LLR Computation and Delivery Algorithm

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Abstract

In this write-up, a simple algorithm for delivering the log-likelihood ratio (LLR) to a designated fusion center to perform optimal statistical inference is proposed. The algorithm requires input of processor assignments and set of links for routing data, explained in detail in [1].

A. Algorithm

Given an input consisting of processor assignment mapping and a packet-operation digraph, the algorithm -A (DFMRF) specifies a sequence of operations and transmissions to compute the LLR and deliver it to the fusion center. There are two phases viz., routing of raw data along the forwarding subgraph to compute potential functions and then aggregation of computed values along the aggregation subgraph. During the raw-data routing phase (steps 5-6), routing of raw data occurs along the FG. At the end of this phase, every processor $i$ computes and stores its local contribution $\text{LocVal}(i)$, given by

$$\text{LocVal}(i) = \sum_{e \in i \cup j, <j,i> \in \text{FG}(G)} \phi_e(Y_e),$$

if $i = \text{Proc}(\phi_e)$. \hfill (1)

This local contribution is then aggregated hierarchically and delivered to the fusion center (steps 7-16), along the aggregation subgraph AG($G$). On receiving the aggregates from all its immediate predecessors in AG($G$), each node $i \neq v_0$ combines them with its local contribution and routes $\text{AggVal}(i)$ to its immediate successor, given by

$$\text{AggVal}(i) = \sum_{<j,i> \in \text{AG}(V)} \text{AggVal}(j) + \text{LocVal}(i).$$

(2)

Note that only the processors have stored contributions. At the end of the aggregation process, the LLR is available at the fusion center and is given by $\text{AggVal}(v_0)$.

We now analyze the class of routing graphs which deliver LLR($Y_n,Y$) to the fusion center, on implementing the DFMRF algorithm.

Lemma 1 (Feasible routing graphs for DFMRF): The class of routing graphs for which DFMRF delivers LLR to the fusion center satisfies the following constraints: every clique has a processor, which is one of its members and all the other members possess mutually-exclusive links to it in the forwarding subgraph ; the aggregation subgraph is a tree containing the set of processors, directed towards the fusion center.

Proof: Since all the cliques potentials are computed at one of the clique members after routing the raw data on FG, it is required that one of the clique members is a processor and other members are strongly connected to it. The aggregation subgraph has to contain the processors, since the sum function of their local contributions has to be delivered to the fusion center. \hfill \Box

REFERENCES

Input: \{Proc, G = \{FG, AG\}\}

\(V\) = set of nodes, \(v_0\) = Fusion center

\(\text{ImmSucc}(i), \text{ImmPred}(i)\) = Immediate successor/predecessor of \(i\)

\(\text{Leaf}(G)\) = Leaf nodes of \(G\)

for each \(<i, j> \in \text{FG}\) do
  Route \(Y_i\) from \(i\) to \(j\)
end for

for each \(i \notin \text{Leaf}(\text{FG})\) do
  Compute \(\text{LocVal}(i)\) from (1)
end for

for each Node \(i\) do
  Initialize \(Tx(i) \leftarrow false\)
end for

for each Node \(i \in \text{Leaf}(\text{AG})\) do
  \(\text{AggVal}(i) \leftarrow \text{LocVal}(i)\)
  Route \(\text{AggVal}(i)\) to \(k : <i, k> \in \text{AG}(V)\)
  \(Tx(i) \leftarrow true\)
end for

while \(\exists\) Node \(i \neq v_0\) with \(Tx(i) = false\) do
  for each Node \(i\) with \(Tx(i) = false\) and \(Tx(j) = true, \forall j : <j, i> \in \text{AG}(V)\) do
    Compute \(\text{AggVal}(i)\) from equation (2)
    Route \(\text{AggVal}(i)\) to \(k : <i, k> \in \text{AG}(V)\)
    \(Tx(i) \leftarrow true\)
  end for
end while

Compute \(\text{AggVal}(v_0)\) from equation (2)

\(\text{LLR}(Y; V) \leftarrow \text{AggVal}(v_0)\)
return \(\text{LLR}(Y; V)\)

Algorithm 1. Data fusion algorithm (DFMRF) to deliver log-likelihood ratio to fusion center, given a packet-operation digraph.