Online Adaptive Reinitialization of the Constant Modulus Algorithm

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***Abstract*—An adaptive reinitialization algorithm for the constant modulus algorithm is proposed that relies on the similarities between the constant modulus and the Wiener equalizer and expands the capabilities of a previous algorithm. The proposed algorithm determines adaptively if the CMA will benefit through reinitialization, and if so, it will calculate new equalizer coefficients leading to the global minimum.**

***Index Terms*—Blind equalization, constant modulus algorithm (CMA).**

I. Introduction

THE CHANNEL-SURFING reinitialization constant modulus algorithm (CSR-CMA), first introduced in [4], is a method that can be used to reinitialize the constant modulus algorithm (CMA), causing it to converge to different local minima. This method exploits the link between CMA and Wiener equalizers [5] to reinitialize the CMA, forcing convergence to approximate Wiener equalizers for different delays. Practical use of the CSR-CMA is impeded by the lack of a systematic method to determine whether the CMA has already reached the global minimum, and if not, which reinitialization option is preferred [4], [5].

We propose in this paper an extension to the CSR-CMA, providing an online adaptive method to reinitialize the CMA after convergence in a manner that can lead to the global minimum. Called the adaptive CSR-CMA (ACSR-CMA), our approach further exploits the link between CMA and Wiener equalizers, treating the CMA as an estimate of a Wiener equalizer for some delay. The mean-squared-error (MSE) cost of the estimated Wiener equalizer is used to assess the performance of the converged CMA equalizer and to predict both if the equalizer would benefit from reinitialization and, if so, how to reinitialize [5].

II. Model

A typical discrete-time linear baseband channel model is shown in Fig. 1.

Assuming the channel has finite impulse response with duration N and equalizing with L taps, we have the following linear model:

$$\mathbf{y}_k = \mathbf{H}\mathbf{s}_k + \mathbf{w}_k \tag{1}$$

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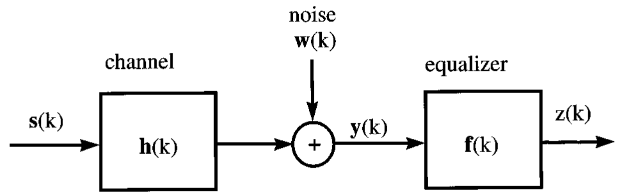

Fig. 1. Discrete-time symbol rate linear communication system.

$$z_k = \mathbf{f}^H \mathbf{y}_k \tag{2}$$

$$\mathbf{y}_k = [y_k, y_{k-1}, \cdots, y_{k-L}]^t \tag{3}$$

$$\mathbf{w}_k = [w_k, w_{k-1}, \cdots, w_{k-L}]^t \tag{4}$$

$$\mathbf{s}_k = [s_k, s_{k-1}, \cdots, s_{k-L-N+1}]^t. \tag{5}$$

The $()^H$ operator indicates Hermetian transpose. The $L\times (L+N)$ channel matrix $\mathbf{H}$ is constructed from the finite impulse response h_k

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_N & 0 & 0 & 0 \\ 0 & h_0 & h_1 & h_2 & \cdots & h_N & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & & \ddots & 0 \\ 0 & \dots & 0 & h_0 & h_1 & h_2 & \cdots & h_N \end{bmatrix}. \tag{6}$$

We consider the case of baud-rate linear equalization. The following assumptions apply.

1) The data sequence s_k is sub-Gaussian, symmetrical, and identically distributed.
2) $\mathbf{s}_k$ is zero mean with identity covariance matrix $E\mathbf{s}_k\mathbf{s}_k^H = \mathbf{I}$.
3) The noise sequence w_k is zero-mean, Gaussian, independent of s_k, and has covariance matrix $E(\mathbf{w}_k\mathbf{w}_k^H) = \sigma^2\mathbf{I}$.

III. CSR-CMA

As discussed in [4] and [5], if a Wiener equalizer for a particular delay has reasonably good MSE performance in estimating $s_{k-\nu}$, there exists a CMA equalizer in its immediate neighborhood. Using the converged CMA equalizer as an estimate for an MMSE equalizer and well-known Wiener results, we have

$$\hat{\mathbf{h}}_\nu = \hat{\mathbf{R}}\mathbf{f}_c \tag{7}$$

where $\mathbf{h}_\nu$ is a column of the channel matrix, $\mathbf{R}$ is the covariance matrix, and $\mathbf{f}_c$ is the converged CMA equalizer (estimating an MMSE equalizer for some unknown delay). Since the channel matrix $\mathbf{H}$ is Toeplitz, an estimate of a different column of $\mathbf{H}$ can be determined simply by shifting the column estimate up or down a number of times and inserting zeros at one end

$$\hat{\mathbf{h}}_{\nu+n} = Z_n\{\hat{\mathbf{h}}_\nu\} \tag{8}$$

where $Z_n\{\}$ is the shifting operator. From this shifted column of $\mathbf{H}$, a reinitialization vector can be obtained by multiplying the shifted column by the estimated covariance matrix inverse

$$\mathbf{f}_{\nu+n} = \hat{\mathbf{R}}^{-1}\hat{\mathbf{h}}_{\nu+n}. \tag{9}$$

Simulation verifies that $\mathbf{f}_{\nu+n}$ is in the neighborhood of the Wiener equalizer for delay ($\nu + n$) [4].

IV. ACSR-CMA

The ACSR-CMA expands the CSR-CMA by introducing a test to compare the current equalizers performance with potential candidates for re initialization. This test relies further on the link between the CMA and the Wiener equalizer, by applying the Wiener cost function to the CMA equalizer. The well-known Wiener MSE cost function (where the subscript m indicates MMSE) for delay ν is

$$J_m^v = 1 - \mathbf{f}_m^{\nu H}\mathbf{h}_\nu. \tag{10}$$

By again treating the CMA equalizer $\mathbf{f}_c$ as an estimate of $\mathbf{f}_m^\nu$, the above equation can be used to give some measure of the performance of the current $\mathbf{f}_c$.

$$\hat{J} = 1 - \mathbf{f}_c^H\hat{\mathbf{h}} = 1 - \mathbf{f}_c^H\hat{\mathbf{R}}\mathbf{f}_c. \tag{11}$$

Simulations show that this estimate accurately predicts the trend of the equalizer MSE versus delay characteristic. Just as the Toeplitz channel matrix can be reconstructed from a single (central) channel column, many of the possible Wiener equalizers corresponding to different delays can be estimated from a single Wiener equalizer estimate. Using (9) and (11), the optimal shift ν_0 over span n can be determined by

$$\min_n \left\{1 - \left(\hat{\mathbf{R}}^{-1}Z_n\{\hat{\mathbf{h}}\}\right)^H Z_n\{\hat{\mathbf{h}}\}\right\}. \tag{12}$$

An effective estimate of Toeplitz and Hermetian symmetric matrix $\mathbf{R}$ is obtained by storing an estimate of as single row

$$\hat{\mathbf{r}}_{\text{row1 } k} = \frac{(k-1)\hat{\mathbf{r}}_{\text{row1 } k-1} + \mathbf{y}_k^t y_k}{k}. \tag{13}$$

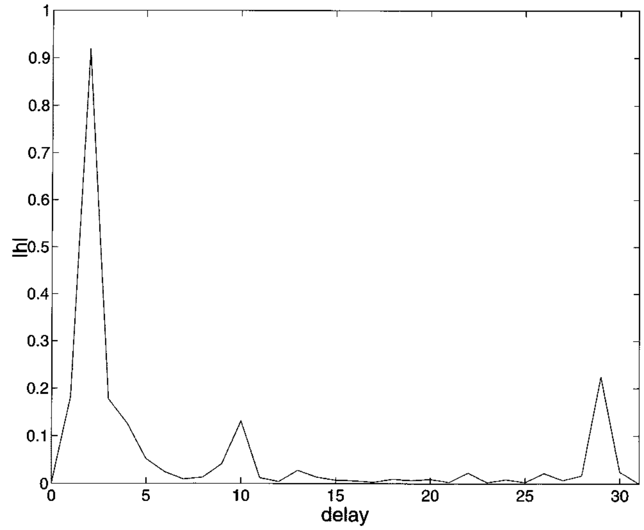

Fig. 2. Complex microwave channel impulse response.

The ACSR-CMA is summarized as follows.

1) Initialize the CMA with any initialization vector. Estimate the first row of $\mathbf{R}$ recursively using (13).

2) Expand the single row estimate into the Toeplitz and Hermetian matrix $\mathbf{R}$ and compute its inverse. Compute the estimate of a column of the channel matrix based on the converged equalizer $\mathbf{f}_c$ using (7).

3) Find optimal delay shift ν_o over a span using (12).

4) Reset $\mathbf{f}_c$ by shifting the estimated $\mathbf{h}_\nu \nu_o$ times and using

$$\mathbf{f}_{\text{new}} = \hat{\mathbf{R}}^{-1}\hat{\mathbf{h}}_{\text{new}}. \tag{14}$$

Remarks:

1) This method offers a systematic approach to evaluate the current equalizer and reinitialize.
2) The algorithm uses the converged CMA equalizer to estimate the MSE of $2^*\text{span} + 1$ Wiener equalizers. The postulate being that the best CMA solution will lie near the best Wiener solution.
3) One must be careful in selecting the span interval to ensure that too much channel information is not shifted out.
4) The ability to predict the optimal delay is determined to some extent by the delay or column of the channel matrix that is provided by (7). If channel information is missing, the estimate will not be accurate for all ν.

V. Simulation Example

A. Complex Microwave Channel

Fig. 2 shows a real world channel derived from Applied Signal Technology's terrestrial microwave channel 3.[1] We

[1][Online.] Available: http://spib.rice.edu/spib/microwave.html

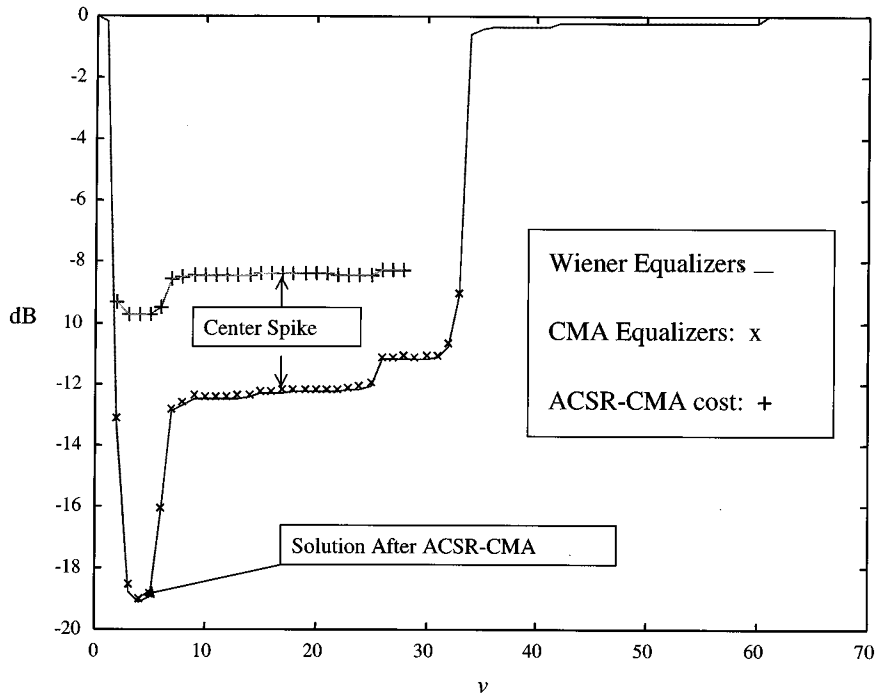

Fig. 3. MSE (decibels) versus delay characteristic for complex microwave channel.

consider an 8-PSK signal constellation at SNR 25 with 32 equalizer taps.

Shown in Fig. 3 is the MSE versus delay characteristic of Wiener equalizers for delays 0–6, and converged the CMA equalizers resulting from spike initializations in each of 32 possible equalizer taps (meaning an initialization with a 1 at one of the 32 taps and zeros elsewhere). The following observations can be made.

1) The MSE of the CMA equalizers are very close to Wiener solutions.
2) Selection of the correct delay is fundamental to optimal performance of the equalizer for this channel.
3) A center spike initialization gives an MSE that is almost 7 dB higher than the best CMA solution (which results from a spike at delay 3).

The ACSR-CMA applied to this channel after convergence with a center spike initialization yields cost estimates for numerous surrounding delays shown in Fig. 3. Although the cost estimates have a bias compared to the actual MSE shown, the trend of the cost is accurate in estimating the CMA equalizers that would result from spikes at delay 1–27. The ACSR-CSA correctly determines that MSE improvement can be gained if the CMA estimates the Wiener equalizer for delay 4 rather than a delay of 17. The channel column estimate $\mathbf{h}_\nu$ is shifted up 13 times ($v_o = -13$) and a reinitialization vector $\mathbf{f}$ is determined using (14). Utilizing the best CMA equalizer improves performance by 7 dB over the center spike.

B. General Simulation Observations

Other simulations indicate that for channels that do not require reinitialization, the ACSR-CMA recommends no reinitialization. When a channel has a wide disparity in the MSE versus delay characteristic as in Fig. 3, the ACSR-CMA reinitializes to the better solution in a fairly robust manner. Robustness is increased by choosing smaller span sizes and possibly using multiple steps to achieve the global minimum. This robustness of the algorithm is dependent upon the ability to reconstruct a portion $\mathbf{H}$ from a single column estimate—thus, better and more central column estimates imply better algorithm performance.

VI. Conclusions

The ACSR-CMA enhances the CSR-CMA by providing an online test for whether or not a converged CMA equalizer should be reinitialized to improve performance and, if so, generates the reinitialization coefficients. The algorithm seeks to determine the relative delay of the optimal Wiener equalizer near which the optimal CMA equalizer is presumed to reside. The robustness of the ACSR-CMA is affected by which Wiener delay the initial CMA solution estimates, the number of delays up or down from the reference delay that are being investigated for possible improvement, and the channel characteristic.

References

[1] S. Evans and L. Tong, "Adaptive channel surfing reinitialization of the constant modulus algorithm," in *Proc. 31st. Annu. Asilomar Conf. Signals, Systems and Computers*, vol. I, Nov. 2–5, 1997, pp. 823–827.
[2] D. N. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," *IEEE Trans. Commun.*, vol. COM-28, pp. 1867–1875, Nov. 1980.
[3] C. R. Johnson Jr. *et al.*, "Blind equalization using the constant modulus algorithm: A review," *Proc. IEEE*, vol. 86, pp. 1927–1950, Oct. 1998.
[4] L. Tong and H. Zeng, "Channel surfing re-initialization for the constant modulus algorithm," *Signal Processing Lett.*, vol. 4, pp. 85–87, Mar. 1997.
[5] H. Zeng, L. Tong, and C. R. Johnson Jr., "Relationships between the constant modulus and Wiener receivers," *IEEE Trans. Inform. Theory*, vol. 44, pp. 1523–1538, July 1998.