Optimal Pricing for Residential Demand Response: A Stochastic Optimization Approach

Liyan Jia and Lang Tong

***Abstract*— The problem of optimizing retail electricity price for residential demand response is considered. A two stage stochastic optimization is formulated in which the retailer optimizes the day ahead price in the first stage, and residential customers schedule their demands optimally in respond to the optimized retail price and in a distributed fashion. For the control of thermal dynamic loads, the optimal residential demand response policy is obtained based on a form of consumer surplus that captures the tradeoff between comfort level and cost. It is shown that the optimal control is an affine function of the retail price with a negative definitive factor matrix. The optimal retail pricing is obtained through a convex program that maximizes average profit or a form of conditional value at risk. Effects of incorporating renewable energy are also considered.**

I. INTRODUCTION

In a conventional demand response program, a residential consumer benefits from reduced electricity price by giving the retailer a level of control of his energy use. The retailer, on the other hand, benefits from such a program by shaping the aggregated load so as to maximize the profit in the presence of operation and wholesale price uncertainties. As an example, the consumer may be offered a lowered electricity price by allowing the retailer to interrupt his services a number of hours in a year [1].

One of the barriers to the wide adoption of demand response is the intrusive nature of such programs. A consumer may feel uncomfortable for letting the utility affect his lifestyle in such a direct and unpredictable fashion. And it is difficult to price the inconvenience caused by interruptions at some unknown time. From a retailer's perspective, the implementation of demand response for a large number of customers is highly nontrivial, even though the underlying technologies have existed for decades.

We consider in this paper an alternative residential demand response framework. The objective of the proposed scheme is twofold. First, it gives the consumer full control in scheduling his own energy usage in responding to a (day-ahead) price from the retailer, which removes the retailer from implementing and managing a control system that involves a large number of distributed components. Second, the retailer optimizes its profit by taking into account the volatility of the wholesale price, the availability of low cost renewable sources, and response behavior of the residential customers.

Because the proposed scheme allows the consumers choose the state of the art technology in home energy management and the retailer adjusts its (daily) price based on the operating conditions of the network, it is hoped that the prosed scheme is more compatible with existing modus operandi, potentially offering a more attractive path to broader adoption.

Furthermore, by shielding the consumer from reacting to real-time wholesale price fluctuations, such a demand response program reduces price volatility and the potential of instability postulated recently in [2].

A. Summary of Results

In this paper, we propose a new retail market mechanism in which the retailer optimizes a day ahead price for residential customers who engage in distributed demand response in scheduling price-elastic load. The proposed market structure is compatible with the current deregulated wholesale market and offers the customers to decide their own energy usage pattern in response to different retail prices.

We formulate the problem of optimal pricing for residential demand response as a two-stage stochastic optimization. To this end, we first consider optimal demand response given a fixed day ahead retail price. Using a criterion based on a linear combination of the cost of electricity and the quadratic deviation of the desired temperature setting, the optimal control is shown to be an affine function of price with a negative definite factor matrix. It is this relationship that leads to a convex optimization of retail price at the retailer end.

We also consider the problem of integrating stochastic renewable generation at the retail level. In particular, we assume that the retailer has access to low cost renewable sources, which allow the retailer reduce the retail price in exchange for a higher volume. We show that the problem of optimal pricing remains convex. We also demonstrate that the accuracy of prediction of renewable generation affects the profit of the retailer in a monotonic fashion.

B. Related Work

Although extensive research has been conducted on the wholesale electricity market, limited attentions have been paid to the retail market. Among the earliest studies of retail electricity market is [3] where the authors present simulation studies of expected profit for retailers. The authors of [4], [5] present a more elaborate formulation of retail markets that incorporate load models, retailers profit functions, and financial risks. Our work is related to [3,4] but different in several important aspects. Specifically, the results in this paper establish the dynamic demand side response to different prices by the retailer. When incorporating random phenomena in stochastic optimization, our approach avoids approximating continuous random variables through quantization. The pricing scheme considered in this paper is also different from and appears to be more flexible than that in [4], [5].

Recent work of Yang, Tang, and Nehorai [6] considers a similar retail market structure. Their work is based on an abstract characterization of static interactions between a retailer and consumers. The scenario considered in this paper incorporates thermal dynamics, resulting a policy involving optimal dynamic demand response. In [7], competition among retailers is considered. Their analysis is based on market share modeling and each retailer is using a mixed strategy.

Given a specific market structure, a key step in obtaining optimal pricing is to establish an optimal or suboptimal demand response policy at the consumer end. To this end, there have been several approaches, all appear to be attacking the problem of optimal demand response without coupling with the optimal pricing problem. For example, authors of [8] use statistical methods to model the household electricity demand, and addresses the fact that accurate modeling is very difficult. In [9], Callaway and Hiskens discussed the some key issues and applications of nondisruptive control strategies for aggregated electric loads. In terms of responding to different prices, the approaches in [10], [11], [12] used MPC (Model Predictive Control) method to control HVAC.

II. MARKET STRUCTURE

The proposed residential electricity retail market structure consists of three main components: the wholesale electricity market, the retail market, and the residential demand, as shown in Fig. 1. We describe in this section briefly models for each component. As a notational convention, we use ω to indicate a sample in the native probability space. When necessary, a random variable is written explicitly as a function of ω, eg. $x(\omega)$.

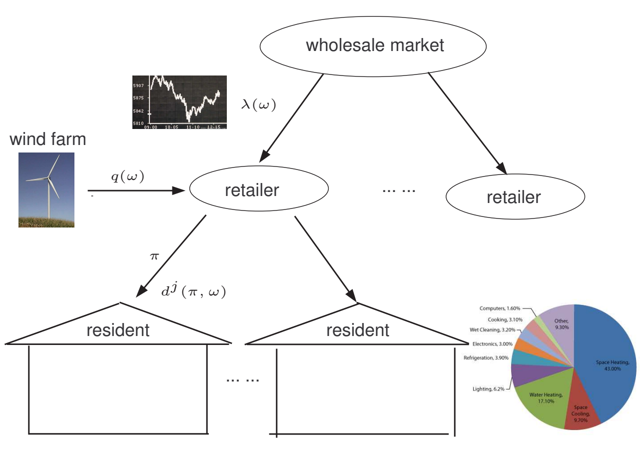

Fig. 1: Market structure

A. The wholesale electricity market

We assume a deregulated wholesale electricity market where the cost of electricity to a retailer is determined in a two settlement system: a day ahead market that generates the day ahead wholesale price and a real-time market that provide necessary adjustments based on the actual operating condition.

Thus the actual cost to a retailer can be modeled as a 24 dimensional random vector $\lambda(\omega)$ with known mean (at the day ahead wholesale price) and variance. Note that the wholesale price in the peak hours might be tens or even hundreds times of the regular price. Such kind of price uncertainty imposes high risk to the retailer who must hedge against its exposure to price hikes when optimizing the retail price.

B. The retail market

Serving as an intermediate agent between the wholesale market and end customers, the retailer has to guarantee power flow to the customer while maintaining a healthy cash flow for itself. In our setting, any level of demand from the end customers has to be satisfied by acquiring power from either the wholesale market or, if possible, from alternatives such as its own renewable generation $q(\omega)$ or through bilateral contracts with independent wind farms.

Previous study shows that allowing real-time response to wholesale market price may cause price instability [2]. In our setting, the retailer shields the end users from the risk of real-time price fluctuations by offering its customers a fixed day ahead price $\pi = (\pi_1, \cdots, \pi_{24})$. In the following, without loss of generality, we assume the retailer provides hourly price. However, the period length is flexible to be generalized.

While this retail market avoids price shocks to consumers who expect the price of electricity stays within a certain nominal range, the challenge of market design is to maximize the retail profit by hedging against uncertainties in the wholesale market, the availability of renewable generation, and exploiting of the demand response behavior of the end users.

C. Residential demand and thermal dynamics

Let the aggregated demand from end customers be denoted as $d(\pi, \omega)$, which is a function of the deterministic retail price π and random factors such as weather conditions.

We partition the load into two components,

$$d(\omega, \pi) = p(\pi, \omega) + p_u(\omega), \tag{1}$$

where $p(\pi, \omega)$ is the price elastic component that can be controlled by the residential customer and $p_u(\omega)$ the price inelastic component that is only affected by random factors in electricity usage.

According to [13] in which the U.S. Energy Information Administration reports that the dominant residential electricity usage comes from space heating and air conditioning, in this paper, we assume that the price elastic demand comes primarily from the control of a certain HVAC unit that maintains the indoor temperature at a certain desirable setting.

To this end, we assume that the thermal energy state of a residential home satisfies a linear state space model as detailed below.

III. OPTIMAL RESIDENTIAL DEMAND RESPONSE

We consider in this section the optimal demand response to the day ahead retail price π by the residential customers. Here we specialize a particular thermal dynamic model involving an HVAC temperature control.

Consider a single residential home. Let x_i be the indoor temperature at hour i. Empirical studies [14], [15], [11], [12] have shown that the dynamic equation that governs the temperature evolution is given by

$$x_i = x_{i-1} + \alpha(a_i - x_{i-1}) - \beta p_i + w_i, \tag{2}$$

where a_i is the outdoor random temperature at hour i, p_i the control variable representing the power drawn by the HVAC unit and w_i the process noise. System parameters α and β model the insolation of the building and efficiency of the HVAC unit. Note that the above equation applies to both heating and cooling scenarios but not simultaneously. We focus herein the cooling scenario and the results apply to heating as well.

To control the HVAC, temperature measurement values need to be collected. We assume thermal meters are implemented both for indoor and outdoor temperatures. The measurement equation is

$$y_i = \begin{bmatrix} x_i & a_i \end{bmatrix}^T + v_i, \tag{3}$$

where v_i is the measurement noise.

Assume at hour i, the resident wants to keep the indoor temperature as t_i. The deviation of actual indoor temperature x_i from t_i can be used to measure the resident's uncomfort level. Hence, a reasonable residential utility function is

$$u(x) = -\mu \sum_i (x_i - t_i)^2, \tag{4}$$

where μ is a weight factor to convert the deviation of x_i from t_i to money.

Given the retail price π, the objective of residential demand response is to maximize the consumer surplus defined as the difference of utility and energy payment. Specifically, the residential optimal stochastic demand response is defined as the solution to following optimal control problem,

$$\begin{array}{rl} \min_p & \mathbb{E}\left(\sum_{i=1}^{24}(\pi_i p_i + \mu(x_i - t_i)^2)\right) \\ \text{s.t.} & x_i = x_{i-1} + \alpha(a_i - x_{i-1}) - \beta p_i + w_i, \\ & y_i = \begin{bmatrix} x_i & a_i \end{bmatrix}^T + v_i. \end{array} \tag{5}$$

For computation convenience, under mild conditions (the price doesn't vary too much during a day and μ is large), we ignore the positive constraint and rate constraint for energy consumption p.

Assume w_i and v_i are jointly Gaussian. Backward induction gives a well structured solution

$$\begin{array}{rcl} p_i^* & = & \frac{1}{\beta}\left(\hat{x}_{i-1|i-1} + \alpha(\hat{a}_{i|i-1} - \hat{x}_{i-1|i-1}) - x_i^*\right), \\ x_i^* & \triangleq & \frac{\pi_i - (1-\alpha)\pi_{i+1}}{2\mu\beta} + t_i, \end{array} \tag{6}$$

where $\hat{x}_{i-1|i-1}$ and $\hat{a}_{i|i-1}$ are the estimated indoor and outdoor temperatures based on observations up to hour $i-1$ respectively, and x_i^* is an ancillary value.

Notice that x_i^* can be viewed as the indoor temperature target for hour i at hour $i-1$. If $w_i = 0$, $\hat{x}_{i-1|i-1} = x_{i-1}$ and $\hat{a}_{i|i-1} = a_{i-1}$, then applying p_i^* will lead the actual indoor temperature at hour i, $x_i = x_i^*$.

This problem is almost the same as the classical LQG (Linear Quadratic Gaussian) control problem. The solution can be viewed as the certainty equivalence [16] with a form of separation principle where the certainty equivalence is implemented by conditional expectation on noisy measurements. Expanding and writing the solution p_i^* matrix form will give us the following theorem

Theorem 1: For fixed retail price π, assume that residential load k has the form

$$d^k(\pi, \omega) = p^k(\pi, \omega) + p_u^k(\omega). \tag{7}$$

Assuming optimal demand response, the aggregated demand

$$d(\pi, \omega) = \sum_k d^k(\pi, \omega) = -G\pi + c(\omega), \tag{8}$$

where matrix $G \geq 0$ is positive definite and deterministic, depending only on the dynamic system parameter.

Proof: Let sup k denote the parameter associated with load k. For individual load k, expanding the form of Eq. (6) will give $p_i^k(\pi,\omega) = \frac{1}{\beta^k}((1-\alpha)\pi_{i-1} - (1+(1-\alpha)^2)\pi_i + (1-\alpha)\pi_{i+1})) + b^k(\omega)$, where $b^k(\omega)$ is independent on π. So the total demand of user k is $d^k(\pi,\omega) = p^k(\pi,\omega) + p_u^k(\omega) = -G^k\pi + c^k(\omega)$, where $c^k(\omega) = b^k(\omega) + p_u^k(\omega)$ and G^k satisfies

$$G_{ij}^k = \begin{cases} 1 + (1-\alpha)^2)/\beta^k & \text{if } i = j \\ -1 + \alpha & \text{if } |i-j| = 1 \\ 0 & \text{o.w.} \end{cases} \tag{9}$$

Notice G^k is deterministic and diagonal dominant with positive diagonal elements. Hence, G^k is positive definite.

On the other hand, the aggregated demand

$$d(\pi,\omega) = \sum_k d^k(\pi,\omega) = \sum_k(-G^k\pi + c^k(\omega)) = -G\pi + c(\omega), \tag{10}$$

where $c(\omega) = \sum c^k(\omega)$, $G = \sum G^k$. Since G^k is positive definite and deterministic, G is also positive definite and deterministic, depending only on the dynamic system parameter. ∎

Eq. (8) gives an affine form of residential demand response. The property that G is positive definite is important to our later discussion.

IV. OPTIMAL RETAIL PRICING

In this section, we focus on optimizing pricing for the retailer. We consider two different objectives: the expected profit and a measure derived from the conditional value at risk (CVaR); the latter represents a form of robustness.

A. Optimal retail price over expected profit

Recall that the wholesale price of electricity is a random vector $\lambda(\omega) = (\lambda_1(\omega), ..., \lambda_{24}(\omega))$, which represents the marginal cost to the retailer. Given the day-ahead price π and the randomness realization ω, the total profit of the retailer can be represented as the product of the demand quantity and the net unit profit, which is the difference between the day-ahead retail price π and the cost λ.

$$\begin{aligned} r(\pi,\omega) \quad &= (\pi-\lambda(\omega))^T d(\pi,\omega) \\ &= (\pi-\lambda(\omega))^T(-G\pi+c(\omega)). \end{aligned} \tag{11}$$

Assume the retailer knows the distribution of the cost $\lambda(\omega)$ and the stochastic demand response $d(\pi,\omega)$, the expected profit of the retailer can be calculated as Eq. (12). Since G is positive definite, $\mathbb{E}[r(\pi,\omega)]$ is a concave function of π.

$$\begin{aligned} \mathbb{E}[r(\pi,\omega)] \quad = \quad & -\pi^T G\pi + (\mathbb{E}[\lambda])^T G\pi \\ & +(\mathbb{E}[c])^T\pi - \mathbb{E}[\lambda^T c]. \end{aligned} \tag{12}$$

On the other hand, the pricing behavior of the electricity retailer will be regulated by many factors. In this paper, we simplify all the regulations into one price cap $\bar{\pi}$, i.e. the retailer's price $\pi_i \leq \bar{\pi}$. Then, the retailer's optimal pricing strategy over expected profit can be formulated as the following quadratic programming

$$\begin{array}{ll} \max & -\pi^T G\pi + (\mathbb{E}[\lambda])^T G\pi + (\mathbb{E}[c])^T\pi - \mathbb{E}[\lambda^T c] \\ \text{s.t.} & \pi_i \leq \bar{\pi}. \end{array} \tag{13}$$

The objective is concave and the constraint is linear. Hence, the optimization is convex. All the classical nonlinear convex programming methods apply to this problem and the solution can be easily found.

B. Optimal retail price over CVaR

In the presence of uncertainty, maximizing the expected profit is not always the best choice, especially when the random variable has a long tail distribution. One alternative is to use risk measures as metric to make decisions. CVaR (Conditional Value at Risk, also know as Expected shortfall) is one commonly used coherent risk measure. The CVaR at γ level is defined as the expected profit in the worst γ of the cases, as shown in Eq. (14).

$$\gamma\text{-CVaR} = \mathbb{E}_\omega[r(\pi,\omega)|r(\pi,\omega) < \tau_\gamma(\pi)], \tag{14}$$

where $\tau_\gamma(\pi) = \inf\{\tau \in \mathbb{R} : \mathbb{P}[r(\pi,\omega) < \tau] \geq \gamma\}$.

Using the equivalent form of CVaR in [17], the retailer's best pricing strategy over CVaR can be formulated as the following optimization problem with the price cap constraint.

$$\begin{array}{ll} \max_{\xi,\pi} & \xi - \frac{1}{\gamma}\mathbb{E}_\omega[\xi - r(\pi,\omega)]^+ \\ \text{s.t.} & \pi_i \leq \bar{\pi}. \end{array} \tag{15}$$

Theorem 2: Problem (15) is a convex programming with π as the variable.

Proof: According to the result of [17], γ-CVaR is a concave function with respect to the decision variable as long as the value function is concave with respect to the decision variable. Since $r(\pi,\omega) = (\pi-\lambda(\omega))^T(-G\pi+c(\omega))$ and G is a positive definite matrix, r is a concave function of π. Hence the objective function is concave. Maximizing a concave function with linear constraints is a convex programming problem. ■

The convex property makes this problem solvable by many nonlinear convex programming methods. Notice that we do not make any assumption that the random variables are discrete, although in practice we can do disretization to simplify the computation. In that case, the objective function will become piecewise linear concave.

V. EFFECTS OF WIND INTEGRATION

We now consider a scenario in which the retailer has access to renewable sources such as a wind farm. We assume that the cost of renewable to the retailer is zero. However, the generation of renewable is stochastic that cannot be controlled by the retailer. The retailer does know the distribution of the renewable generation. This knowledge will be exploited for maximizing its profit. The wind power is used to supply the residential load, but if it is larger than needed, extra wind power cannot be sold back to the wholesale market.

Denote the wind generation for the next day as a 24 dimensional random vector $q(\omega) = (q_1(\omega), ..., q_{24}(\omega))$. Similar to Eq. (11), given the day-ahead price π and the demand response $d(\pi,\omega)$, the retailer's profit with wind $r_w(\pi,\omega)$ can be represented as

$$\begin{aligned} r_w(\pi,\omega) = \quad & \pi^T d(\pi,\omega) - \lambda(\omega)^T(d(\pi,\omega)-q(\omega))^+ \\ = \quad & \pi^T(-G\pi+c(\omega)) \\ & -\lambda(\omega)^T(-G\pi+c(\omega)-q(\omega))^+. \end{aligned} \tag{16}$$

The following theorem gives the nice property of the retailer's objectives after incorporating wind power.

Theorem 3: $\mathbb{E}[r_w]$ and γ-CVaR are both concave functions of π.

Proof: For each ω, $(-G\pi+c(\omega)-q(\omega))^+$ is equivalent to $\max\{-G\pi+c(\omega)-q(\omega), 0\}$. Both of $-G\pi+c(\omega)-q(\omega)$ and 0 are linear functions hence convex. The max of two convex function is convex. So $-\lambda(\omega)^T(-G\pi+c(\omega)-q(\omega))^+$ is concave. Since G is positive definite, $\pi^T(-G\pi+c(\omega)$ is concave. As the sum of two concave functions, r_w is concave. So both of $\mathbb{E}[r_w]$ and γ-CVaR are concave functions of π.

■

Hence, the retailer's optimization problems in terms of both expected profit and CVaR remain as convex programming, which are solvable by numerical nonlinear programming methods. In the simulation part, we will test how the uncertainty of wind power affects the retailer's objective values.

VI. SIMULATION RESULTS

In order to illustrate the optimality of the proposed pricing scheme, two alternative commonly used pricing schemes are compared to the optimal pricing: constant pricing scheme and constant mark-up pricing scheme. Constant pricing means the retailer will offer constant price for the next day, i.e. $\pi_1 = \pi_2 = ... = \pi_{24}$. The constant mark-up pricing scheme is to design the price according to the prediction of next day's cost λ, and each hour's price has the same mark-up over the predicted cost, i.e. $\frac{\pi_1}{\mathbb{E}\lambda_1} = \frac{\pi_2}{\mathbb{E}\lambda_2} = ... = \frac{\pi_{24}}{\mathbb{E}\lambda_{24}}$.

We use the optimal pricing scheme as benchmark. The y-axis is the expected profit of constant pricing or constant mark-up pricing divided by the expected profit of optimal pricing scheme. The x-axis is the average price of the two compared pricing schemes, i.e. $\frac{\sum_i \pi_i}{24}$. The parameter for testing is the same as [12]. The result is shown in Fig. 2, from which we can see that the proposed pricing scheme is much better than the two dummy pricing schemes.

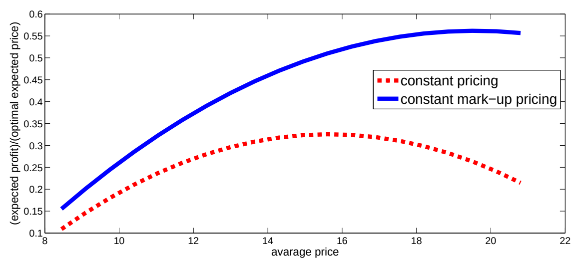

Fig. 2: Expected profit comparison of three pricing schemes

The same simulation is conducted for the pricing scheme over CVaR, as shown in Fig. 3. In terms of CVaR, the advantage of using optimal pricing scheme over dummy alternatives is more significant.

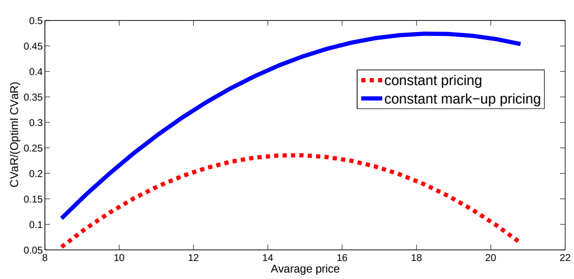

Fig. 3: CVaR comparison of three pricing schemes

After incorporating wind power, we show that the problem can still be formulated as a convex program. Now we want to test how the uncertainty of the wind power will affect the retailer's objective, both expected profit and CVaR. Assume the wind power is uniformly distributed over $[\bar{q} - \Delta/2, \bar{q} + \Delta/2]$. Fix $\bar{q}$, simple calculation can give us that $\frac{\partial r_w}{\partial \Delta} \leq 0$. So better prediction will give the retailer more profit, as shown in Fig. 4. The x-axis is the uncertainty Δ, and the y-axis is the expected profit.

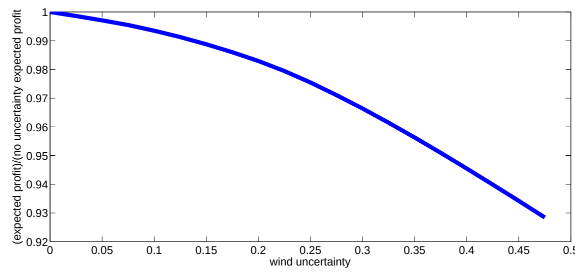

Fig. 4: Wind uncertainty vs. retailer's expected profit

As for CVaR, although analytical result is hard to show, intuitively, large uncertainty of wind will cause more loss for the worst cases. Hence CVaR will be decrease as the uncertainty increases as shown in Fig. 5. Comparing Fig. 4 and Fig. 5, the uncertainty affects the retailer's CVaR more significantly than expected profit.

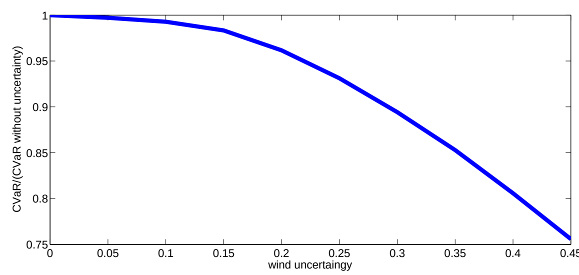

Fig. 5: Wind uncertainty vs. retailer's CVaR

Then we fix the uncertainty Δ but increase the average wind level, $\bar{q}$. The result is shown in Fig. 6. The x-axis is the average wind divided by the base case and the y-axis is the corresponding CVaR divided by the CVaR at the base case. The CVaR increases monotonically as expected since more wind production will increase the retailer's profit for each realization.

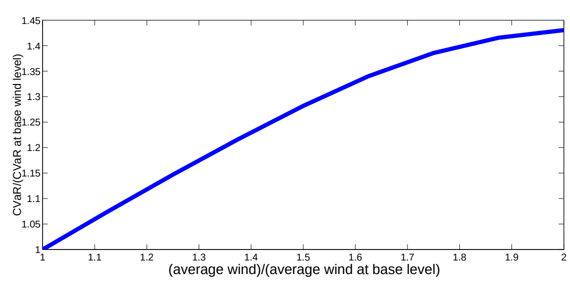

Fig. 6: Average wind vs. retailer's CVaR

VII. CONCLUSION AND FUTURE WORK

In this paper, we propose a new retail market structure with fixed day-ahead retail price from retailer and real-time stochastic demand response from the residents. The problem of optimal retail pricing for residential demand response is formulated as a two-stage stochastic optimization. A closed form solution to the optimal residential demand response is shown to be an affine function of price with a negative definite factor matrix. At the retailer side, optimal pricing strategy is given as the solution to a convex program, over both expected profit and CVaR. Finally, effect of incorporating wind power is considered. It is shown that the problem of optimal pricing remains convex. We also demonstrate that the accuracy of prediction of renewable generation affects the profit of the retailer in a monotonic fashion.

The framework in this paper is compatible with the current deregulated wholesale market and flexible for generalizations. More comprehensive resident side model will help to get more precise stochastic demand form. Deferrable loads, energy storage, and other controllable loads need to be considered. In addition, competition among different retailers in a more competitive environment can be formulated within the current framework. These issues are currently under investigation.

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