A Cognitive Framework for Improving Coexistence Among Heterogeneous Wireless Networks

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***Abstract*—The proliferation of wireless systems requires that the coexistence between heterogeneous technologies be addressed. This paper presents a cognitive framework in which sensing-based resource management of an infrastructure system effectively suppresses interference to close-by ad-hoc or peer-to-peer links. By utilizing its superior communication resources the infrastructure system estimates interference conditions and judiciously allocates transmission power such as to minimize interference. Despite adapting its transmission behavior, a rate constraint ensures that the infrastructure system continues to meet a specified quality-of-service level.**

The problem of optimal coexistence is formulated as a convex program. Structured solutions similar to classical water filling are derived and a solution method with guaranteed convergence is developed. An average-rate formulation extends the results to water filling across frequency and time. Numerical results corroborate our analysis and demonstrate a promising interference reduction.

***Index Terms*—Cognitive Radio, Coexistence in Heterogeneous Networks; Interference Management.**

I. Introduction

Cognitive radio (CR) enables heterogeneous systems to share the same frequency band by opportunistically orthogonalizing transmissions based on spectrum sensing. This is the foundation of dynamic spectrum access (DSA) which has recently received increasing attention and refers to methods in which a secondary system shares allocated frequency bands with a primary user subject to not inflicting significant interference. For a recent survey of DSA see [1].

The notion of primary and secondary users implies a certain hierarchy. In licensed bands, primary users correspond to spectrum licensees whereas secondary users try to exploit spectrum opportunities in which primary users are vacant. In unlicensed bands, on the other hand, primary users may be those who demand (and are willing to pay for) services with a higher level of quality-of-service (QoS) and secondary users those who are interested in low-cost, best-effort services. In both cases, the issue of coexistence arises naturally.

This paper focuses on cognitive techniques for the coexistence of two different types of networks: the infrastructure (IS) network and the ad-hoc (AH) or peer-to-peer network. Users in an IS network communicate with a base station (as in a cellular setup) whereas users in the AH network communicate locally among themselves. See Fig. 1.

We propose a cognitive coexistence framework in which the IS system dynamically adapts its resource allocation based on interference conditions while maintaining rate constraints for its users. This approach differs from standard DSA setups [2]. Instead of maximizing one system's throughput subject to not inflicting interference upon the other, this approach minimizes interference subject to satisfying a rate requirement across the IS link. This could be viewed as a "best effort" approach toward interference mitigation.

This framework encompasses numerous practical problems. For example, consider the coexistence of license-exempt IEEE 802.16 systems with IEEE 802.11 wireless local area networks (WLANs) [3], [4], [5]. Given the distributed nature of WLAN, it appears natural to use the flexibility of the 802.16 system toward interference mitigation. A similar problem arises in the military domain, where infrastructure radio links (*e.g.,* those connecting to a central command) interfere with short-range communication (*e.g.,* sensor networks or other local transmissions) [6].

Main contribution: This paper develops a cognitive coexistence framework for two different types of networks. In particular, an IS system with superior communication resources uses channel sensing to accommodate AH links as long as this does not impact the rate at which it is required to serve its own users.

- A convex optimization problem is formulated, which leads to the optimal frame-level solution. A binary search technique is used to find remaining parameters with guaranteed convergence and low complexity.
- Rate constraints are relaxed to the long term average and it is shown that water filling in both frequency and time can further reduce interference.

Related work: Resource management in cognitive radio networks is more difficult than in conventional systems because interference constraints need to be respected. Primary users are typically protected by designing the secondary system subject to interference power constraints [7], [8]. Furthermore, secondary users need to be able to effectively share the spectrum left over by the primary system [9], [10].

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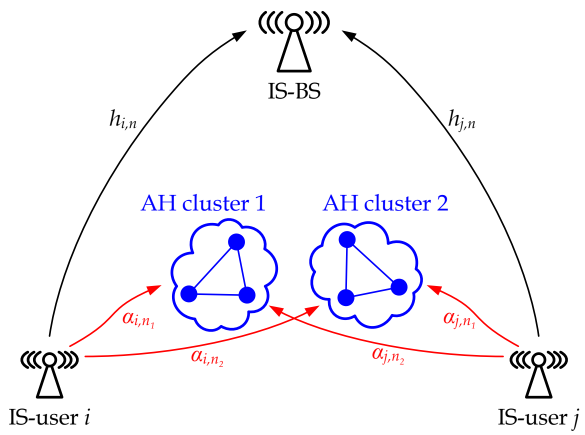

Fig. 1. System setup. IS uplink transmissions across channel $h_{i,n}$ interfere with local, low-power transmissions in AH clusters 1 and 2. The interference channel is modeled by α_{i,n_1} and α_{i,n_2}.

Cognitive coexistence among local and personal area networks has been addressed in [11], [12] based on Bluetooth and WLAN coexistence. Quite different from our approach are information theoretic approaches that allow cognitive networks to communicate without interfering with the primary systems [13], [14].

II. System Setup

The system setup is shown in Fig. 1. The IS network is a multicarrier system, which evolves in frames of fixed duration. IS clients operate on mutually exclusive subsets of sub-channels, assigned by their base station. Finding the optimal, interference-aware power allocation on these sub-carriers is the objective of this paper.

The AH system consists of distributed nodes, which evolve passively in the same frequency band. AH transmissions are low power, limited to small clusters, and are assumed not to interfere with the uplink of the IS system. However, they are being interfered with through the channels $\alpha_{i,n}$. Our analysis focuses on the scenario in which uplink transmission of the IS clients interfere with AH links; extensions to downlink scenarios are possible but not considered in this paper.

Throughout this paper, the interference channels $\alpha_{i,n}$ are assumed to be known. In practice there are several ways to estimate or model $\alpha_{i,n}$. For example, IS users could dedicate a portion of the frame to spectrum sensing, capture AH transmissions and (assuming channel reciprocity and that $\alpha_{i,n}$ varies reasonably slowly) infer the channel condition (note that only relative values are needed; any common scaling of $\alpha_{i,n}$ can be disregarded). Alternatively, if the location of AH nodes is known (as may be the case in some military scenarios), geolocation information can be combined with path-loss models to approximate $\alpha_{i,n}$.

Based on $\alpha_{i,n}$, IS users minimize interference power while maintaining an uplink rate constraint across channel $h_{i,n}$. The rate on sub-carrier n is given by

$$r_{i,n} = \log_2\left(1+\kappa\frac{p_{i,n}|h_{i,n}|^2}{N_0}\right) = \log_2\left(1+\beta_{i,n}p_{i,n}\right), \quad (1)$$

where N_0 is the noise power, κ is a normalization factor, and $\beta_{i,n}$ is introduced for notational convenience[1].

In summary, the cognitive allocation method operates as follows. In every frame, IS users request a rate R from their base station and are assigned a set of sub-carriers accordingly. IS users perform spectrum sensing to estimate the interference channel $\alpha_{i,n}$ and determine the optimal power allocation, which minimizes interference while maintaining rate and power constraints. Note that the optimal sub-carrier allocation is not dealt with; this remains an item of future work.

III. Frame-Level Formulation

In this section we derive the optimal power allocation for IS users, show that it admits a special structure, and use this property to derive an efficient solution algorithm.

A. Problem formulation

The objective of this paper is to find a power allocation $p_{i,n}$ which minimizes interference subject to the constraint that IS users meet rate and power constraints. Based on the assumption that IS users operate on pre-assigned, orthogonal sets of sub-carriers, the problem reduces to finding the optimal power allocation for each of the IS users individually. Without loss of generality we can therefore consider a single IS user, denote its set of sub-carriers $\mathbb{A}$, and for convenience drop the first index of the power allocation and channel coefficients.

The optimization problem **P1** is then formulated mathematically as ($\mathbf{p} = [p_1, \ldots, p_N]^T$)

$$\min_{\mathbf{p}} \sum_{n\in\mathbb{A}} \alpha_n p_n \quad (2)$$

$$\text{subject to} \sum_{n\in\mathbb{A}} \log_2(1+\beta_n p_n) \geq R \quad (3)$$

$$\sum_{n\in\mathbb{A}} p_n \leq P \quad (4)$$

$$p_n \geq 0, \ \forall n \in \mathbb{A}, \quad (5)$$

with rate constraint (3) and uplink power constraint (4). It is straightforward to show that **P1** is a convex optimization problem since, once rewritten in standard form, both the objective function and the constraints are convex [16].

The inequalities (3) and (4) may not have a common solution because for any power constraint P there exists a sufficiently large rate constraint R that renders the problem infeasible. To avoid trivial complications we therefore assume that (3) and (4) have a solution. Fundamentally, the power constraint (4) makes the problem interesting; in its absence the problem reduces to classical water filling[2][16].

[1]The above formulation encompasses a channel capacity formulation (for $\kappa = 1$) as well as the case of variable-rate M-QAM in which case $\kappa = 1.5/(-\ln \mathsf{BER})$ is chosen such that a target BER is met [15].

[2]To see this, introduce new variables $\bar{p}_n = \alpha_n p_n$ to convert the problem into classical water filling over the equivalent channel β_n/α_n.

B. Solution structure

The optimization problem **P1** admits a structured solution that can be developed in a similar fashion to classical water filling. The Lagrangian of the problem is given by

$$L(\mathbf{p},\gamma,\epsilon) = \sum_{n\in\mathbb{A}} \alpha_n p_n + \gamma\left(R - \sum_{n\in\mathbb{A}} \log_2(1+\beta_n p_n)\right) + \epsilon\left(\sum_{n\in\mathbb{A}} p_n - P\right), \quad (6)$$

where $\gamma \geq 0$ and $\epsilon \geq 0$ denote the Lagrange multipliers for the rate and power constraint, respectively. The non-negativity constraints (5) will be absorbed into the optimality conditions and do not require separate Lagrange multipliers.

The Karush-Kuhn-Tucker (KKT) conditions [16] for problem **P1** consist of its constraints (3)-(5), non-negativity constraints for the Lagrange multipliers, $\gamma \geq 0$ and $\epsilon \geq 0$, the slackness conditions

$$\gamma\left(R - \sum_{n\in\mathbb{A}} \log_2(1+\beta_n p_n^*)\right) = 0 \quad (7)$$

$$\epsilon\left(\sum_{n\in\mathbb{A}} p_n^* - P\right) = 0, \quad (8)$$

and the condition

$$\left.\frac{\partial L(\mathbf{p},\gamma,\epsilon)}{\partial p_n}\right|_{p_n=p_n^*} \begin{cases} =0 & p_n^*>0 \\ >0 & p_n^*=0 \end{cases}. \quad (9)$$

Note that the non-negativity constraints (5) have been absorbed into (9).[3] The above equation can be interpreted by noting that, for p_n^* to minimize $L(\mathbf{p},\gamma,\epsilon)$, its partial derivative must vanish at p_n^* unless it lies at the boundary of the feasible set.

Substituting (6) into (9) and solving for p_n^* we obtain the solution structure

$$p_n^* = \left[\frac{\gamma}{(\alpha_n+\epsilon)\ln 2} - \frac{1}{\beta_n}\right]^+, \quad (10)$$

where $(\cdot)^+ = \max\{0,\cdot\}$ denotes the positive part.

In order to find the optimal solution, we need to find the optimal Lagrange multipliers γ and ϵ in (10) such that both power and rate constraints are satisfied. With this objective, first substitute (10) into (3) and arrive at

$$\sum_{n\in\mathbb{A}} \left[\log_2 \frac{\gamma\beta_n}{(\alpha_n+\epsilon)\ln 2}\right]^+ \geq R. \quad (11)$$

In order to find a closed-form expression for γ, we introduce the set

$$\mathbb{P} = \left\{ n\in\mathbb{A} : \frac{\gamma\beta_n}{(\alpha_n+\epsilon)\ln 2} \geq 1 \right\}, \quad (12)$$

to express the γ for which (11) holds with equality as

$$\gamma = \ln 2 \left[\frac{2^R}{\prod_{n\in\mathbb{P}} \frac{\beta_n}{\alpha_n+\epsilon}}\right]^{1/|\mathbb{P}|}. \quad (13)$$

[3] This can be verified by introducing Lagrange multipliers for the inequality constraints. The non-negativity constraints for those Lagrange multipliers together with their slackness conditions lead to (9).

For any value of $\epsilon \geq 0$ the above expression specifies a $\gamma(\epsilon)$ such that the rate constraint (3) is met with equality. In the following define $\mathbf{p}(\epsilon)$ by (13) for a given ϵ. To fully determine the optimal solution to **P1** we require a value of ϵ such that the power constraint (4) is satisfied. First note that if $\epsilon = 0$ leads to a feasible solution then $\mathbf{p}(\epsilon = 0)$ is in fact optimal. To see this, consider the slackness condition (8), and note that if $\epsilon = 0$ is feasible then the power constraint is not active at the optimal solution. Hence for any $\epsilon' > 0$,

$$f(\mathbf{p}(\epsilon')) := \sum_{n\in\mathbb{A}} \alpha_n p_n(\epsilon'), \quad (14)$$

can be no less than $f(\mathbf{p}(0))$.

Assume now that $\epsilon > 0$. Substituting (13) into (10) we obtain

$$p_n(\epsilon) = \frac{2^{R/|\mathbb{P}|}\ln 2}{\left(\prod_{i\in\mathbb{P}}\beta_i\right)^{1/|\mathbb{P}|}} \left[\prod_{i\in\mathbb{P}} \frac{\alpha_i+\epsilon}{\alpha_n+\epsilon}\right]^{1/|\mathbb{P}|} - \frac{1}{\beta_n} \quad (15)$$

if $n\in\mathbb{P}$ and zero otherwise. We now show that $f(\mathbf{p}(\epsilon))$ is a non-increasing function in ϵ. To see this assume without loss of generality that $\alpha_1 \leq \cdots \leq \alpha_N$. Then, $p_1(\epsilon)$ is clearly a non-increasing function in ϵ, and consequently less power gets allocated to the first sub-channel as ϵ increases. Given the ordering of α_i and the fact that the total allocated power remains constant (the power constraint is active for $\epsilon > 0$) the interference power also increases with ϵ. We hence have that the optimal solution represents a feasible solution where ϵ is as small as possible, *i.e.*, if ϵ were decreased any further, $\mathbf{p}(\epsilon)$ would no longer be feasible. We use this result to derive an algorithm for finding the optimal ϵ.

C. Solution algorithm

Based on the results of the previous section, we formulate a solution algorithm to find the optimal value of the Lagrange multiplier ϵ. A flow chart of the algorithm is shown in Fig. 2.

The algorithm starts with calculating $\mathbf{p}(0)$. If this solution is feasible, *i.e.*, if it satisfies the power constraint, we know that it is optimal and the algorithm terminates. Otherwise, we find an ϵ_2 such that the power constraint is satisfied. Such an ϵ_2 exists as long as (3) and (4) have strictly feasible solutions, which we assume hereafter.

Having obtained an infeasible $\mathbf{p}(\epsilon_1)$ and a feasible $\mathbf{p}(\epsilon_2)$, binary search is used to find the optimal solution. The power allocation at the mid-point $\epsilon' = (\epsilon_1+\epsilon_2)/2$ is computed. If it is feasible we set $\epsilon_2 \leftarrow \epsilon'$; otherwise we continue with $\epsilon_1 \leftarrow \epsilon'$. The algorithm terminates once a desired accuracy $\Delta\epsilon$ is reached.

The above algorithm converges to the optimal solution $\mathbf{p}^*$. In fact, in every step, the algorithm keeps an infeasible $\mathbf{p}(\epsilon_1)$ which represents a lower bound to the optimal solution, and a feasible $\mathbf{p}(\epsilon_2)$, which serves as an upper bound due to the fact that $f(\mathbf{p}(\epsilon))$ is non-decreasing in ϵ. Hence,

$$f(\mathbf{p}(\epsilon_1)) \leq f(\mathbf{p}^*) \leq f(\mathbf{p}(\epsilon_2)), \quad (16)$$

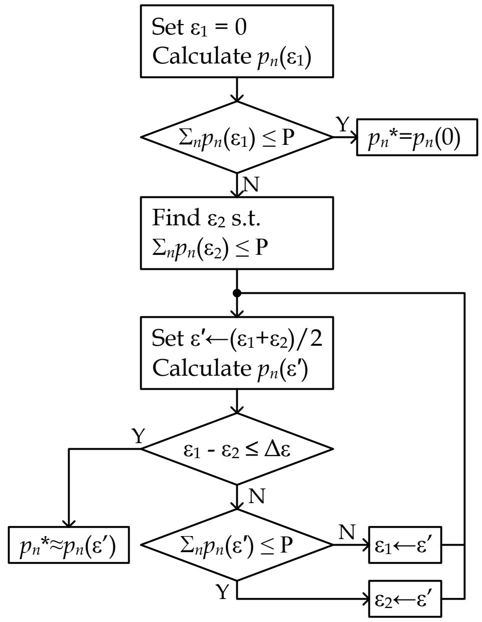

Fig. 2. Flow chart of the solution algorithm. A solution is approximated by upper and lower bounding the optimal value of ϵ and using binary search.

and since upper and lower bound converge as the algorithm progresses, the optimal solution is approximated arbitrarily well.

IV. Average Formulation

In most applications, imposing rate constraints on a frame level is unnecessary; instead average rate guarantees suffice. This section presents an average problem formulation which is based on knowing the long term statistics of β or both α and β (in addition to measuring both α and β on a frame-level basis). For both cases, the additional flexibility allows for further interference reduction and leads to water filling in both frequency and time [17], [18].

A. Average formulation for random β

First, we address the case where β is a random variable but α is a known deterministic constant. This leads to

$$\min_{p_n(\boldsymbol{\beta})} \int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} \alpha_n p_n(\boldsymbol{\beta}) dF(\boldsymbol{\beta}) \tag{17}$$

$$\text{s.t.} \int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} \log_2(1+\beta_n p_n(\boldsymbol{\beta}))dF(\boldsymbol{\beta}) \geq R \tag{18}$$

$$\int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} p_n(\boldsymbol{\beta}) dF(\boldsymbol{\beta}) \leq P, \tag{19}$$

$$p_n(\boldsymbol{\beta}) \geq 0,\ \forall n, \beta, \tag{20}$$

where $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_N]^T$ and $F(\boldsymbol{\beta})$ is the cumulative density function (cdf) of $\boldsymbol{\beta}$. Note that there are infinitely many decision variables $p_n(\boldsymbol{\beta})$ (for each realization of β). It is straightforward to see that the structure of the optimal solution is still given by (10). Since the optimality conditions have to be satisfied for each realization of $\boldsymbol{\beta}$ we obtain that

$$p_n(\boldsymbol{\beta}) = \left[\frac{\gamma}{(\alpha_n+\epsilon)\ln 2} - \frac{1}{\beta_n}\right]^+, \tag{21}$$

where γ and ϵ are Lagrange multipliers for the rate and power constraint, as before.

The optimal values of γ and ϵ need to satisfy the average constraints (18) and (19). We derive closed-from solutions for the integral expressions assuming that all sub-channels are independently flat Rayleigh fading, $\beta_i \sim \exp(1)$.

For a specific sub-channel i the rate constraint evaluates to

$$\int_{\beta_i=0}^{\infty} \left[\log_2 \tfrac{\gamma\beta_i}{(\alpha_i+\epsilon)\ln 2}\right]^+ e^{-\beta_i} d\beta_i = -\frac{1}{\ln 2}\text{Ei}\left(\tfrac{-(\alpha_i+\epsilon)\ln 2}{\gamma}\right), \tag{22}$$

where $\text{Ei}(z) = -\int_{-z}^{\infty} e^{-t}/t\, dt$ denotes the exponential integral function [19]. The rate constraint can hence be written as

$$\frac{1}{\ln 2}\sum_{n\in\mathbb{A}} \text{Ei}\left(\frac{-(\alpha_n+\epsilon)\ln 2}{\gamma}\right) \geq R. \tag{23}$$

In a similar way we obtain for the power constraint

$$\sum_{n\in\mathbb{A}} \frac{\gamma}{(\alpha_n+\epsilon)\ln 2} e^{-\frac{(\alpha_n+\epsilon)\ln 2}{\gamma}} + \text{Ei}\left(\frac{-(\alpha_n+\epsilon)\ln 2}{\gamma}\right) \leq P. \tag{24}$$

Based on the expressions (23) and (24), the optimal values of γ and ϵ can again be found through binary search similar to Sec. III-C.

B. Average formulation for random α, β

If the statistics of both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are known, interference can be reduced further. This leads to the optimization problem

$$\min_{p_n(\boldsymbol{\alpha},\boldsymbol{\beta})} \int_{\boldsymbol{\alpha}}\int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} \alpha_n p_n(\boldsymbol{\alpha},\boldsymbol{\beta}) dF(\boldsymbol{\alpha})dF(\boldsymbol{\beta}) \tag{25}$$

$$\text{s.t.} \int_{\boldsymbol{\alpha}}\int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} \log_2(1+\beta_n p_n(\boldsymbol{\alpha},\boldsymbol{\beta}))dF(\boldsymbol{\alpha})dF(\boldsymbol{\beta}) \geq R \tag{26}$$

$$\int_{\boldsymbol{\alpha}}\int_{\boldsymbol{\beta}} \sum_{n\in\mathbb{A}} p_n(\boldsymbol{\alpha},\boldsymbol{\beta})dF(\boldsymbol{\alpha})dF(\boldsymbol{\beta}) \leq P, \tag{27}$$

$$p_n(\boldsymbol{\alpha},\boldsymbol{\beta}) \geq 0,\ \forall n, \boldsymbol{\alpha}, \boldsymbol{\beta}, \tag{28}$$

in which both $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_N]^T$ and $\boldsymbol{\beta}$ are random variables. This problem can again be solved by evaluating the integrals in (26) and (27) and using binary search.

V. Numerical Results

In this section we present numerical performance results for frame-level and average formulations. We assume that there are a total of five sub-channels and that α_n, β_n are i.i.d. Rayleigh distributed in both frequency and time and compare the proposed methods to a reference method in which the IS system minimizes transmit rather than interference power.

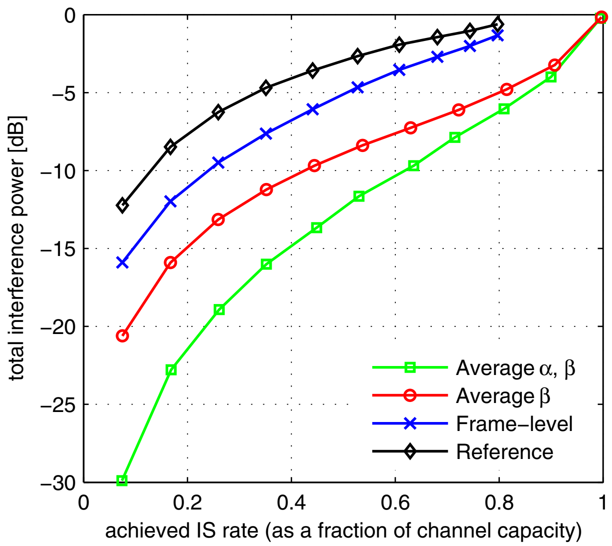

Fig. 3. Performance result. The total interference power (normalized by the uplink power constraint P and shown in dB) is plotted with respect to the IS rate (as a fraction of the maximum IS capacity). At low IS rates, resource allocation is flexible enough to accommodate ad-hoc links.

The result is shown in Fig. 3 and plots the interference power with respect to the achieved IS rate. For better comparison the rate is shown as a fraction of the IS channel capacity and the total interference power is normalized by P and shown on a logarithmic scale. The plot shows that more interference is generated at higher rates of the IS link since there is less flexibility in allocating IS resources.

The performance ordering reflects our expectations. The reference scheme is dominated by the frame-level formulation, which is in turn outperformed by the average formulations. The average formulations show better performance because of their additional flexibility in assigning resources (the doubly average formulation outperforms the case of average β since additional diversity of the channel α can be leveraged).

In comparing average and frame-level formulations, note that the latter does not achieve the same capacity as the average case because in some frames **P1** will be infeasible (in such frames we maximize frame rate subject to the power constraint to ensure a fair comparison). This complication is avoided in the average formulations because water filling is performed in both frequency and time. The performance gain associated with the average schemes is large and amounts to more than 9 dB for an IS system operating at 50% of its capacity (compared to the reference scheme).

VI. Conclusion

This paper has developed a novel framework for the coexistence of two systems that operate on different scales: an infrastructure system, which adapts its resource management such as to accommodate ad-hoc transmissions. Based on a frame-level formulation, we investigated the optimal solution structure and then extended results to the average case, in which interference can be further reduced by performing water filling in both frequency and time. Numerical performance results corroborated our analysis and showed that a promising interference reduction can be achieved. In future work, we will investigate the problem of joint sub-channel and power allocation for a multi-user scenario.

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