Channel Estimation for Frequency Hopping Systems via Multiple Invariances

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Abstract— **The problem of channel estimation for a frequency hopping system is considered in this paper. Under a discrete multipath fading model, the path gains and delays are estimated separately. By exploiting fixed (but unknown) patterns in the data packet, the delay estimation problem is formulated in an ESPRIT-like format by identifying multiple rotational invariance structures in the received data. Since ESPRIT is limited to exploitation of single invariance, the recently proposed SPECC algorithm is employed for the multipath delay estimation. Performance results based on simulations are presented.**

I. Introduction

Interest in spread spectrum communication has been phenomenal over the past several years. One particular spread spectrum scheme that has generated much interest in the communications community is the code division multiple access (CDMA) scheme which manifests in most discussions in the form of either direct sequence (DS)-CDMA or frequency hopping (FH)-CDMA. FH has been extensively studied as a spread spectrum technique for interference avoidance as opposed to interference attenuation achieved by other spread spectrum techniques [1].

A conventional receiver in an FH system conceptually consists of a bank of filters arranged so that each filter in the bank is responsible for a portion of the total bandwidth [2]. Acquisition, the process of achieving time synchronization, has been the main focus of receiver design for many of the frequency hopping systems [3], [4], [5] with the matched filter or the decorrelator being the usual detection schemes. However, this approach is neither optimal nor does it exploit the diversity inherent in multipath reception of the FH signal. Conventionally, FH systems are considered narrow-band and the so called flat-fading model is invoked thereby ignoring the effect of the multipath delays to a considerable extent.

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In general, optimal detection requires the estimation of channel parameters: the delays and amplitudes associated with each multipath. The characterization of a signal as narrow-band is channel dependent and depends upon the coherence bandwidth of the channel. It was shown in [11], [6] that the flat-fading model is not valid for an FH system with bandwidth comparable to the coherence bandwidth of the multipath channel. Thus estimation of delays assumes significance. An estimation algorithm based on the well known ESPRIT [10] algorithm was developed in [6] and was shown to provide robust estimates of the delays. Unfortunately, the ESPRIT algorithm exploits only single invariance structures in the data. An algorithm similar to but more general than the ESPRIT algorithm is presented in this paper. The capability of this algorithm to exploit multiple invariances will be harnessed to obtain better estimates of the delays in a frequency hopping system.

II. Multipath Delay Estimation

In this paper, we consider the baseband model of the FH received signal in a multipath environment of the form

$$y^{(n)}(t) = \sum_{i=1}^{P} \beta_i e^{-j2\pi f_n \tau_i} s(t-\tau_i; \mathbf{b}_n) + w^{(n)}(t) \qquad (1)$$

where $y^{(n)}(t)$ is the received signal in the nth hop, f_n the frequency in the nth hop, and $\mathbf{b}_n$ the vector of transmitted bits. The transmitted baseband signal $s_n(t, \mathbf{b}_n)$ is distorted by multipath fading with unknown amplitudes β_i and delays τ_i, and white Gaussian noise $w^{(n)}(t)$. We assume that τ_i is sufficiently large so that the flat fading model is no longer accurate [11].

An optimal scheme (not in the sense of minimum error probability) at the receiver for any channel would be maximum likelihood (ML) detection, which performs the following minimization:

$$e(\mathbf{b}_n, \beta_i, \tau_i) := y^{(n)}(t) - \sum_{i=1}^{P} \beta_i e^{-j2\pi f_n \tau_i} s(t - \tau_i; \mathbf{b}_n)$$

$$\{\hat{\mathbf{b}}_n\} = \arg \min_{\mathbf{b}_n, \beta_i, \tau_i} \int_0^{T_h} |e(\mathbf{b}_n, \beta_i, \tau_i)|^2 dt \quad (2)$$

where T_h is the hop period, and $\{\hat{\mathbf{b}}_n\}$ are estimates of the transmitted bits in the nth hop. This minimization is difficult to accomplish particularly due to the nonlinear dependence of the received signal on the delay parameters τ_i. However, the ML-minimization problem becomes tractable if the delays τ_i are assumed known. Since the dependence on the channel gain parameters $\{\beta_i\}$ is linear, the minimization over $\{\beta_i\}$ and $\mathbf{b}_n$ can be separated. This motivates us to seek robust algorithms for the estimation of the delay parameters.

The following assumptions are made throughout the paper:

A1. τ_i's remain constant or vary slowly.

Notice that the channel itself is frequency selective but that does not prevent us from assuming that the delays remain constant over a range of frequencies. Frequency selectivity results from the fact that these delays are large and is only worsened if the delays change with frequency.

A2. Frequency hops from packet to packet.

We assume slow frequency hopping (SFH) and further assume that the frequency does not change within a packet. Commercial use of FH demands that the receiver have a simple structure, and this prohibits frequency changes within a packet.

A3. A string of K' symbols remains constant for all packets.

This string of symbols may be the header (or midamble) of the packet that contains address bits or could even be just sync bits. This can be expected to remain the same for consecutive packets. For example, the Bluetooth standard [8] employs SFH and specifies the use of 72 bits as access code and 54 bits as header information in each packet. The access code is used for synchronization and is the same for all packets originating in the same group. The header consists of the member address, link flow bits, and other control information. Another example is the SFH based PCS system described in [7] which has a total of 16 QPSK symbols or 32 bits in each packet as sync and guard bits.

III. Multiple Invariance Structure

The problem of finding the delays $\{\tau_i\}$ will be cast in the multiple invariance [13] format in this section. The sampled version of (1) can be written as

$$y^{(n)}(kT) = \sum_{i=1}^{P} \beta_i e^{-j2\pi f_n \tau_i} s(kT - \tau_i; \mathbf{b}_n) + w^{(n)}(kT), \quad (3)$$

where T is the sampling period. We now restrict ourselves to the set of samples that correspond to the string of symbols that remain constant for all hops. The discrete time version of the received signal can be written as

$$y^{(n)}[k] = \sum_{i=1}^{P} \beta_i e^{-j2\pi f_n \tau_i} s_i[k] + w^{(n)}[k], k = 1, 2, \cdots, K \quad (4)$$

where $s_i[k]$ correspond to the transmitted signal that is delayed by τ_i and is constant throughout all hops. We make the following remarks about the above model:

1. By restricting to the part of data that are constant among all hops, the signal samples $s_i[k]$ no longer depend on n. However, $s_i[k]$ are still unknown even if the string of symbols is known because they are functions of the unknown delays.
2. The model in (3) is almost identical to the well known directions-of-arrival (DoA) estimation problem in array processing. Here, the hop index n would correspond to the sensor index in DoA estimation. What is different in our formulation is that n varies with time, which corresponds to the DoA estimation problem with a varying number of sensors.

Suppose that we collect data from a number of hops, and partition the frequencies of the received packets into $M + 1$ subsets $\{\mathcal{F}_0, \cdots, \mathcal{F}_M\}$, each with N elements. In particular, $\mathcal{F}_i = \{f_{i1}, \cdots, f_{iN}\}$. Assume also that the partition is such that

$$f_{0i} = f_{1i} + \Delta_f = f_{2i} + 2\Delta_f = \cdots = f_{Mi} + M\Delta_f. \quad (5)$$

In other words, frequency sets $\mathcal{F}_i$ and $\mathcal{F}_{i+1}$ differ by a constant Δ_f. Note that this does not preclude the possibility of some frequencies occurring in more than one set. The proposed estimation algorithm applies also to other types of partitions.

Within the frequency set $\mathcal{F}_i = \{f_{i1}, \cdots, f_{iN}\}$, let f_{ij} be obtained at the n_{ij}th hop. Define

$$\mathbf{y}_i[k] \triangleq [y^{(n_{i1})}[k], \cdots, y^{(n_{iN})}[k]]^T, \quad (6)$$

$$\mathbf{Y}_i \triangleq [\mathbf{y}_i[1], \cdots, \mathbf{y}_i[K]], \quad (7)$$

$$\mathbf{x}[k] \triangleq [\beta_1 s_1[k], \cdots, \beta_P s_P[k]]^T, \quad (8)$$

$$\mathbf{X} \triangleq [\mathbf{x}_i[1], \cdots, \mathbf{x}_i[K]]. \quad (9)$$

With $\mathbf{w}_i[k]$ and $\mathbf{W}_i$ similarly defined for the noise, we have the following multiple invariance structures

$$\begin{aligned}\mathbf{Y}_0 &= \mathbf{A}\mathbf{X} + \mathbf{W}_0\\ \mathbf{Y}_1 &= \mathbf{A}\Phi\mathbf{X} + \mathbf{W}_1\\ \vdots \quad &\vdots \quad \vdots\\ \mathbf{Y}_M &= \mathbf{A}\Phi^{\mathbf{M}}\mathbf{X} + \mathbf{W}_M,\end{aligned}$$

where

$$\mathbf{A} = \begin{pmatrix} e^{-j2\pi f_{p_1}\tau_1} & \dots & e^{-j2\pi f_{p_1}\tau_P}\\ \vdots & \vdots & \vdots\\ e^{-j2\pi f_{p_N}\tau_1} & \dots & e^{-j2\pi f_{p_N}\tau_P}\end{pmatrix}, \quad (10)$$

$$\Phi = diag\{e^{j2\pi\Delta_f\tau_1}, \cdots, e^{j2\pi\Delta_f\tau_P}\}. \quad (11)$$

It is worthwhile noting that Φ is a diagonal matrix and the only unknown parameters in the diagonal elements are the delays. We arrived at this particular structure by imposing condition (5).

The task now is to estimate Φ assuming that data matrices $\mathbf{Y}_0, \cdots, \mathbf{Y}_M$ are the only measurements available. This is accomplished by separating the complex vector space to which the received signals belong into orthogonal subspaces, the signal and noise spaces. Define $\mathbf{Y} \triangleq [\mathbf{Y}_0^T \mid \cdots \mid \mathbf{Y}_M^T]^T$. The auto-correlation matrix $\mathbf{R}_{yy}$ is given by

$$\mathbf{R}_{yy} = E[\mathbf{Y}\mathbf{Y}^H] = \begin{bmatrix} \mathbf{R}_{00} & \cdots & \mathbf{R}_{0M}\\ \vdots & \vdots & \vdots\\ \mathbf{R}_{M0} & \cdots & \mathbf{R}_{MM}\end{bmatrix} + \sigma^2\mathbf{I},$$

where σ^2 is the noise variance. Here $\mathbf{R}_{ij}$ represents the correlation between data from $\mathcal{F}_i$ and that from $\mathcal{F}_j$. In particular, we have

$$\mathbf{R}_{ij} = E[\mathbf{Y}_i\mathbf{Y}_j^H] = \mathbf{A}\Phi^i\mathbf{R}_{xx}(\Phi^H)^j\mathbf{A}^H, \quad (12)$$

where $\mathbf{R}_{xx} = \mathbf{X}\mathbf{X}^H$, which is assumed to be full column rank. This translates to the condition that the various delayed versions of the signal are not fully correlated.

The signal and noise subspace separation is achieved by considering the eigen-decomposition of $\mathbf{R}_{yy}$ as

$$\mathbf{R}_{yy} = \mathbf{U}_s\Sigma_s\mathbf{U}_s^H + \sigma^2\mathbf{U}_n\mathbf{U}_n^H, \quad (13)$$

where $\mathbf{U}_s = [\mathbf{u}_1|\cdots|\mathbf{u}_P]$ and $\mathbf{U}_n = [\mathbf{u}_{P+1}|\cdots|\mathbf{u}_N]$ represent the signal and noise subspaces respectively. Note that the signal space is represented by the eigenvectors corresponding to the P largest eigenvalues of the covariance matrix, while the noise space is represented by the rest. The signal space corresponding to each data set can be obtained by splitting the signal space matrix $\mathbf{U}_s$ as

$$\mathbf{U}_s = \begin{bmatrix}\mathbf{U}_0\\ \vdots\\ \mathbf{U}_M\end{bmatrix}, \quad (14)$$

where $\{\mathbf{U}_0, \cdots, \mathbf{U}_M\}$ are $N \times P$ unitary matrices representing the signal subspaces of the data from the collection of frequency sets $\{\mathcal{F}_0, \cdots, \mathcal{F}_M\}$ respectively. With the covariance matrices related as in (12), the unitary matrices $\{\mathbf{U}_0, \cdots, \mathbf{U}_M\}$ all represent the same subspace, and this subspace is well known to be the same as span$\{\mathbf{A}\}$ [9]. This implies the existence of a $P \times P$ full rank matrix T satisfying

$$\mathbf{U}_0 = \mathbf{A}\mathbf{T}, \quad \mathbf{U}_1 = \mathbf{A}\Phi\mathbf{T}, \quad \cdots, \quad \mathbf{U}_M = \mathbf{A}\Phi^{\mathbf{M}}\mathbf{T}. \quad (15)$$

Defining $\mathbf{B} = \mathbf{A}\mathbf{T}$ and $\Psi = \mathbf{T}^{-1}\Phi\mathbf{T}$, the structure can be rewritten as

$$\mathbf{U}_0 = \mathbf{B}, \quad \mathbf{U}_1 = \mathbf{B}\Psi, \quad \cdots, \quad \mathbf{U}_M = \mathbf{B}\Psi^{\mathbf{M}}. \quad (16)$$

The matrix Φ can be estimated if $\{\mathbf{U}_0, \cdots, \mathbf{U}_M\}$ are known. In practice, the presence of noise and only a finite data set implies that only estimates of the signal subspaces $\{\hat{\mathbf{U}}_0, \cdots, \hat{\mathbf{U}}_M\}$ are available. The search for the true Φ now assumes the form

$$\hat{\Phi} = \arg\min_{\Phi,\mathbf{A},\mathbf{T}}\left(\begin{bmatrix}\hat{\mathbf{U}}_0\\ \vdots\\ \hat{\mathbf{U}}_M\end{bmatrix} - \begin{bmatrix}\mathbf{A}\mathbf{T}\\ \vdots\\ \mathbf{A}\Phi^{\mathbf{M}}\mathbf{T}\end{bmatrix}\right). \quad (17)$$

An equivalent representation is

$$\hat{\Psi} = \arg\min_{\mathbf{B},\Psi}\left(\begin{bmatrix}\hat{\mathbf{U}}_0\\ \vdots\\ \hat{\mathbf{U}}_M\end{bmatrix} - \begin{bmatrix}\mathbf{B}\\ \vdots\\ \mathbf{B}\Psi^{\mathbf{M}}\end{bmatrix}\right). \quad (18)$$

Note that the eigenvalues of Ψ are the diagonal elements of Φ.

When $M = 1$, the above structure reduces to the ESPRIT format and closed-form estimates of the delays can be obtained [6]. In general, the structure described above is in the multiple invariance format and can be solved by invoking the multiple invariance ESPRIT(MI-ESPRIT) scheme [13]. Unfortunately, the solution involves a multidimensional search for the minimizer as opposed to the closed-form ESPRIT solution.

Conventionally, the ESPRIT solution is considered to be a closed-form solution despite the fact that it requires computation of the eigenvalues of a matrix. The SPECC algorithm proposed in [12] exploits the multi-invariance in an alternate form to obtain a closed-form solution for the diagonal elements of Φ. The algorithm is closed-form in the same sense as the ESPRIT algorithm and involves a search for the roots of a polynomial. We exploit this algorithm for the delay estimation problem.

IV. SPECC BASED DELAY ESTIMATION

The SPECC algorithm was based on the following observation. Let $p(\lambda) = \sum_{i=0}^{P} b_i \lambda^i$ be the characteristic polynomial of the $P \times P$ square matrix Ψ. The Cayley-Hamilton theorem enforces the following constraint on Ψ [14]:

$$p(\Psi) = b_0 \mathbf{I}_P + b_1 \Psi + \ \ldots + b_P \Psi^P = \mathbf{0}, \tag{19}$$

where $\mathbf{I}_P$ is the identity matrix of size P. This, along with (16), translates to the following constraint on the signal space:

$$b_0 \mathbf{U}_0 + \ldots + b_P \mathbf{U}_P = \mathbf{0}. \tag{20}$$

The eigenvalues of Ψ can be obtained as roots of $p(\lambda)$ if the coefficients $\{b_i,\ i = 0, \ldots, P\}$ are known. It was shown in [12] that given $\mathbf{U}_i$, the characteristic polynomial is uniquely determined from (20) through the following theorem.

Theorem 1: Let $\mathbf{U}_i$, $\mathbf{B}$ and Ψ be as defined above. Let $p(\lambda)$ be the characteristic polynomial of Ψ. Let $\mathcal{Z}(g)$ denote the set of roots of a polynomial $g(\lambda)$. So the set $\mathcal{Z}(p) = \{\lambda_1, \ldots, \lambda_P\}$ is the set of eigenvalues of Ψ. Consider the minimization

$$\{a_i\} = \arg \min_{\alpha_i, \sum \alpha_i^2 = 1} \|\alpha_0 \mathbf{U}_0 + \ldots + \alpha_P \mathbf{U}_P\|_F. \tag{21}$$

Let $a(\lambda) = a_0 + a_1 \lambda + \ldots + a_P \lambda^P$. Then we have the following
1. If $\mathbf{B}$ is full column rank, $\mathcal{Z}(p) \subseteq \mathcal{Z}(a)$.
2. If Ψ is diagonalizable as $\Psi = \mathbf{MDM}^{-1}$ and $\mathbf{BM}$ has no zero columns, then $\mathcal{Z}(p) \subseteq \mathcal{Z}(a)$.
3. If Ψ has distinct eigenvalues and $\mathbf{BM}$ has no zero columns, then $\mathcal{Z}(p) = \mathcal{Z}(a)$. □

Since the delays we intend to estimate are assumed distinct, the roots of the polynomial $a(\lambda)$ are precisely the eigenvalues of Ψ, from which the delays can be estimated. Note that if Ψ is diagonalizable, then the full rank condition on $\mathbf{B}$ can be relaxed. It suffices to have the row space of $\mathbf{B}$ orthogonal to no eigenvectors of Ψ, which translates to the no zero column condition on $\mathbf{BM}$. Since the Ψ matrix for the estimation problem turns out to be diagonalizable with $\mathbf{M} = \mathbf{T}^{-1}$, this removes the ESPRIT restriction that the invariance be present among at least P data packets in each set and makes it possible to exploit invariances even among data sets with just one packet.

A more general version of the SPECC algorithm has been developed in [15]. Consider a partition of received frequencies into subsets such that the resulting data samples have the following structure

$$\mathbf{Y}_{k_i} = \mathbf{A} \Phi^{k_i} \mathbf{X} + \mathbf{W}_{k_i}, \qquad i = 0, 1, \cdots, M. \tag{22}$$

We assume that $k_i \in \mathcal{Z}^+$, the set of non-negative integers. Following the same development as in section III, the signal subspaces, $\mathbf{U}_{k_i}$ corresponding to the data sets $\mathbf{Y}_{k_i}$ are related as

$$\mathbf{U}_{k_i} = \mathbf{A} \Phi^{k_i} \mathbf{T} = \mathbf{B} \Psi^{k_i}, \qquad i = 0, \cdots, M, \tag{23}$$

where $\mathbf{T}, \mathbf{B}$ and Ψ are again as defined in section III.

The task now is to estimate the eigenvalues of Ψ assuming that the signal subspaces are known. This was shown to be possible in [15] whenever the k_i's are distinct and larger than $P+1$ in number. In other words, distinct invariances greater than P in number can be exploited to obtain delay estimates. The delay estimation algorithm based on SPECC is presented in the next section.

V. THE DELAY ESTIMATION ALGORITHM

Consider a partition of the frequencies of the received packets into subsets $[\{f_{0i}\}, \{f_{k_1 i}\}, \cdots, \{f_{k_M i}\}]$, such that

$$f_{0i} = f_{k_1 i} + k_1 \Delta_f = f_{k_2 i} + k_2 \Delta_f = \cdots = f_{k_M i} + k_M \Delta_f$$

which is a generalization of (5). Since the frequencies of the received packets are randomly distributed, the choice of $\{k_i\}$ dictates the minimum number of packets that have to be received before we can form the subsets of frequencies with the above structure. The data samples corresponding to the frequencies in each set share the rotational invariance structure shown in (22). An estimate of the auto-covariance of the received data is obtained as

$$\hat{\mathbf{R}}_{yy} = \frac{1}{K} \mathbf{Y} \mathbf{Y}^H, \tag{24}$$

where $\mathbf{Y} = [\mathbf{Y}_0 \ |\mathbf{Y}_{k_1} \ | \cdots | \ \mathbf{Y}_{k_M}]^T$. The signal subspace estimates $\{\hat{\mathbf{U}}_{k_i}, i = 0, \cdots, M\}$ can be obtained by considering the eigen-decomposition of $\hat{\mathbf{R}}_{yy}$ as before.

Consider the minimization

$$\{a_{k_i}\} = \arg \min_{\{\alpha_{k_0}, \cdots, \alpha_{k_M}\}} \|\alpha_{k_0} \hat{\mathbf{U}}_{k_0} + \ldots + \alpha_{k_M} \hat{\mathbf{U}}_{k_M}\|_F, \tag{25}$$

subject to the normalization constraint. Note that the structure we constructed has $k_0 = 0$. Obtain roots of the polynomial $a(x) = a_{k_0} x^{k_0} + \cdots + a_{k_M} x^{k_M}$. The estimates $\{\hat{\lambda}_i, i = 1, \cdots, P\}$ of the eigenvalues of Ψ are the

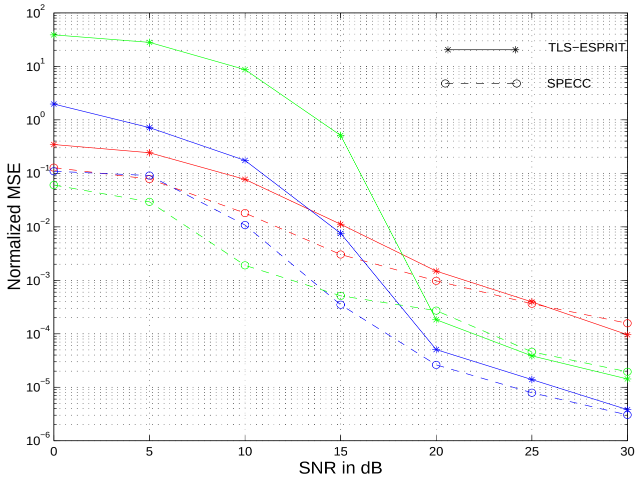

Fig. 1. *Comparison of the MSE for SPECC and TLS-ESPRIT, delays:* $\tau = [1.8\ \ 4.3\ \ 9.0]\mu s$.

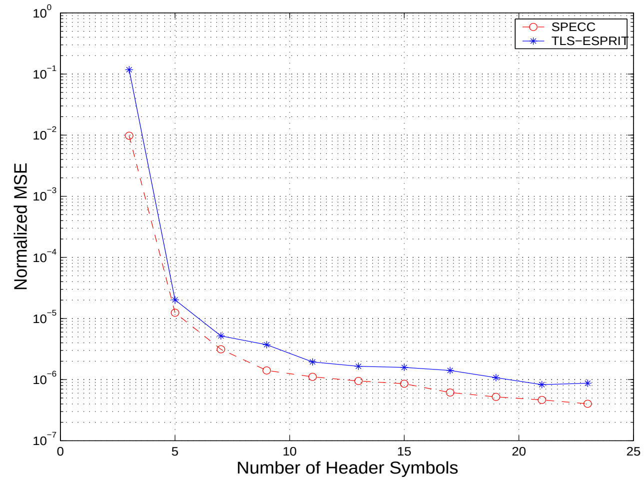

Fig. 2. *Normalized MSE plot for SPECC and TLS-ESPRIT vs. number of header symbols, delays:* $\tau = [\ 1.8\ \ 4.3\ \ 9.0]\mu s$, *SNR* $= 30dB$.

P roots nearest to the unit circle. The delay parameters are estimated as

$$\hat{\tau}_i = \ln \hat{\lambda}_i / (2\pi \Delta_f). \tag{26}$$

Note that the minimization is quadratic in nature and hence has a unique solution that can be obtained by computing the singular value decomposition (SVD) of the appropriate matrix. The particular case $\{M = P, k_0 = 0, \cdots, k_P = P\}$ corresponds to the stated theorem. In this case, the polynomial $a(x)$ is an estimate of the characteristic polynomial of Ψ, and hence the P roots are the required eigenvalue estimates.

VI. Simulation Results

The results in this section are presented by comparing the performance of the SPECC algorithm with that of the ESPRIT. The scenario considered is similar to the one considered for ESPRIT performance evaluation in [6]. Transmission was supposed to occur in the frequency range $1899-1929$ MHz and a QPSK modulation scheme achieving a bit rate of 500 Kbps was assumed. A symbol period of $T = 4\mu s$ was assumed and delays were assumed to be significant compared to the symbol period. In particular, the delay profile was assumed to be $\tau = [1.8,\ 4.3,\ 9.0]\mu s$. The assumption of 3 multipaths is for the sake of convenience; results tend to be identical for larger number of multipaths.

Twenty packets were assumed available at the receiver. We assume that the hop frequencies in these packets have a uniform linear arrangement. This amounts to assuming that the hop frequencies form an arithmetic series so that as many as 19 packets are made available to each data set in the ESPRIT formulation. With $P = 3$ multipaths, the SPECC algorithm needs a minimum of $P + 1 = 4$ data sets and hence the 20 available packets are divided into 4 sets consisting of 5 packets each. Labeling the available hop frequencies as $\{1, 2, \cdots, 20\}$, the data sets considered for the two algorithms are

- ESPRIT: $\{1, 2, \cdots, 19\},\ \ \{2, 3, \cdots, 20\}$
- SPECC: $\{1-5\},\ \{6-10\},\ \{11-15\},\ \{16-20\}$

This structure results in $\{k_0 = 0, k_1 = 1, k_2 = 2, k_3 = 3\}$. Note that the uniform linear variation is a highly unrealistic situation and is being considered only to favor the ESPRIT estimation algorithm. In practice, the situation is more complicated and the estimation is possible only after the receiver has accumulated a sufficient number of packets with the above structure. The header part in each packet was assumed to consist of 8 symbols.

A comparison of the behavior of the normalized mean square against SNR for the SPECC and total-least-squares ESPRIT (TLS-ESPRIT) algorithms is shown in Fig. 1. The MSE is calculated for the delays expressed as a fraction of the symbol period and is hence termed *normalized*. The SPECC algorithm clearly outperforms the TLS-ESPRIT algorithm in estimating the channel parameters. The MSE achieved by the SPECC algorithm is considerably lower than that of the ESPRIT algorithm at low SNRs. The two MSEs approach each other at high SNRs.

Fig. 2 is a comparison plot between the two algorithms with a varying number of header symbols. Since the number of signal samples available is limited by the number of header symbols, signal space estimation tends to be inaccurate. It is clear from the plots that the SPECC algorithm is more robust against inaccurate

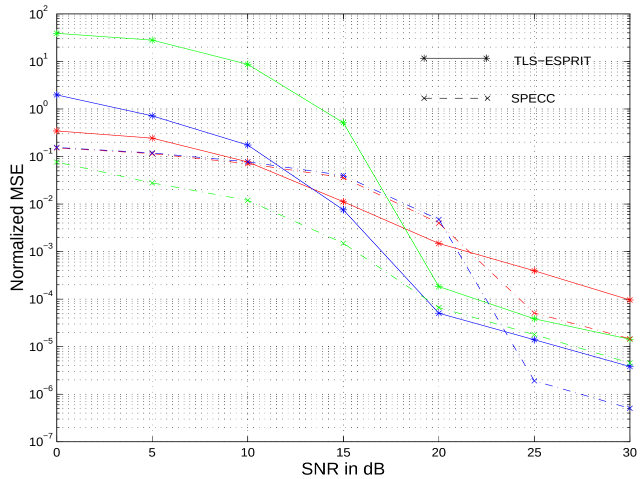

Fig. 3. *Comparison of the MSE for generalized SPECC and TLS-ESPRIT, delays:* $\tau = [1.8\ 4.3\ 9.0]\mu s,\ k_0 = 0,\ k_1 = 2,\ k_2 = 3,\ k_3 = 4.$

subspace estimation when compared to the ESPRIT algorithm. Applications that employ the ESPRIT scheme require a sufficient number of samples to be available for a reliable estimation of the signal subspace. The subspace estimation is further hindered by the presence of noise. The SPECC algorithm exploits a linear constraint on the subspaces as opposed to the rotational invariance exploited by the ESPRIT algorithm and hence is better suited to handle inaccurate estimates of signal subspaces.

To evaluate the performance of the generalized SPECC algorithm, assume that a total of 25 packets are available at the receiver. The packet hop frequencies continue to have a uniform linear arrangement. With the available hop frequencies labeled as $\{1, 2, \cdots, 25\}$, the data partitions considered for the two algorithms are

- ESPRIT: $\{1, 2, \cdots, 19\}, \quad \{2, 3, \cdots, 20\}$
- SPECC: $\{1-5\},\ \{11-15\},\ \{16-20\},\ \{21-25\},$

The total number of packets used by SPECC is still 20, and hence the ESPRIT scheme was made available the same number of packets. Note that this partition corresponds to $M = P$ and $\{k_0 = 0, k_1 = 2, k_2 = 3, k_3 = 4\}$.

The plot of MSE vs. SNR is shown in Fig. 3. The performance is similar to the basic SPECC algorithm and provides better estimates than the ESPRIT algorithm especially at low SNRs.

VII. Conclusion

It was shown that the maximum likelihood detection scheme for a frequency hopping system is much simplified if the multipath delays are known. The estimation of the multipath delays was cast in the multiple invariance format and the SPECC algorithm was employed to obtain robust estimates of the multipath delays. The algorithm was shown to perform better than the single invariance ESPRIT scheme. It was also demonstrated that the SPECC algorithm is more robust against inaccurate estimation of the signal subspaces that results as a consequence of limitations in the number of header symbols available for estimation.

The SPECC algorithm has been applied for the channel estimation problem where a small number of packets share multiple invariances. In general, the SPECC algorithm can be applied in all scenarios where ESPRIT is applicable and, in particular, SPECC is suitable for applications with small data sets sharing multiple invariances.

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