# Flat Fading Approximation Error

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***Abstract*—Flat fading approximation assumes that the system bandwidth is small compared to the coherence bandwidth. An upper bound is derived for an approximation to the error involved in flat fading approximation thereby giving us a measure of the degradation. The bound is found to vary as the square of the system bandwidth to coherence bandwidth ratio.**

***Index Terms*—Coherence bandwidth, flat-fading, frequency-selective fading, multipath.**

## I. Introduction

Wireless channels are subject to serious impairments which manifest as both time and frequency distortion. A prerequisite to combating these impairments is accurate characterization of the behavior of the wireless channel. To this end, two parameters are defined that characterize the behavior of the wireless channel, coherence time ($T_c$) and coherence bandwidth ($B_c$) [1]–[3].

The channel is assumed to suffer frequency dispersion and time selective fading if the duration of transmission is large compared to $T_c$ [1]. This effect may not be significant in present day systems where the data rate is high enough to prevent time selective fading. The converse effect, time dispersion and frequency selective fading, is what matters more in high data rate wireless systems envisaged in the future.

Frequency selective fading is observed in wireless channels when the system bandwidth is high compared to the coherence bandwidth of the channel. For systems with smaller bandwidths, time dispersion is usually neglected resulting in frequency-flat fading approximation of the received signal.

Let $s(t)$ be the baseband representation of the transmitted signal constrained to bandwidth $B$ and assumed deterministic. The baseband representation of the received signal for a fading channel is given by

$$y(t) = \sum_{i=1}^{P} \beta_i e^{-j\psi_i} s(t-\tau_i) \tag{1}$$

where $\psi_i = 2\pi f_c \tau_i + \theta_i$ [1], [3]. $\theta_i$ is the phase distortion due to reflections and diffractions [4] associated with the $i$th ray, $f_c$ is the carrier frequency and $\beta_i$ is the amplitude attenuation factor. $P$ is the number of significant rays assumed.

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Flat fading approximation neglects time dispersion and assumes that $s(t-\tau_i) \approx s(t)$. The received signal can now be represented as

$$\hat{y}(t) = s(t) \sum_{i=1}^{P} \beta_i e^{-j\psi_i} \tag{2}$$

The approximation error is given by

$$e(t) \stackrel{\Delta}{=} \hat{y}(t) - y(t) \tag{3}$$

We wish to bound the approximation error $e(t)$ and provide a basis for a concrete definition of narrowband signals.

## II. Signal Modeling

A good modeling scheme is to assume $\tau_i$ to be distributed uniformly in the range [0, $\tau_{\max}$]. We further assume the following channel model [5] to demonstrate the bound.

$$\begin{aligned} \theta_i &\sim \mathcal{U}(0, 2\pi) \quad \text{i.i.d} \\ \tau_i &\sim \mathcal{U}(0, \tau_{\max}) \quad \text{i.i.d} \\ \beta_i &\sim \text{Rayleigh},\ E[\beta_i^2 | \tau_i] = g_i^2 = Ge^{-\tau_i/\gamma} \end{aligned} \tag{4}$$

where $\gamma$ is dependent on the environment considered. Note that the delay spread, $\tau_{\max}$, is intimately connected with $\gamma$ with rays of attenuation larger than $e^{-\tau_{\max}/\gamma}$ being rejected from the summation. The assumption of $\tau_{\max} \sim 10\gamma$ is suitable for most cases [5].

## III. Flat Fading Approximation Error

Consider the squared magnitude of the approximation error

$$\begin{aligned} e(t) &\stackrel{\Delta}{=} \hat{y}(t) - y(t) \\ &= \sum_i \beta_i e^{-j\psi_i} [s(t) - s(t-\tau_i)]. \\ |e(t)|^2 &= \sum_k \sum_l \beta_k \beta_l e^{-j2\pi f_c \tau_k} e^{j2\pi f_c \tau_l} e^{-j\theta_k} e^{j\theta_l} \\ &\quad \times [s(t) - s(t-\tau_k)][s^*(t) - s^*(t-\tau_l)] \end{aligned}$$

Under the assumption on the $\theta_k$'s, $E[\exp(j\theta_k - j\theta_\ell)] = \delta_{k,\ell}$. Hence,

$$E[|e(t)|^2] = \sum_k E[\beta_k^2 |s(t) - s(t-\tau_k)|^2]$$

Assuming good behavior of $s(t)$ and resorting to Taylor series expansion, we have

$$s(t) - s(t-\tau_k) = s'(t)\tau_k + O(\tau_k^2)$$

Considering the fact that the baseband signal $s(t)$ is low-pass, we ignore higher order terms and write

$$s(t) - s(t-\tau_k) \approx s'(t)\tau_k$$

which leads to

$$E[|e(t)|^2] = |s'(t)|^2 \sum_k E[\beta_k^2 \tau_k^2] \tag{5}$$

The expectation is evaluated as

$$\begin{aligned} E[\beta_k^2\tau_k^2] &= E[E[\beta_k^2|\tau_k]\tau_k^2] = E[g_k^2\tau_k^2] \\ &= GE[e^{-\tau_k/\gamma}\tau_k^2] \\ &\simeq G \times 2\gamma^3/\tau_{\max} \end{aligned}$$

where the approximation is valid as long as $\tau_{\max}/\gamma \gg 1$, which is usually the case. This enables us to write (5) as

$$E[|e(t)|^2] \simeq G\frac{2P\gamma^3}{\tau_{\max}}\,|s'(t)|^2.$$

Integrating both sides over the time axis, we get

$$\int E[|e(t)|^2]\,dt \approx GP\frac{2\gamma^3}{\tau_{\max}}\int |s'(t)|^2\,dt$$

A similar evaluation of the received signal energy gives

$$\int E[|y(t)|^2]\,dt \approx GP\frac{\gamma}{\tau_{\max}}\int |s(t)|^2\,dt \tag{6}$$

The normalized error energy (NEE) is given by

$$\frac{\int E[|e(t)|^2]\,dt}{\int E[|y(t)|^2]\,dt} = 2\gamma^2\frac{\int |s'(t)|^2\,dt}{\int |s(t)|^2\,dt} \tag{7}$$

Given a particular modulation, NEE may be evaluated explicitly. If the specific form of $s'(t)$ is not known,

$$\begin{aligned} \int |s'(t)|^2\,dt &= \int_{-B}^{B} |j2\pi f S(f)|^2\,df \\ &\le 4\pi^2 B^2 \int_{-B}^{B} |S(f)|^2\,df \end{aligned}$$

where $S(f)$ is the Fourier Transform of $s(t)$. Using the fact that the energy in the time domain is the same as that in the frequency domain, we finally write down the bound as

$$\begin{aligned} \frac{\int E[|e(t)|^2]\,dt}{\int E[|y(t)|^2]\,dt} &\le 4\pi^2 B^2 2\gamma^2 \\ &\simeq 8\pi^2(0.1)^2(B/B_c)^2 \end{aligned} \tag{8}$$

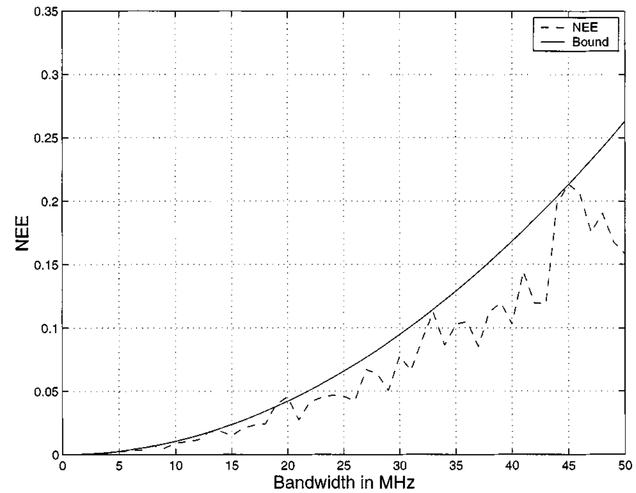


Fig. 1. Plot of NEE against bandwidth.

where $B_c = \tau_{\max}^{-1}$ is the coherence bandwidth. The expression on the left in equation (8) is the normalized error energy (NEE) at the receiver. We summarize the result with the following proposition.

*Proposition 1:* Under suitable assumptions, the Normalized Error Energy (NEE) for flat fading approximation in a multipath propagation scenario is proportional to the square of the System Bandwidth-to-Coherence Bandwidth Ratio.

Fig. 1 plots NEE against bandwidth for a sinc baseband transmitted pulse. The derived bound is plotted on the same axis for reference. A carrier frequency of 3 GHz and a maximum delay of 20 ns was assumed. It can be observed that the bound is less tight at higher bandwidths due to the approximation involved. As mentioned before, a tighter bound is possible if the specific form of the transmitted pulse is known. The plot indicates that the NEE is as high as 30% at higher bandwidths. These parameters are typical of present day communication systems and the analysis suggests that the usually invoked flat fading approximation may not always be valid.

## REFERENCES

[1] A. A. M. Saleh and R. Valenzuela, *Mobile Radio Communications*. London, U.K.: Pentech, 1992.
[2] J. G. Proakis, *Digital Communications*. New York, NY: McGraw Hill, 1995.
[3] T. S. Rappaport, *Wireless Communications: Principles & Practice*. Englewood Cliffs, NJ: Prentice Hall, 1996.
[4] J. D. Krauss, *Electromagnetics*. New York, NY: McGraw Hill, 1984.
[5] A. A. M. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 128–137, Feb. 1987.