THE IMPACT OF MAC DESIGN ON ESTIMATION OF SPATIAL MARKOV PROCESS IN SENSOR NETWORKS

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Abstract— We investigate the impact of medium access control (MAC) design on the reconstruction performance of one-dimensional signal field in large scale sensor networks. Two types of MAC schemes are considered: random access MACs and deterministic MACs. We show that, in the high SNR regime, the deterministic MAC with uniform sampling is favored. Specifically, for noiseless measurements, we show that the ratio of reconstruction distortion under random access MACs to that under the deterministic MAC with uniform sampling grows as $O(logM)$, where M is the number of received packets.

1. INTRODUCTION

We consider measuring some phenomena of interest (such as temperature) in a field using a large scale reachback sensor network [1]. Specifically, sensors are densely deployed in a field for local measurements. At a prearranged time, sensors take local measurements and form a snapshot of the information in the field. At the collection time, sensors transmit their packets containing measurement data along with their locations to the access point through a common channel. Based on the received data samples, the access point reconstruct the signal field. In such networks, how to coordinate data transmissions from sensors to the access point through a common channel, *i.e.*, medium access control (MAC), is one of the key issues. Unlike conventional communication networks, in sensor networks, data transmitted from each sensor are highly correlated and contribute as a part of information to obtain the source information in the whole field. Consequently, the pattern of received data sample locations affects the source reconstruction. Therefore, the MAC design not only affects the network performance, but also directly affects the final reconstruction performance. Different from the conventional MAC design performance criterion which usually aims at high throughput, we try to link the MAC layer to the application performance, and investigate the impact of MAC design on the final source reconstruction performance. Specifically, we ask if one should pay effort to carefully schedule sensor transmission in order to form the desired data sampling pattern, or simply allow random access which results in random sampling patterns; and how much gain, if any, one can obtain by doing the former one.

2. THE MAIN RESULT

Consider a one-dimensional field of length D, denoted by $A = [0, D]$. Let $S(x)$ $(x \in A)$ be the source of interest in A at the measurement time. We assume that the spatial dynamic of $S(x)$ is a one-dimensional homogeneous Gaussian random field governed by the following linear stochastic differential equation: $dS_t(x) = -fS_t(x)dx + \sigma dW_t(x)$, where $f > 0$, $\sigma \in \mathbf{R}$ are

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known, $W(x)$ is a standard Brownian motion, and $S(x) \sim \mathcal{N}(0, \frac{\sigma^2}{2f})$. We assume that sensor measurements contains no noise. At the collection time, sensors transmits their data using a specific MAC protocol. We consider two types of MACs: random access MACs (π_r) and deterministic MAC with uniform sampling (π_u). For random access MACs (such as ALOHA), sensors contend the channel with equal priority, and have equal chance to get through. In contrast, the deterministic MAC with uniform sampling schedules transmissions according to a fixed pattern, *i.e.*, obtaining M packets from uniformly spaced locations in A. To avoid the boundary effect for signal reconstruction, we assume that the access point always obtains the packets from the two sensors closest to the two boundaries of A. After collecting M packets originated from some M points $\mathbf{p}_M = [P_1, \cdots, P_M]$ in A, we reconstruct the signal field using the MMSE estimator based on these samples. We define the maximum source reconstruction distortion by the maximum estimation error in A: $\mathcal{E}(\mathbf{p}_M) = \max_{x \in A} E\{|\hat{S}(x) - S(x)|^2 \mid \{S(p), p \in \mathbf{p}_M\}\}$, where $\hat{S}(x)$ is the estimate of the source. A MAC scheme π specifies how packets should be transmitted. It, therefore, specifies the distribution of sample points $\mathbf{p}_M$. The average maximum distortion is then given by $\bar{\mathcal{E}}(M;\pi) \stackrel{\Delta}{=} E\{\mathcal{E}(\mathbf{p}_M) \mid M, \pi\}$, where the expectation is taken over $\mathbf{p}_M$ for a given M. We assume the sensor density in the network goes to infinity. Under no measurement noise, it can be shown that $\mathcal{E}(\mathbf{p}_M)$ is only a function of the maximum distance between any two location-adjacent data samples received,*i.e.*, $\mathcal{E}(\mathbf{p}_M) = \mathcal{E}(d_{\max}; M)$. Therefore, for the MAC with uniform sampling π_u, we have $\bar{\mathcal{E}}(M;\pi_u) = \mathcal{E}(\frac{D}{M+1}; M) = \frac{1-e^{-f\frac{D}{M+1}}}{1+e^{-f\frac{D}{M+1}}}\frac{\sigma^2}{2f}$. For random access MACs π_r, we only need to find the pdf of $d_{\max}$.

Theorem 1 *The probability distribution of the maximum distance $d_{\max}$ in π_r is given by*

$$F_{d_{\max}}(x|M) = \begin{cases} 0 & \text{if } 0 \le x < \frac{D}{M+1} \\ \sum_{i=0}^{k}(-1)^i\binom{M+1}{i}\left[(M-i+1)\frac{x}{D} - 1\right]^M & \text{if } \frac{D}{M-k+1} \le x < \frac{D}{M-k}, \text{for } k = 0, \cdots, M-2 \\ 1-(M+1)(1-\frac{x}{D})^M & \text{if } \frac{D}{2} \le x \le D. \end{cases}$$

Using Theorem 1, we can calculate $\bar{\mathcal{E}}(M;\pi_r)$ by: $\bar{\mathcal{E}}(M;\pi_r) = \int \mathcal{E}(x;M)dF_{d_{\max}}(x|M)$.

As $M \to \infty$, we expect that the maximum distortion $\bar{\mathcal{E}}(M;\pi)$ decreases to 0 under both π_u and π_r, but at different rates. Define the distortion ratio between π_r and π_u as $r(M) = \bar{\mathcal{E}}(M;\pi_r)/\bar{\mathcal{E}}(M;\pi_u)$. The asymptotic behavior of distortion ratio $r(M)$ as a function of M is shown as following.

Theorem 2 *For estimating the one-dimensional spatial Markov process in a sensor network, when there is no measurement noise, the ratio of the average maximum distortion under random access MACs to that under the MAC with uniform sampling is $r(M) = O(logM)$ as M increases.*

Theorem 2 indicates that the MAC designs can have significant impact on the source field reconstruction performance. When sensors have no measurement noise, the MAC with uniform sampling can provide a large application-wised performance gain over the random access MACs, as the number of received packets becomes large. In this case, the benefit from carefully scheduling transmission in stead of random access is substantial.

3. REFERENCES

[1] L. Tong, Q. Zhao, and S. Adireddy, "Sensor networks with mobile agents," in *Proc. IEEE MILCOM'03*, Boston, MA, October 2003.