the original $\{s_i\}$, we have

$$\tilde{s}_{q_i} = s_i, \; i = 0, \cdots, M_s - 1. \tag{10}$$

Then the interleaver that leads to the lowest $C_\mathbf{h}(\mathbf{h}, \mathbf{s}_d, \mathbf{P})$ is such that the sequence q_i is obtained from

$$\max_{q(\cdot)} \min |q_i - q_j| \quad i, j = 0, 1, \cdots, M_s - 1; i \neq j. \tag{11}$$

Unfortunately, it is difficult to obtain a closed-form expression of $C_\mathbf{h}(\mathbf{h}, \mathbf{s}_d, \mathbf{P})$ as a function of $\mathbf{P}$. We assort to simulations for numerical evaluation under different interleavers.

IV. The CRB of MMSE Equalization of Block Interleaved Channel

A. *Complex CRB for Transformations*

For a complex parameter $\boldsymbol{\theta} \in \mathcal{C}^{p\times 1}$, define $\mathbf{f} = \mathbf{g}(\boldsymbol{\theta})$, where $\mathbf{g}$ is an r-dimensional function. Let $\boldsymbol{\theta}_R = \begin{bmatrix} Re(\boldsymbol{\theta})^H & Im(\boldsymbol{\theta})^H \end{bmatrix}^H$ be the corresponding real parameter for $\boldsymbol{\theta}$. It is known that, to estimate the real parameter $\mathbf{f}_R$, which is similarly defined as above, CRB is given by

$$\mathbf{\Lambda}_{\mathbf{f}_R}(\boldsymbol{\theta}) = \frac{\partial \mathbf{f}_R}{\partial \boldsymbol{\theta}} \mathbf{J}^{-1}_{\theta_R} \frac{\partial \mathbf{f}_R^T}{\partial \boldsymbol{\theta}_R}. \tag{12}$$

Given a complex parameter vector $\boldsymbol{\theta}$, it is desirable to find the equivalent complex CRB for the transform $\mathbf{f} = \mathbf{g}(\boldsymbol{\theta})$. Assuming $\mathbf{J}_{\theta\theta^*} = 0$, we obtain the CRB for $\mathbf{f}_R$:

$$\mathbf{\Lambda}_{\mathbf{f}_R}(\boldsymbol{\theta}) = \frac{1}{2} \begin{bmatrix} Re(\mathbf{\Lambda}_{\boldsymbol{\theta}_1}) + Re(\mathbf{\Lambda}_{\boldsymbol{\theta}_2}) & -Im(\mathbf{\Lambda}_{\boldsymbol{\theta}_1}) + Im(\mathbf{\Lambda}_{\boldsymbol{\theta}_2}) \\ Im(\mathbf{\Lambda}_{\boldsymbol{\theta}_1}) + Im(\mathbf{\Lambda}_{\boldsymbol{\theta}_2}) & Re(\mathbf{\Lambda}_{\boldsymbol{\theta}_1}) - -Re(\mathbf{\Lambda}_{\boldsymbol{\theta}_2}) \end{bmatrix} \tag{13}$$

where

$$\mathbf{\Lambda}_{\boldsymbol{\theta}_1} = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} \mathbf{J}^{-1}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}}\right)^H + \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*} \mathbf{J}^{-*}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*}\right)^H \tag{14}$$

$$\mathbf{\Lambda}_{\boldsymbol{\theta}_2} = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} \mathbf{J}^{-1}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*}\right)^{*H} + \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*} \mathbf{J}^{-*}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}}\right)^{*H} \tag{15}$$

and $\partial \mathbf{f}/\partial \boldsymbol{\theta}$ is the Jacobian matrix of $\mathbf{f}$. The real CRB $\Lambda_{\mathbf{f}_R}(\boldsymbol{\theta})$ is then completely determined by

$$\mathbf{\Lambda}_\mathbf{f}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{\Lambda}_{\boldsymbol{\theta}_1} & \mathbf{\Lambda}_{\boldsymbol{\theta}_2} \\ \mathbf{\Lambda}^*_{\boldsymbol{\theta}_2} & \mathbf{\Lambda}^*_{\boldsymbol{\theta}_1} \end{bmatrix}, \tag{16}$$

when $\mathbf{J}_{\theta\theta^*} = 0$. Hence $\mathbf{\Lambda}_\mathbf{f}(\boldsymbol{\theta})$ is the equivalent complex CRB for $\mathbf{f}$. A simple expression for the lower bound on the MSE of $\mathbf{f}$ is

$$C_\mathbf{f}(\boldsymbol{\theta}) = tr(\mathbf{\Lambda}_{\boldsymbol{\theta}_1}) = tr\left[\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} \mathbf{J}^{-1}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}}\right)^H + \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*} \mathbf{J}^{-*}_{\theta\theta} \left(\frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}^*}\right)^H\right]. \tag{17}$$

B. *The Bound on Equalizer*

For a given receiver structure, the presence of interleavers also affect the receiver performance. We now consider the linear equalizer of the form

$$\mathbf{f} = (\mathcal{H}(\mathbf{h})\mathcal{H}^H(\mathbf{h}) + \lambda\mathbf{I})^{-1}\mathcal{H}(\mathbf{h})\mathbf{e}_d \tag{18}$$

where $\lambda > 0$, d is the detection delay and $\mathbf{e}_d$ is the dth column of the identity matrix $\mathbf{I}$ with the size of M_s. When $\lambda = 0$, $\mathbf{f}$ is the Zero-Forcing equalizer, otherwise, it resembles the the noise-to-signal ratio in the MMSE equalizer.

For our model with interleavers in Figure 1, each equalizer is a function of $\mathbf{P}$, denoted as $\mathbf{f}_i(\mathbf{P}), i = 1, \cdots, K$. Define

$$\mathbf{f}(\mathbf{P}) = [\mathbf{f}_1^H(\mathbf{P}), \cdots, \mathbf{f}_K^H(\mathbf{P})]^H. \tag{19}$$

The linear MMSE equalizer, as a function of channel coefficients, is then given by

$$\mathbf{f}_m(\mathbf{P}) = [\mathcal{H}(\mathbf{h},\mathbf{P})\mathcal{H}^H(\mathbf{h},\mathbf{P}) + \lambda\mathbf{I}]^{-1}\mathcal{H}(\mathbf{h},\mathbf{P})\mathbf{e}_d. \tag{20}$$

Define $\mathcal{F}(\boldsymbol{\alpha})$ a Hankel matrix generated from $\boldsymbol{\alpha} = [\alpha_0^H, \cdots, \alpha_{M_s-1}^H]^H$ and $\mathcal{T}(\boldsymbol{\beta})$ a Toeplitz matrix generated from $\boldsymbol{\beta} = [\beta_0^H, \cdots, \beta_{M_s-L-1}^H]^H$

$$\mathcal{F}(\boldsymbol{\alpha}) = \begin{bmatrix} \alpha_0 & \cdots & \alpha_{L+1} \\ \vdots & Hankel & \vdots \\ \alpha_{M_s-L-1} & \cdots & \alpha_{M_s-1} \end{bmatrix}, \tag{21}$$

$$\mathcal{T}(\boldsymbol{\beta}) = \begin{bmatrix} \beta_0 & & \\ \vdots & \ddots & \\ \beta_{M_s-L} & & \beta_0 \\ & \ddots & \vdots \\ & & \beta_{M_s-L-1} \end{bmatrix}. \tag{22}$$

Also define $\boldsymbol{\zeta}(\mathbf{P}) = \mathbf{e}_d - \mathcal{H}^H(\mathbf{h},\mathbf{P})\mathbf{f}_m(\mathbf{P})$. Then the bound for the equalizer estimators is given by

$$\begin{aligned} C_{\mathbf{f}}(\mathbf{h},\mathbf{s}_d,\mathbf{P}) &= tr\{\frac{\partial\mathbf{f}}{\partial\mathbf{h}}\boldsymbol{\Lambda}_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})(\frac{\partial\mathbf{f}}{\partial\mathbf{h}})^H + \\ &\quad \frac{\partial\mathbf{f}}{\partial\mathbf{h}^*}\boldsymbol{\Lambda}_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})^*(\frac{\partial\mathbf{f}}{\partial\mathbf{h}^*})^H\} \end{aligned} \tag{23}$$

where

$$\frac{\partial\mathbf{f}}{\partial\mathbf{h}} = [\mathcal{H}(\mathbf{h},\mathbf{P})\mathcal{H}^H(\mathbf{h},\mathbf{P}) + \lambda\mathbf{I}]^{-1}\mathbf{D}, \tag{24}$$

$$\mathbf{D} = \begin{bmatrix} \mathcal{F}(\boldsymbol{\zeta}(\mathbf{P})) & & & \\ & \mathcal{F}(\mathbf{P}_1\boldsymbol{\zeta}(\mathbf{P})) & & \mathbf{0} \\ & & \ddots & \\ & \mathbf{0} & & \mathcal{F}(\mathbf{P}_{K-1}\boldsymbol{\zeta}(\mathbf{P})) \end{bmatrix}, \tag{25}$$

$$\frac{\partial\mathbf{f}}{\partial\mathbf{h}^*} = -[\mathcal{H}(\mathbf{h},\mathbf{P})\mathcal{H}^H(\mathbf{h},\mathbf{P}) + \lambda\mathbf{I}]^{-1}\mathcal{H}(\mathbf{h},\mathbf{P})\mathbf{F}, \tag{26}$$

$$\mathbf{F} = [\ \mathcal{T}(\mathbf{f}_{m_1}(\mathbf{P})) \vdots \mathbf{P}_1^H\mathcal{T}(\mathbf{f}_{m_2}(\mathbf{P})) \vdots \cdots \vdots \mathbf{P}_{K-1}^H\mathcal{T}(\mathbf{f}_{m_K}(\mathbf{P}))\ ], \tag{27}$$

and $\boldsymbol{\Lambda}_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ is the CRB for the channel coefficients defined in equation (9).

As in channel estimation, different choices of $\mathbf{P}$ affect $C_{\mathbf{f}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$. It is not hard to see that, the more accurate the channel estimator gets, the more accurate the equalizer will be and thus the lower detection error. Therefore the $\mathbf{P}$ that leads the lowest bound $C_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ for channel estimators also leads the lowest bound $C_{\mathbf{f}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ for equalizer estimators.

V. Simulation Results

Let $\Delta_{\mathbf{h}} = C_{\mathbf{h}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{I}) - C_{\mathbf{h}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ and $\Delta_{\mathbf{f}} = C_{\mathbf{f}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{I}) - C_{\mathbf{f}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ be the difference between the bounds for interleaved and no interleaved cases in channel estimation and equalization respectively.

We first considered two channels with one interleaver case, $K = 2$ and channel order $L = 5$. Both channels were complex and generated as 3-ray multipath channels. The input symbols were randomly drawn from QPSK sequence. The SNR was defined as $\frac{||\mathbf{h}||^2\sigma_s^2}{K\sigma_n^2}$. The known symbols were grouped in the middle of the sequence. The CRBs were averaged over 1000 realizations.

Figure 3 shows $C_{\mathbf{h}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ and $C_{\mathbf{f}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ vs. different interleaver length over different percentage η of known symbols respectively. The packet size was $M_s = 100$ and SNR was fixed at 10dB. The block interleaver length n was drawn from all possible integers that are factors of the number of unknown symbols $(M_s - M_p)$. Notice that the two end-points of each curve($n = 1$, $n = Ms$) correspond to the case with no interleavers. From each curve, we observed that both $C_{\mathbf{h}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ and $C_{\mathbf{f}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ were the highest at the two end points while the lowest bounds were reached when the interleaver length n was such that the interleaved symbols were as far away from their original neighbors as possible. In other words, $C_{\mathbf{h}}(\boldsymbol{\theta},\mathbf{s}_d,\mathbf{P})$ was the lower when the $m \times n$ block interleaver is near a square size. For example, at $\eta = 30\%$, the lowest bound was achieved at $n = 10$. For 10% known symbols case, there were about $\Delta_{\mathbf{h}} = 3dB$ and $\Delta_{\mathbf{f}} = 4dB$ between the values of bounds corresponding to $n = 1$ and $n = 10$. Also, we observed that $\Delta_{\mathbf{h}}$ increased as η decreased. This showed that, the smaller the percentage of known symbols, the more interleaving affect channel estimation.

Figure 4 showed $C_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ and $C_{\mathbf{f}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ vs. interleaver length n over different data packet lengths at $SNR = 10dB$, $K = 2$ and $\eta = 10\%$. The plots for the channel estimation on the left showed that $\Delta_{\mathbf{h}}$ was inversely proportional to the data length; the longer the data length, the smaller the performance gain. Similar result for $\Delta_{\mathbf{f}}$ is also shown on the right.

We then increased the number of channels and interleavers and tested the effect of the depth of interleavers on CRB. Figure 5 plots the result of $C_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ vs. K—the number of diversity channels—when $SNR = 10$, $\eta = 10\%$, $M_s = 60$ and $L = 4$. We assumed that $\mathbf{P}_i, i = 1 \cdots K-1$ are the same and compared the CRBs for $n = 1$ (no interleaver) and $n = 10$. We observed that interleaver again provided improved performance although the decrease in CRB appeared to level off after $K = 10$.

We plotted the normalized MSE(NMSE): $NMSE = ||\hat{\mathbf{h}} - \mathbf{h}||^2/||\mathbf{h}||$ of the ML channel estimator vs. SNR averaged over 100 Monte-Carlo runs in Figure 6, and compared it with $C_{\mathbf{h}}(\mathbf{h},\mathbf{s}_d,\mathbf{P})$ vs. interleaver length n over different data packet lengths at $SNR = 10dB$, $K = 2$ and $\eta = 10\%$. The plots show that $\Delta_{\mathbf{h}}$ was inversely proportional to the data length; the longer the data length, the smaller the performance gain.

VI. Conclusion

Channel estimation and equalization problems of multiple channels with block interleavers is considered. In this model, CRB for the semi-blind channel estimators and linear MMSE equalizers is derived. The effects of interleaving compared with no interleaving case are investigated through simulations. ML method for channel estimation is derived in this case. The simulations showed that block interleaving on the source improves channel estimation and equalization in terms of MSE.

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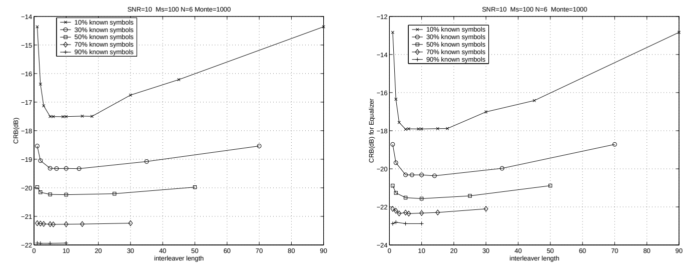

Figure 3: CRB vs. interleaver length n for different percentage of known symbols, $Ms = 100$. Left: Left: For channel estimation; Right: For equalization.

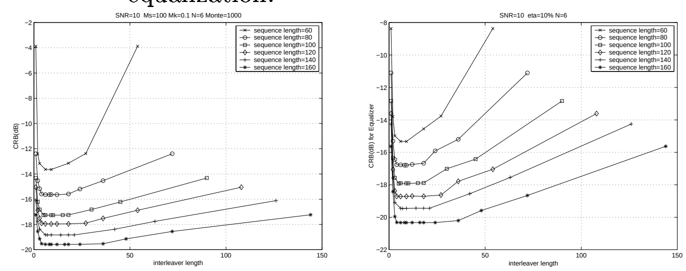

Figure 4: CRB vs. interleaver length n for different data length with 10% of known symbols. Left: For channel estimation; Right: For equalization.

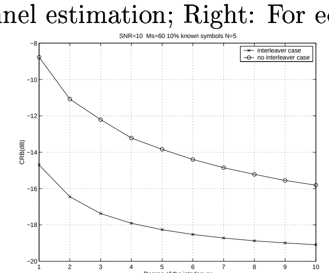

Figure 5: CRB vs. interleaver level.

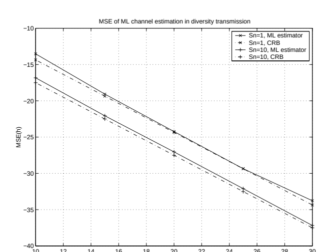

Figure 6: NMSE for ML with 10% of known symbols.