The Nose of a Bloodhound: Target Chasing Aided by a Static Sensor Network

Oliver Kosut and Lang Tong

***Abstract*— The benefits of a large static sensor network to the problem of target chasing are explored. It is shown that a chaser is able to track down a moving target much more quickly with the aid of the static sensor network than without it. These sensors need not communicate with each other, only with the chaser when it comes near. In fact, it is enough for the sensors to remember only when the target was last nearby in order for the chaser to efficiently intercept the target. This can be likened to the target leaving a trail from which the chaser is able to determine how long it has been since the target has been to a particular location, much like how a bloodhound is able to detect the scent of its prey. This trail gives no information about which direction the target was going, only when it was there, but this is still enough to find it.**

I. Introduction

Consider the following target tracking problem. A target performs a simple random walk in an infinite two dimensional grid, moving to one of the four adjacent grid points with equal probability every second. A chaser, also able to move one grid point every second, wishes to find the target. Assuming the chaser has only local sensing capabilities, its only choice is to wander the field with no knowledge of the target's location until it eventually stumbles into the target. Under these conditions, the chaser cannot find the target quickly. Indeed, due to the nature of the random walk, the best the chaser can do is to find the target with probability one, but in infinite expected time.

Now suppose that sitting on each grid point is a static sensor, able to detect when the target comes to its point. We assume that these sensors cannot communicate with one another, only the chaser. In particular, a static sensor can deliver to the chaser its accumulated information about when the target was present only if the chaser stands on the same grid point as the sensor. In fact, it is enough for the sensors to remember only how long it has been since the last time the target visited them, and for the chaser to have no sensing capability at all except to know whether it has met the target exactly. One can imagine that as the target moves through the field, it leaves a trail of information embedded in the static sensors' memory, a trail that can be detected by the chaser. We will show that with this additional capability, the chaser is able to meet the target in finite expected time. Moreover, the chaser can meet the target in approximately $\mathcal{O}(d^{\frac{4}{3}})$ expected time, where d is the initial distance between target and chaser, but no faster.

The architectural concept of the mobile agent employing a static sensor network to accomplish some task originated in the Sensor Network with Mobile Access (SENMA) Testbed [1]. This work differs primarily from many other investigations into tracking with wireless sensors networks, such as [2], in the assumption that there is no communication between static sensors. This assumption dramatically limits the performance of the network, because direct communication between sensors would allow them to quickly inform the chaser of the target's location simply by relaying information through the network. Therefore this work would only have applications in situations in which power conservation is of the utmost importance. Indeed, RFID can be used to implement communication and memory in the static sensor requiring no local power whatsoever [3]. Thus the sensors could be implemented as devices that have powered sensing and unpowered communication.

The rest of the paper is organized as follows. Section II formally introduces the model and gives the main result. In Section III, we propose a chasing strategy and prove a bound on its performance. In Section IV, we prove a lower bound on the possible performance of any chasing strategy. Section V presents some simulation results. Finally, in Section VI we conclude and give some directions for future work.

II. Main Result

We assume that when the target begins walking on the field at time $n = 0$, the chaser has no knowledge or prior distribution of its initial whereabouts. Therefore the chasing strategy must be designed to handle any possible starting location, and the performance of the strategy will depend on the relative starting locations of the target and chaser. Since the infinite grid is invariant to translation, we may assume that the chaser starts at the origin. Let $\vec{s} \in \mathbb{Z}^2$ be the starting location of the target.

Let Γ be the set of all possible chasing strategies. For any $\gamma \in \Gamma$, let $N_\gamma(\vec{s})$ be the expected time for the chaser to find the target using strategy γ if the target starts at $\vec{s}$. Let

$$\tilde{N}_\gamma(d) \triangleq \sup_{\vec{s}:\|\vec{s}\|\leq d} N_\gamma(\vec{s})$$

where $\|\cdot\|$ is the Euclidean norm. That is, $\tilde{N}_\gamma(d)$ is the worst expected performance of γ when the target and chaser begin at most d distance apart.

Theorem 1: There exists a chasing strategy $\gamma \in \Gamma$ such that

$$\tilde{N}_\gamma(d) = \mathcal{O}(d^\alpha)$$

for any $\alpha > \frac{4}{3}$. Moreover, for all $\gamma \in \Gamma$,

$$\tilde{N}_\gamma(d) = \Omega\left(\frac{d^{\frac{4}{3}}}{\sqrt{\log\log d}}\right).$$

The first part of the theorem is proved in Section III by proposing a chasing strategy and then proving that it achieves

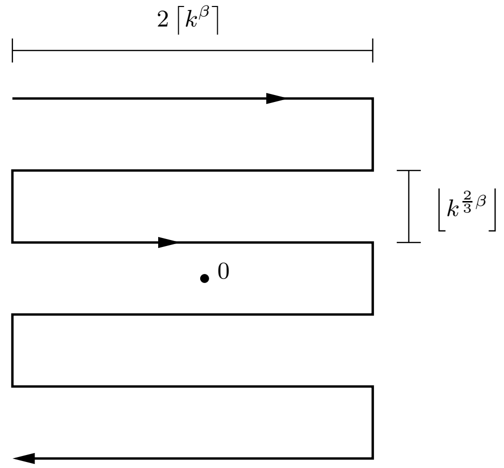

Fig. 1. The path followed by the chaser in the first stage for some k. What is shown here is followed by the same pattern rotated vertically, thereby forming a complete gridding of the square.

$\mathcal{O}(d^{\alpha})$ for any $\alpha > \frac{4}{3}$. The second part is proved in Section IV by invoking the law of the iterated logarithm and using it to argue that the chaser must come within a certain distance of every point, which requires at least $\Omega\left(\frac{d^{\frac{4}{3}}}{\sqrt{\log\log d}}\right)$ time.

The bounds on performance given by Theorem 1 are obviously not tight. It is unknown whether there exists a strategy $\gamma \in \Gamma$ for which $\tilde{N}_{\gamma}(d) = \mathcal{O}(d^{\frac{4}{3}} f(d))$, where $f(d)$ is any sub-power-law function such that $f(d) = \Omega(1/\sqrt{\log\log d})$.

III. Chasing Strategy

Fix $\alpha > \frac{4}{3}$. We propose a chasing strategy that achieves $\tilde{N}_{\gamma}(d) = \mathcal{O}(d^{\alpha})$. This strategy is composed of two stages, the first meant to find the target's trail, the second meant to catch up to the target once it has done so. First we describe the two stages. Then, in Section III-A, we introduce some analytical tools necessary to prove upper bounds on the expected time to complete our proposed strategy. In Section III-B, we show that the expected time to perform the first stage is $\mathcal{O}(d^{\alpha})$, and then in Section III-C that the expected time to perform the second stage is $\mathcal{O}(n)$, where n is the time taken to perform the first stage.

In the first stage, the chaser performs the following procedure until it comes upon a point on the target's path. Let $\beta \triangleq \frac{1}{\alpha - 4/3}$. Beginning with $k = 1$, and then for successive positive integers, the chaser traverses the interior of a square centered at the origin with radius $\lceil k^{\beta}\rceil$ (i.e. side length $2\lceil k^{\beta}\rceil$) in the following manner. It begins by moving to the nearest corner of this square, then moves back and forth across the length of the square, tracing out horizontal lines $\lfloor k^{\frac{2}{3}\beta}\rfloor$ apart until it reaches the opposite edge (see Figure 1). Then it repeats this procedure for vertical lines. Thus, after this traversal, the chaser has drawn out a grid with separation $\lfloor k^{\frac{2}{3}\beta}\rfloor$ on the square of radius $\lceil k^{\beta}\rceil$. Finally it increments k and repeats.

The second stage begins with the chaser having just found a point on the target's path. Whenever the chaser comes upon such a point, we define the *time lag* as the time since the target came to this point, i.e. the age of this part of the trail. The chaser travels $\lfloor\sqrt{n}\rfloor$ grid points in some direction, where n is the time lag, then travels along the edge of a square of radius $\lceil\sqrt{n}\rceil$ centered at the point it has just found (see Figure 2). It continues moving around the edge of this square until it finds a point on the target's path that is more recent than the one it has already found. Once it does, it recalculates the time lag n and begins a new square. The chaser repeats this procedure until it meets the target exactly.

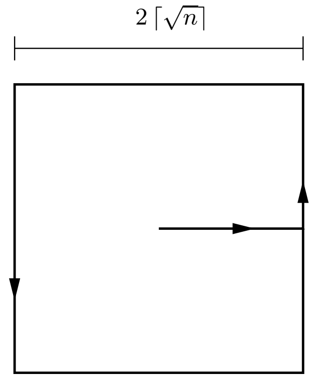

Fig. 2. The stage two chasing procedure. The chaser continues around the edge of this square until it comes upon a more recent point on the target's path.

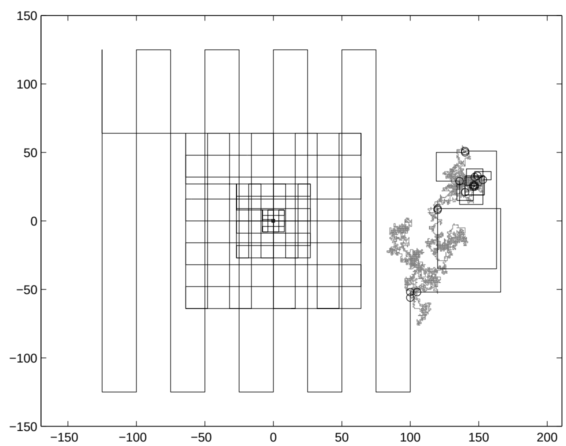

Fig. 3. An example run of our proposed chasing strategy. The black line is the chaser's path and the gray line is the target's path. Circles represent points where the chaser found a more recent point on the target's path.

Figure 3 shows a simulated example run of this strategy.

A. Analytical Preliminaries

Define the random variable U_a as the time it takes for a one dimensional simple random walk to move a steps from its initial position. It can be shown [4, Theorem 2.13] that if a_j and n_j are positive sequences with $\frac{n_j^{1/3}}{a_j} = o(1)$, then for any $\epsilon > 0$ and sufficiently large j,

$$\Pr(U_{a_j} > n_j) \leq \frac{4(1+\epsilon)}{\pi} \exp\left\{-\frac{\pi^2 n_j}{8a_j^2}\right\} \tag{1}$$

and

$$\Pr(U_{a_j} > n_j) \geq \frac{4(1-\epsilon)}{\pi}\left[\exp\left\{-\frac{\pi^2 n_j}{8a_j^2}\right\} - \frac{1}{3}\exp\left\{-\frac{9\pi^2 n_j}{8a_j^2}\right\}\right]. \quad (2)$$

Now consider a two dimensional simple random walk. Such a walk can be thought of as two simultaneous one dimensional walks operating perpendicular to one another but diagonal to the two dimensional grid. In particular, if S_n and S'_n are two independent one dimensional walks, then $\left(\frac{S_n+S'_n}{2}, \frac{S_n-S'_n}{2}\right)$ is a two dimensional walk. Consider the "diamond" made up of all points with Manhattan distance a from the starting point of a two dimensional random walk, and let V_a be the time for the walk to strike this diamond. This occurs when either of its two constituent one dimensional walks moves a steps from its start position. Therefore V_a is the minimum of two independent copies of U_a, so $\mathbb{E}[V_a] \leq \mathbb{E}[U_a] = a^2$ and $\Pr(V_a > n) = \Pr(U_a > n)^2$. Now let Z_a be the time for the two dimensional walk to strike a square centered at the walk's start position with radius a. Observe that $V_a \leq Z_a \leq V_{2a}$, so

$$\Pr(U_a > n)^2 \leq \Pr(Z_a > n) \leq \Pr(U_{2a} > n)^2$$

and $\mathbb{E}[Z_a] \leq \mathbb{E}[V_{2a}] \leq \mathbb{E}[U_{2a}] = 4a^2$.

B. Time of Stage One

Recall that the kth traversal in stage one covers draws a grid on the square with radius $\lceil k^\beta \rceil$ such that the distance between grid lines is $\lfloor k^{\frac{2}{3}\beta} \rfloor$. Therefore, if $\|\vec{s}\| \leq \lceil k^\beta \rceil$ and the target has struck a square of radius $\lfloor k^{\frac{2}{3}\beta} \rfloor$ centered on its start location by the end of the $k-1$st traversal, then the chaser will find the target's trail during the kth traversal.

The kth traversal involves first moving to the edge of the square to be searched, which takes $2(\lceil k^\beta \rceil - \lceil (k-1)^\beta \rceil)$ time, then drawing two copies of the path illustrated in Figure 1, each taking

$$\lceil k^\beta \rceil \left(\left\lceil \frac{\lceil k^\beta \rceil}{\lfloor k^{\frac{2}{3}\beta} \rfloor} \right\rceil + 2 \right)$$

time to complete. Therefore the time to perform the first k traversals is

$$\sum_{j=1}^{k} 2\lceil j^\beta \rceil \left(\left\lceil \frac{\lceil j^\beta \rceil}{\lfloor j^{\frac{2}{3}\beta} \rfloor} \right\rceil + 3 - \frac{\lceil (j-1)^\beta \rceil}{\lceil j^\beta \rceil} \right) = \Theta(k^{\frac{4}{3}\beta+1}).$$

In particular, there exist constants c_1 and c_2 such that $c_1 \leq c_2$ and the time to perform the first k traversals is between $c_1 k^{\frac{4}{3}\beta+1}$ and $c_2 k^{\frac{4}{3}\beta+1}$. Therefore if $\|\vec{s}\| \leq \lceil k^\beta \rceil$ and

$$Z_{\lfloor k^{\frac{2}{3}\beta} \rfloor} \leq c_1 (k-1)^{\frac{4}{3}\beta+1}$$

then the chaser will find the target's path by $c_2 k^{\frac{4}{3}\beta+1}$. Letting $P(\vec{s})$ be the expected time to find the target's path if it begins at $\vec{s}$, if $\|\vec{s}\| \leq \lceil k^\beta \rceil$, we have

$$\begin{aligned}
&\Pr(P(\vec{s}) > c_2 k^{\frac{4}{3}\beta+1}) \\
&\leq \Pr(Z_{\lfloor k^{\frac{2}{3}\beta} \rfloor} > c_1 (k-1)^{\frac{4}{3}\beta+1}) \\
&\leq \Pr(U_{2k^{\frac{2}{3}\beta}} > c_1 (k-1)^{\frac{4}{3}\beta+1})^2 \\
&\leq \frac{16(1+\epsilon)^2}{\pi^2} \exp\left\{-\frac{\pi^2 c_1 (k-1)^{\frac{4}{3}\beta+1}}{16 k^{\frac{4}{3}\beta}}\right\} \qquad (3) \\
&= \mathcal{O}(\exp\{-c_1 \pi^2 k/16\}) \qquad (4)
\end{aligned}$$

where our use of (1) in (3) is justified because

$$\frac{(c_1(k-1)^{\frac{4}{3}\beta+1})^{1/3}}{2k^{\frac{2}{3}\beta}} = \mathcal{O}(k^{-\frac{2}{9}\beta+\frac{1}{3}}) = o(1)$$

for $\beta > 3/2$.

Consider an arbitrary random variable X and an increasing sequence x_m. Assume by convention that $x_{-1} = -\infty$. Then

$$\begin{aligned}
\mathbb{E}(X) &\leq \sum_{m'=0}^{\infty} x_{m'} \Pr(x_{m'-1} < X \leq x_{m'}) \\
&= \sum_{m'=0}^{\infty} \left(x_0 + \sum_{m=0}^{m'-1} (x_{m+1} - x_m) \right) \Pr(x_{m'-1} < X \leq x_{m'}) \\
&= x_0 + \sum_{m=0}^{\infty} (x_{m+1} - x_m) \sum_{m'=m+1}^{\infty} \Pr(x_{m'-1} < X \leq x_{m'}) \\
&= x_0 + \sum_{m=0}^{\infty} (x_{m+1} - x_m) \Pr(X > x_m).
\end{aligned}$$

We wish to apply this with $X = P(\vec{s})$. Recall that (4) only applies if $\|\vec{s}\| \leq \lceil k^\beta \rceil$, so we let $x_m = c_2(\lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil + m)^{\frac{4}{3}\beta+1}$ for $m = 0, 1, \cdots$. Observe that $x_{m+1} - x_m = \mathcal{O}((\lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil + m)^{\frac{4}{3}\beta})$, so

$$\begin{aligned}
&\sum_{m=0}^{\infty} (x_{m+1} - x_m) \Pr(P(\vec{s}) > x_m) \\
&= \sum_{m=0}^{\infty} \mathcal{O}\Big((\lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil + m)^{\frac{4}{3}\beta} \\
&\qquad \cdot \exp\{-c_1 \pi^2 (\lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil + m)/16\} \Big) \\
&= \exp\{-c_1 \pi^2 \lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil / 16\} \\
&\qquad \cdot \sum_{m=0}^{\infty} \mathcal{O}\left((\lceil \|\vec{s}\|^{\frac{1}{\beta}} \rceil + m)^{\frac{4}{3}\beta} \exp\{-c_1 \pi^2 m/16\} \right) \\
&= o(1).
\end{aligned}$$

Consequently

$$\sup_{\vec{s}: \|\vec{s}\| \leq d} \mathbb{E}[P(\vec{s})] \leq c_2 \lceil d^{\frac{1}{\beta}} \rceil^{\frac{4}{3}\beta+1} + o(1) = \mathcal{O}(d^{\frac{4}{3}+\frac{1}{\beta}}) = \mathcal{O}(d^\alpha).$$

C. Time of Stage Two

The chaser's path in stage two is defined by the moments when the chaser comes upon a more recent point in the target's path, and so begins traveling along a new square. We call these events *square-starts*. Let T_k be the sequence of intervals between successive square-starts, and let N_k be time lag associated with the kth square-start. Recall that by time lag we mean that the kth time the chaser finds a more recent point on the path, the target came to this point N_k seconds ago. Let $K = \inf\{k : N_k = 0\}$, the number of square-starts until the chaser finds the target exactly. Then the total time to perform stage two is $\sum_{k=1}^{K-1} T_k$.

Given N_k, N_{k+1} and T_k are independent of all past history. Recall that the radius of the square traveled along by the chaser after the kth square-starts is $R_k \triangleq \lfloor\sqrt{N_k}\rfloor$. Let W_k be the time from when the target made the point on the trail found by the chaser in the kth square-start to when both the target and chaser have struck the square of radius R_k. Therefore $W_k = \max\{Z_{R_k}, N_k + R_k\}$ since it takes R_k seconds for the chaser to get from the center to edge of the square. Once both chaser and target have reached the square, it will be at most $8R_k$ seconds before the next square-start, and the point that the chaser finds on the target's path will be from no sooner than Z_{R_k} seconds after the previous square-start. Therefore

$$\begin{aligned} N_{k+1} &\le W_k - Z_{R_k} + 8R_k \\ &= \max\{Z_{R_k}, N_k + R_k\} - Z_{R_k} + 8R_k \\ &= \max\{8R_k, N_k - Z_{R_k} + 9R_k\}. \end{aligned} \tag{5}$$

We can also get a looser bound on N_{k+1} by noting that $Z_{R_k} \ge R_k$, so

$$N_{k+1} \le \max\{8R_k, N_k - 8R_k\} \le N_k + 8R_k. \tag{6}$$

Furthermore

$$\begin{aligned} T_k &\le W_k + 8R_k - N_k \\ &= \max\{Z_{R_k}, N_k + R_k\} + 8R_k - N_k \\ &= \max\{Z_{R_k} - R_k, N_k\} + 9R_k - N_k \\ &\le Z_{R_k} + 8R_k. \end{aligned} \tag{7}$$

Fix any $0 < \delta < 1$. By (5), for n large enough that $n > 8R_k$,

$$\begin{aligned} &\Pr(N_{k+1} < \delta N_k | N_k = n) \\ &\ge \Pr(N_k - Z_{R_k} + 9R_k < \delta N_k | N_k = n) \\ &= \Pr(Z_{R_k} > (1-\delta)N_k + 9R_k | N_k = n) \\ &\ge \Pr(U_{\lfloor\sqrt{n}\rfloor} > (1-\delta)n + 9\lfloor\sqrt{n}\rfloor)^2 \\ &\ge \frac{16(1-\epsilon)^2}{\pi^2}\left[\exp\left\{-\frac{\pi^2((1-\delta)n + 9\lfloor\sqrt{n}\rfloor)}{8\lfloor\sqrt{n}\rfloor^2}\right\}\right. \\ &\qquad \left. - \frac{1}{3}\exp\left\{-\frac{9\pi^2((1-\delta)n + 9\lfloor\sqrt{n}\rfloor)}{8\lfloor\sqrt{n}\rfloor^2}\right\}\right]^2 \end{aligned} \tag{8}$$

where in (8) we have used (2), which is allowable because

$$\frac{((1-\delta)n + 9\lfloor\sqrt{n}\rfloor)^{1/3}}{\lfloor\sqrt{n}\rfloor} = \mathcal{O}(n^{-1/6}) = o(1).$$

Therefore

$$\begin{aligned} &\lim_{n\to\infty} \Pr(N_{k+1} < \delta N_k | N_k = n) \ge \frac{16(1-\epsilon)^2}{\pi^2} \\ &\cdot \left[\exp\left\{-\frac{\pi^2(1-\delta)}{8}\right\} - \frac{1}{3}\exp\left\{-\frac{9\pi^2(1-\delta)}{8}\right\}\right]^2 > 0. \end{aligned}$$

In particular, there is some $p > 0$ and $\tilde{n}$ such that for all $n \ge \tilde{n}$, $\Pr(N_{k+1} < \delta N_k | N_k = n) \ge p$. Observe that the process N_k now has the following behavior as long as $N_k \ge \tilde{n}$. Given N_k, with probability p, $N_{k+1} < \delta N_k$, and with probability $1-p$, by (6), $N_{k+1} \le N_k + 8R_k \le N_k + 8\sqrt{N_k} \le \left(1 + \frac{8}{\sqrt{\tilde{n}}}\right) N_k$. Thus the sequence N_k effectively performs a geometric random walk, which we can ensure decreases by making $\tilde{n}$ large enough. More specifically, let $\tilde{K} = \inf\{k : N_k \le \tilde{n}\}$ and observe that for any $j > k$,

$$\mathbb{E}[N_k | N_1 = n, \tilde{K} = j] \le \left[p\delta + (1-p)\left(1 + \frac{8}{\sqrt{\tilde{n}}}\right)\right]^{k-1} n.$$

Let $\lambda \triangleq \left[p\delta + (1-p)\left(1 + \frac{8}{\sqrt{\tilde{n}}}\right)\right]$. We assume that $\tilde{n}$ is sufficiently large that $\lambda < 1$. Applying (7) gives

$$\begin{aligned} \mathbb{E}[T_k | N_1 = n, \tilde{K} = j] &\le \mathbb{E}[Z_{R_k} + 8R_k | N_1 = n, \tilde{K} = j] \\ &\le \mathbb{E}[4R_k^2 + 8R_k | N_1 = n, \tilde{K} = j] \\ &\le \mathbb{E}\left[\left(4 + \frac{8}{\sqrt{\tilde{n}}}\right) N_k \Big| N_1 = n, \tilde{K} = j\right] \\ &\le \left(4 + \frac{8}{\sqrt{\tilde{n}}}\right)\lambda^{k-1} n. \end{aligned}$$

Thus given $N_1 = n$, the expected time until the lag drops below $\tilde{n}$ is

$$\begin{aligned} &\mathbb{E}\left[\sum_{k=1}^{\tilde{K}-1} T_k \Big| N_1 = n\right] \\ &= \sum_{j=1}^{\infty} \Pr(\tilde{K} = j | N_1 = n)\, \mathbb{E}\left[\sum_{k=1}^{j-1} T_k \Big| N_1 = n, \tilde{K} = j\right] \\ &\le \sum_{j=1}^{\infty} \Pr(\tilde{K} = j | N_1 = n) \sum_{k=1}^{j-1} \left(4 + \frac{8}{\sqrt{\tilde{n}}}\right)\lambda^{k-1} n \\ &\le \sum_{k=1}^{\infty} \left(4 + \frac{8}{\sqrt{\tilde{n}}}\right)\lambda^{k-1} n = \mathcal{O}(n). \end{aligned}$$

Therefore, if the initial lag is n, the time until the lag falls below the constant $\tilde{n}$ is $\mathcal{O}(n)$. Once this occurs, the process has essentially forgotten its initial state, so the additional expected time is no longer a function of the initial time lag. Specifically, the length of excursions during which the lag is greater than $\tilde{n}$ are i.i.d., so a constant fraction of time is spent with time lag less than $\tilde{n}$. Furthermore, there is some constant $q > 0$ such that when the time lag N_k is less than $\tilde{n}$, the probability that $N_{k+1} = 0$ is at least q. Thus, after the time lag falls below $\tilde{n}$, the additional expected time until the chaser finds the target exactly is $\mathcal{O}(1)$, meaning that the expected time of stage two is $\mathcal{O}(n)$. Since the initial lag n is at most the time of stage one, which as shown above has expected value $\mathcal{O}(d^\alpha)$, the total expected time of the entire strategy is $\mathcal{O}(d^\alpha)$.

IV. Lower Bound on Performance

Any conceivable chasing strategy can be divided into two stages, much like our proposed strategy in Section III. The first stage takes place before the chaser comes upon a point in the target's path, during which the chaser's motion must be independent from the target, since it does not yet have any knowledge of its location or behavior. Then the second stage takes place after the chaser finds a point in the target's path.

We will show that the expected time to perform the first stage must be $\Omega\left(\frac{d^{\frac{4}{3}}}{\sqrt{\log\log d}}\right)$.

We will show that this bound holds for any deterministic chaser path $\vec{p}_n$, which means it holds also for any random path. Let H_n be the distance at time n from the target to its starting location $\vec{s}$, and recall the law of the iterated logarithm [5] for the simple random walk in two dimensions

$$\Pr(H_n > c\sqrt{n\log\log n} \text{ i.o.}) = 0$$

for $c > 1$. Let $J_n \triangleq \max_{n' \leq n} H_n$; that is, the furthest the target has gotten from its starting location by time n. Since $\sqrt{n\log\log n}$ is increasing,

$$\Pr(J_n > c\sqrt{n\log\log n} \text{ i.o.}) = 0$$

for $c > 1$ as well. In particular, there exists an integer n_0 such that

$$\zeta \triangleq \Pr(J_n \leq c\sqrt{n\log\log n} \text{ for all } n \geq n_0) > 0.$$

If $\|\vec{s}\| \geq 2n_0$, then certainly the chaser cannot find the path earlier than n_0, meaning that with probability ζ, the chaser will not find the path until it comes within $c\sqrt{n\log\log n}$ of $\vec{s}$. Thus if $n(d)$ is the first time after which, for all $\vec{s}$ such that $d \geq \|\vec{s}\| \geq 2n_0$, and some $n \leq n(d)$,

$$\|\vec{p}_n - \vec{s}\| \leq c\sqrt{n\log\log n}$$

then $\tilde{N}_\gamma(d) \geq \zeta n(d)$. Consider how quickly $\vec{p}_n$ can come to within $c\sqrt{n\log\log n}$ of all points $\vec{s}$ with $d \geq \|\vec{s}\| \geq 2n_0$. In time n, the number of points the chaser covers (i.e. comes to within $c\sqrt{n\log\log n}$) is at most the number of points it covers when moving in a straight line, which is

$$\Big|\Big\{\vec{p} \in \mathbb{Z}^2 : \exists n' : 0 \leq n' \leq n,$$
$$\|\vec{p} - (n', 0)\| \leq c\sqrt{n'\log\log n'}\Big\}\Big| = \mathcal{O}\left(n^{\frac{3}{2}}\sqrt{\log\log n}\right).$$

Therefore $\mathcal{O}\left(n(d)^{\frac{3}{2}}\sqrt{\log\log n(d)}\right) = \Omega(d^2)$ meaning for all $\gamma \in \Gamma$,

$$\tilde{N}_\gamma(d) \geq \zeta n(d) = \Omega\left(\frac{d^{\frac{4}{3}}}{\sqrt{\log\log d}}\right).$$

V. Simulation Results

Figure 4 shows the average time for the chaser to find the target over 300 Monte Carlo runs each for initial separations between 10 and 200 grid points. The chasing scheme was implemented with $\beta = 3$, which by our analysis should achieve $\mathcal{O}(d^{\frac{5}{3}})$. The average time is shown along with a best fit curve of the form $y = ax^{\frac{5}{3}} + b$, which appears to match fairly well.

VI. Conclusion and Future Work

The lower bound on the expected time of any chasing strategy that we gave in Section IV put a bound on the time to find the target's trail for the first time. However, we have no lower bound on the expected time for the chaser to find the target exactly once it has found the trail. Obtaining such a bound would help evaluate stage two of our proposed strategy.

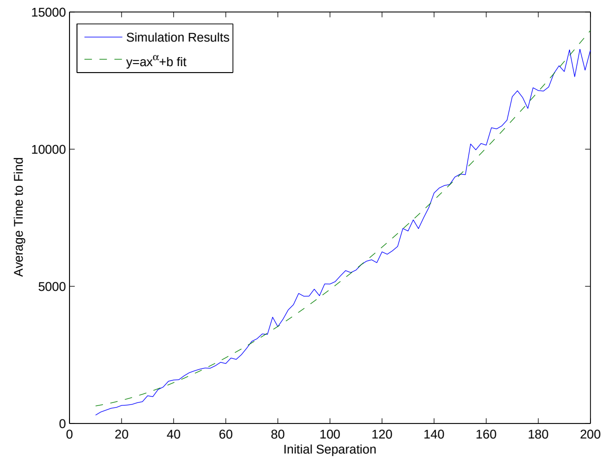

Fig. 4. Simulation results showing the average over 300 Monte Carlo runs with $\beta = 3$.

In addition, in practice the chaser may have some initial information about the location of the target, if only because it is constrained to a finite area. In that case, improving the estimate of the target's location will dominate the search time, rather than finding an initial estimate, so a provably optimal stage two strategy may be more useful in practice.

Additionally, extending our results to more general target motion would be worthwhile. The simplest generalization would be to consider non-simple random walks in the plane, such that the probability that the target moves to an adjacent grid point depends on the direction. Initial simulations have shown that our proposed chasing strategy works well for walks that are close to simple, but if the walk moves too quickly in any one direction, the chaser does not always succeed in finding the target. This is hardly surprising, as our proposed strategy was tailored to the simple random walk, but it should be possible do design a more flexible strategy in which the chaser estimates the target's behavior and adapts to it.

Finally, one could consider random walks in more than two dimensions. The simple random walk in three or more dimensions is transient, meaning that without sensors, a chaser cannot even find the target with probability one. Whether the addition of sensors would allow the chaser to find the target in finite expected time, or even with probability one, is unknown.

References

[1] Y. Sung, Z. Yang, and L. Tong, "Multitone acoustic sensor network with mobile access: An experimental testbed," in *Proc. of MILCOM*, (Monteray, CA), Oct. 2004.

[2] T. Pham and H. C. Papadopoulos, "Distributed tracking in AD-HOC sensor networks," *IEEE/SP 13th Workshop on Statistical Signal Processing*, pp. 1226–1231, July 2005.

[3] R. Want, "Enabling ubiquitous sensing with RFID," *Computer*, vol. 37, no. 4, pp. 84–86, April 2004.

[4] P. Révész, *Random Walk in Random and Non-Random Environments*. World Scientific, 2005.

[5] P. Erdös and S. J. Taylor, "Some problems concerning the structure of random walk paths," *Acta Math. Aead. Sci. Hung.*, vol. 11, pp. 137–162, March 1960.