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Optimal Cognitive Access of Markovian Channels under Tight Collision Constraints

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***Abstract*—The problem of cognitive access of channels of primary users by a secondary user is considered. The transmissions of primary users are modeled as independent continuous-time Markovian on-off processes. A secondary cognitive user employs slotted transmissions, and it senses one of the possible channels before transmission. The objective of the cognitive user is to maximize its throughput subject to collision constraints imposed by the primary users. The optimal access strategy is in general a solution of a constrained partially observable Markov decision process, which involves a constrained optimization in an infinite dimensional functional space. It is shown in this paper that, when the collision constraints are tight, the optimal access strategy can be implemented by a simple memoryless access policy with periodic channel sensing. Numerical results are presented to validate and extend the analysis for different practical scenarios.**

***Index Terms*—Cognitive radio. Dynamic spectrum access. Markov decision processes.**

I. Introduction

We consider a hierarchical overlay cognitive network with N parallel communication channels shared by primary and secondary users [1]. The Primary Users (PUs) communicate through dedicated channels, oblivious to the presence of Secondary Users (SUs). On the other hand, a secondary user (SU) transmits opportunistically by first sensing a candidate channel. Based on the sensing outcome, the SU will decide if and where to transmit. The SU must also obey certain collision constraints so that its transmission will not interfere the communication of PUs beyond the acceptable levels. The first such cognitive access scheme for slotted primary and secondary users was proposed in [2].

In this paper, we model the occupancy of PUs as independent continuous-time on-off Markov processes with "on" indicating that the channel is being used by a PU and "off" for the case when the channel is idle. This model is an approximation of existing wireless access applications (*e.g.* 802.11 WiFi) [3]. The aim of the SU is to maximize its throughput subject to collision constraints. For example, the network designer may want to assure the PUs that, whenever they transmit, the probability of colliding with an opportunistic SU is below a threshold, say, less than 5%. With such a guarantee, the PU may be willing to allow cognitive transmissions if the PU is compensated accordingly.

Obtaining the optimal access policy for the SU appears to be intractable at the first glance. In particular, the optimal policy includes both sensing and access policies, and the design of these two policies are in general not separable. Because the SU can only observe one channel at a time, the problem falls into the category of partially observable Markov decision process (POMDP) with constraints for which there are no general practical solutions available [4].

A. Main Results

In this paper, we show that, when the the collision constraints are tight, the optimal cognitive access policy can be implemented by a simple memoryless policy with linear complexity. Referred to as periodic sensing and memoryless access (PS-MA), the policy senses channels in a round robin fashion and transmits with a probability that is a function of the collision constraints and traffic statistics of the primary users. To establish the optimality of PS-MA, we take an indirect approach by analyzing PS-MA and an optimal policy that assumes full spectrum observation. These two policies give performance lower and upper bounds on that of the optimal policy, respectively. The main results of this paper is to show that, when the collision constraints are tight, the lower and upper bounds match.

B. Related Work

We will restrict comments on related work to the problem of cognitive access in a hierarchical network of primary and secondary users. In this context, we refer to a recent survey by Zhao and Sadler [1]. Related work in a broader context can be found in [5].

The joint design of sensing and access policy that maximizes the throughput of a cognitive SU subject to collision constraints is difficult in general, and it becomes tractable only when certain structures are imposed on the primary and secondary users. In [2], [6], Zhao *et al.* consider the case when all users follow a slotted transmission structure: when a PU has packets to transmit, it will do so at the beginning of the slot. Imposing a slotted structure simplifies the problem considerably, thanks to the separation principle [7] and the optimality of myopic policies [8], [9].

The problem of cognitive access of multiple continuous-time Markovian channel is first considered in [10]. The authors simplify the problem by restricting the sensing policy to a periodic sensing scheme, which changes the problem from a partially observable Markov decision process with constraint to a constrained finite state Markov decision process. The resulting algorithm, referred to as optimal spectrum access with periodic sensing (PS-OSA), can be obtained from a linear program. Other related work assuming un-slotted PUs can be found in [3], [11], [12], [13].

The fundamental limit and structure of cognitive access of a single continuous-time Markovian channel is investigated in [12], [14] where the authors considered the optimal transmission policy with arbitrarily small sensing and transmission periods. The optimal transmission is probabilistic. For the single channel case, PS-MA reduces asymptotically to the access policy considered in [12] as the duration of the transmission period approaches to zero.

The two algorithms analyzed in this paper are first presented in [10] and used as benchmark (numerical) comparisons. The full observation and optimal spectrum access (FO-OSA) policy that provides the throughput upper bound is used in [10] as a performance upper bound in simulations. The PS-MA policy presented here is slightly different from that in [10]. In particular, the policy presented in [10] has a bias, which is removed in this paper.

II. Models, Performance, and Optimality

Assume that there are N parallel channels (indexed from 0 to $N-1$) available for transmissions by the PUs. The occupancy of each channel by a PU is assumed to evolve independently according to a homogeneous continuous-time Markov chain with idle ($X_i = 0$) and busy state ($X_i = 1$), respectively. The holding times are exponentially distributed with parameters λ_i^{-1} for the idle and μ_i^{-1} for the busy states, respectively. The state transition rate matrix (Q-matrix) under the continuous-time Markov process is given by

$$Q_i \triangleq \begin{pmatrix} -\lambda_i & \lambda_i \\ \mu_i & -\mu_i \end{pmatrix} \quad i = 0, 1, \ldots, N-1.$$

The stationary distribution of the ith PU process can then be determined as

$$v_i(0) = \frac{\mu_i}{\lambda_i + \mu_i}, \quad v_i(1) = \frac{\lambda_i}{\lambda_i + \mu_i}. \tag{1}$$

The set of admissible access policies $\mathscr{P}$ for the SU is defined as follows. We assume that the SU employs slotted transmissions. At the beginning of each slot, the SU chooses one of the N channels to sense and makes a decision to transmit in one of the N channels or not to transmit at all. Therefore, the action space of the SU is a product of sensing and transmission spaces $\mathcal{S} \times \mathcal{A}$ where $\mathcal{S} = \{0, \cdots N-1\}$ is the set of channels to sense and $\mathcal{A} = \{-1, 0, \cdots N-1\}$ the set of transmission actions with i indicating the transmission on the ith channel and -1 indicating no transmission.

Fig 1 illustrates a realization of an access policy of the SU. If the SU transmits on a particular channel and it does not collide with the PU during its transmission, then the SU receives a reward of 1 (successfully transmitted packet). If a collision happens, the SU receives no reward. Note that for continuous-time Markov channels, if the SU transmits on a channel that is idle initially, because the PU may start the transmission at any time, the expected reward received by the SU is $\exp(-\lambda_i T)$ where λ_i is the rate that the channel i stays idle and T the slot duration.

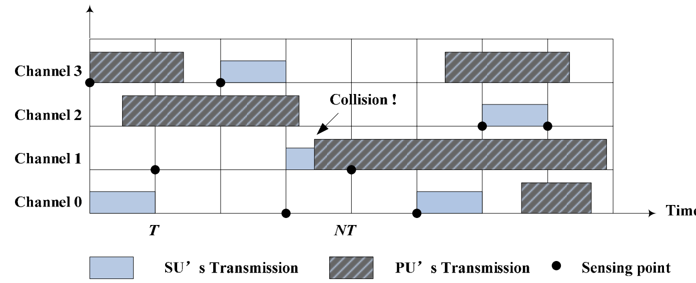

Fig. 1. Illustration of a general admissible access protocol. The transmissions are in general probabilistic. Collisions may happen over channels that are sensed idle.

We use average throughput as the performance metric. For a fixed policy π, let R_k^π be the expected reward received in the kth slot, the average throughput of π can be defined by

$$J(\pi) = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} E_\pi(R_k^\pi) \tag{2}$$

where the expectation is taken over the probability distributions induced by π.

We will impose collision constraints on SU's transmissions, using a conditional interference as a metric. Specifically, we denote $C_i(\pi)$ as the fraction of the PU transmission time that SU collides with PU.

We define the optimal access policy π^* as the solution of the following constrained optimization

$$\max_{\pi\in\mathscr{P}} J(\pi) \ \text{ subject to } C_i(\pi) \le \gamma_i, \quad i = 0, \ldots, N-1, \tag{3}$$

where $0 \le \gamma_i \le 1$ are given constants representing tolerable collision levels.

III. Cognitive Access Policies

With the optimal policy π^* defined by (3), we present two policies in this section. The first policy is the optimal spectrum access with full observation (FO-OSA) denoted by π^{FO}. FO-OSA assumes the simultaneous sensing of *all channels*. Therefore, it does not belong to the set of admissible policies. Because it maximizes the SU throughput subject to the same collision constraints as that for π^*, it performs better than π^*.

The second policy is the memoryless access with periodic sensing (PS-MA), denoted by π^{MA}. PS-MA is a policy that senses each channel periodically (round robin) and it makes the transmission decision based on the current sensing outcome. PS-MA belongs to the set of admissible policies, thus in general suboptimal. To summarize, we have

$$J(\pi^{\text{MA}}) \leq J(\pi^*) \leq J(\pi^{\text{FO}}). \quad (4)$$

A. Full observation and optimal spectrum access: π^{FO}

Full observation means that all the channels can be sensed simultaneously at the beginning of every time slot. Thus the access decision will be made based on the full channel state information $X(k) = x, x \in \{0,1\}^N$ in time slot k, *i.e.*, the SU chooses the action $i \geq 0$ based on the observation $x = (x_i)$ with the probability $\beta_i(x)$ transmitting on channel i. The FO-OSA policy is a one-step policy, and the immediate reward when channel i is used for transmission is given by

$$g(x,i) = \begin{cases} \exp(-\lambda_i T), & \text{if } x_i = 0; \\ 0, & \text{if } x_i = 1. \end{cases} \quad (5)$$

Let $f(x)$ be the stationary distribution of the state vector $X(k)$. It is easy to show that any stationary optimal access policy π^{FO} can be fully specified by its transmission probabilities for all observation vector x. In the following, we also use a vector $\beta(\pi)$ or simply β to represent a stationary policy π. Furthermore, it follows from the similar argument in [10] that the probability vector β corresponding to π^{FO} can be determined by solving the following linear programming:

$$\max_{\beta \in [0,1]^{N \times 2^N}} \sum_{x \in \{0,1\}^N} f(x) \sum_{i=0}^{N-1} g(x,i)\beta_i(x) \quad (6)$$

subject to

$$\sum_{x \in \{0,1\}^N} \frac{f(x)(1-g(x,i))\beta_i(x)}{(1-v_i(0)\exp(-\lambda_i T))} \leq \gamma_i, \quad \forall i \quad (7)$$

$$\sum_{i=0}^{N-1} \beta_i(x) \leq 1, \quad \forall x. \quad (8)$$

Denote $J(\beta)$ as the objective function in (6), i.e.,

$$J(\beta) \equiv \sum_{x \in \{0,1\}^N} f(x) \sum_{i=0}^{N-1} g(x,i)\beta_i(x). \quad (9)$$

B. Periodic sensing and memoryless access: π^{MA}

The policy PS-MA, first introduced in [10], decouples sensing and access. The SU senses the channel in an increasing order at the beginning of each slot, starting from the smallest index (say, channel 0). In slot k, the SU senses channel $q = k \bmod N$.

Policy PS-MA has no memory, and the transmission will only depend on the current sensing outcome. Specifically, in the kth slot, if the SU senses a busy channel $q = k$ mod N, no transmission is made. Otherwise it will transmit in the sensed channel q with probability β_q^{MA} given in [10],

$$\beta_q^{\text{MA}} = \min(\frac{\gamma_q N \phi_q}{v_q(0)}, 1), \quad (10)$$

where

$$\phi_i \equiv \frac{(1-v_i(0)\exp(-\lambda_i T))}{(1-\exp(-\lambda_i T)}.$$

The access policy in (10) is decided such that collision constraints are satisfied while maximizing the throughput for the SU when channel q is sensed idle, and the collision constraint of channel q is γ_q. We note that β_q^{MA} here is slightly different from the policy showed in the previous work [10] by adding $v_q(0)$ in the denominator in (10).

The throughput of this policy is:

$$J(\pi^{\text{MA}}) = \frac{1}{N}\sum_{i=0}^{N-1} v_i(0)\beta_i^{\text{MA}} \exp(-\lambda_i T). \quad (11)$$

IV. Optimality of PS-MA

We establish in this section that, under tight collision constraints, $J(\pi^{\text{FO}}) = J(\pi^{\text{MA}})$, which necessarily means that $J(\pi^*) = J(\pi^{\text{MA}})$, *i.e.* PS-MA is optimal.

We assume a heterogeneous network setting in which each channel has different traffic statistics and collision constraints. Specifically, the on-off traffic on the ith channel is given by the holding times λ_i^{-1} and μ_i^{-1} for idle and busy states, respectively. The (conditional) collision probability on channel i is upper bounded by γ_i. Under these conditions, the performance of PS-MA is given by the following Proposition. Let

$$W_i \equiv \phi_i \exp(-\lambda_i T), \; \phi_i \equiv \frac{1-v_i(0)\exp(-\lambda_i T)}{1-\exp(-\lambda_i T)}. \quad (12)$$

Proposition 1: In heterogeneous networks, if the collision constraints are tight, i.e., $\gamma_i \in [0, \gamma_i^{\text{MA}}], \forall i$, the throughput of PS-MA is

$$J(\pi^{\text{MA}}) = \sum_{i=0}^{N-1} W_i \gamma_i, \quad (13)$$

where

$$\gamma_i^{\text{MA}} \equiv \frac{v_i(0)}{N\phi_i}. \quad (14)$$

For general cases, the throughput of PS-MA is

$$J(\pi^{\text{MA}}) = \sum_{i=0}^{N-1} W_i(\gamma_i^{\text{MA}} 1_{[\gamma_i > \gamma_i^{\text{MA}}]} + \gamma_i 1_{[\gamma_i \leq \gamma_i^{\text{MA}}]}). \quad (15)$$

The proof of the above proposition is omitted due to space limitations. See [15] for details.

To obtain similar results for policy FO-OSA, introduce subsets S_i^k of $\{0,1\}^N$

$$S_i^k \equiv \{x | x \in \{0,1\}^N, \; x_i = 0, \; \text{there are } k \text{ 0s in } x\}$$

for $k = 1, \ldots, N$ and denote

$$F_i^k \equiv \sum_{x \in S_i^k} f(x).$$

We define

$$\gamma_i^{\text{FO}} \equiv \frac{\sum_{j=1}^N (F_i^j / j)}{\phi_i}. \quad (16)$$

Then for the performance of the FO-OSA policy in this case, we can give the following proposition.

Proposition 2: In heterogeneous networks, if the collision constraints are tight, i.e., $\gamma_i \in [0, \gamma_i^{\rm FO}], \forall i = 0, 1, \cdots, N-1$, the throughput of the optimal FO-OSA policy can be given by

$$J(\pi^{\rm FO}) = \sum_{i=0}^{N-1} W_i \gamma_i. \tag{17}$$

If the collision constraints are loose (not tight), the throughput of the optimal FO-OSA policy is upper bounded,

$$J(\pi^{\rm FO}) \le \min(U, \sum_{i=0}^{N-1} W_i \gamma_i), \tag{18}$$

where

$$U \equiv v_{(0)}(0)\exp(-\lambda_{(0)}T) + \sum_{i=1}^{N-1} (\prod_{j=0}^{i-1} v_{(j)}(1)) v_{(i)}(0) \exp(-\lambda_{(i)}T).$$

Here we sort the channels in the decreasing order of $\exp(-\lambda_i T)$, and the parameters with subscript "(i)" means the sorted channel number.

Proof: Details of the proof can be found in [15]. We give a brief proof here due to the space limitation. First, U can be proved as an upper bound of $J(\pi^{\rm FO})$ by assuming no collision constraints and it is not difficult to conclude the second item in (18) that $\sum_{i=0}^{N-1} W_i \gamma_i$ serves an upper bound of the objective function defined in (9). When $\gamma_i \in [0, \gamma_i^{\rm FO}], \forall i = 0, 1, \cdots, N-1$, we can also construct a solution to make this upper bound achieved, and thus (17) is proved. ∎

We are now in the position to establish the optimality of policy PS-MA. From (14) and (16), we can conclude $\forall i = 0, 1, \cdots, N-1$,

$$\gamma_i^{\rm FO} = \frac{N \sum_{j=1}^{N} (F_i^j / j)}{N \phi_i} \ge \frac{\sum_{j=1}^{N} F_i^j}{N\phi_i} = \frac{v_i(0)}{N\phi_i} = \gamma_i^{\rm MA}. \tag{19}$$

Therefore from Proposition 1 and Proposition 2, we can easily establish the following theorem:

Theorem 1: In heterogeneous networks, if $\gamma_i \in [0, \gamma_i^{\rm MA}], \forall i$, the throughput of PS-MA equals to the throughput of FO-OSA

$$J(\pi^{\rm FO}) = J(\pi^{\rm MA}). \tag{20}$$

If $\gamma_i \in [0, \gamma_i^{\rm FO}], \forall i$, the performance gap between the FO-OSA and PS-MA polices can be written as

$$J(\pi^{\rm FO}) - J(\pi^{\rm MA}) = \sum_{i=0}^{N-1} W_i (\gamma_i - \gamma_i^{\rm MA}) 1_{[\gamma_i > \gamma_i^{\rm MA}]}. \tag{21}$$

For the general cases,the performance gap between the FO-OSA and PS-MA polices is upper bounded by

$$J(\pi^{\rm FO}) - J(\pi^{\rm MA}) \le \min(U, \sum_{i=0}^{N-1} W_i \gamma_i) - \sum_{i=0}^{N-1} W_i (\gamma_i^{\rm MA} 1_{[\gamma_i > \gamma_i^{\rm MA}]} + \gamma_i 1_{[\gamma_i \le \gamma_i^{\rm MA}]}). \tag{22}$$

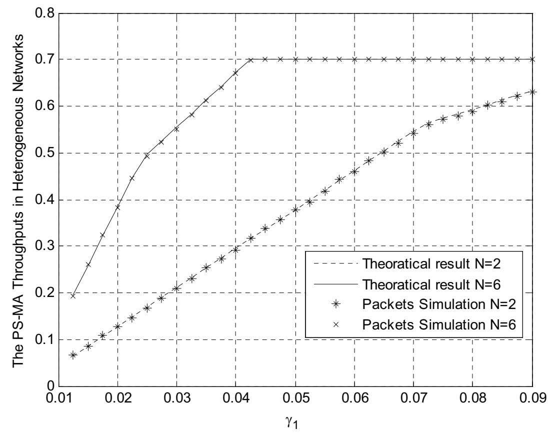

Fig. 2. The PS-MA throughputs of the SU in heterogeneous networks with N=2 and N=6 respectively. The parameters are setting as $N = 2: \lambda_0^{-1} = 4.20$ ms, $\lambda_1^{-1} = 3.23$ ms, $\mu_0^{-1} = 1.00$ ms, $\mu_1^{-1} = 1.43$ ms, and $\gamma_0 = \gamma_1 - 0.01$; $N = 6: \lambda_0^{-1} = \lambda_2^{-1} = \lambda_4^{-1} = 4.20$ ms, $\lambda_1^{-1} = \lambda_3^{-1} = \lambda_5^{-1} = 3.23$ ms, $\mu_0^{-1} = \mu_2^{-1} = \mu_4^{-1} = 1.00$ ms, $\mu_1^{-1} = \mu_3^{-1} = \mu_5^{-1} = 1.43$ ms and$\gamma_0 = \gamma_2 = \gamma_4 = \gamma_0(N=2), \gamma_1 = \gamma_3 = \gamma_5 = \gamma_1(N=2)$;

Theorem 1 shows that when all the channel collision constraints are tight, i.e. $\gamma_i \in [0, \gamma_i^{\rm MA}], \forall i$, we have

$$J(\pi^{\rm FO}) = J(\pi^{\rm MA}), \gamma_i \in [0, \gamma_i^{\rm MA}], \forall i. \tag{23}$$

This means that when each channel collision constraint lies in the special interval, $\gamma_i \in [0, \gamma_i^{\rm MA}], \forall i$, the policy PS-MA also performs the same to the policy FO-OSA. Thus we have the similar conclusion that when it is heterogeneous network, we can also view the policy PS-MA as the one of the optimal policy π^*.

V. Numerical Simulation

We now show numerical results of our analytical conclusion. To this end, we consider some generic parameters as in [10]. The selections of these parameters are not crucial, and our simulation results are representative of the general behavior of the network. Specifically, we assume the slot size $T = 0.25$ ms. We consider up to $N = 6$ channels in our simulation. Specific parameters are listed in the figure captions.

We perform the packet-level simulations to validate the numerical results of our analytical conclusion in Proposition 1 and Proposition 2. The running time length of the packet-level simulation is set as 10000ms. We also performed the comparison of the performances of PS-MA and FO-OSA to validate the result of Theorem 1. We focus on the case $N = 2$ and $N = 6$, and consider the trend of throughput variation as we loosen the constraints, because the constraint parameters are all set with linear relationship.

In Fig. 2, it shows that our analytical results of the PS-MA policy agree with the packet simulation results and thus we validate Proposition 1. Fig. 3 shows that the matching results of analytical expressions and packet simulations for the FO-OSA policy. Through computation, we have $\gamma_0^{\rm FO}(N =$

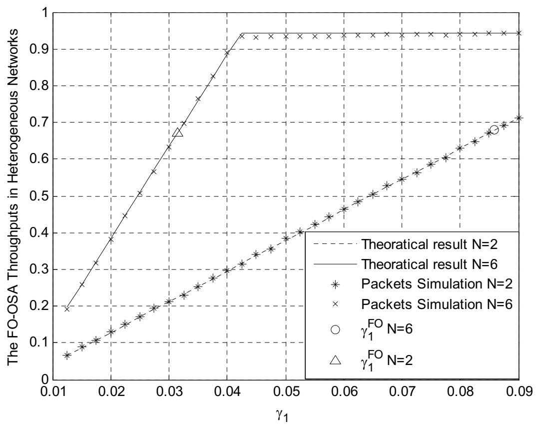

Fig. 3. The FO-OSA throughputs of the SU in heterogeneous networks with $N = 2$ and $N = 6$ respectively. The parameters setting is as in Fig. 2.

$2) = 0.1276, \gamma_1^{\mathrm{FO}}(N = 2) = 0.0859, \gamma_0^{\mathrm{FO}}(N = 6) = 0.0366$, and $\gamma_1^{\mathrm{FO}}(\mathrm{N} = 6) = 0.0315$. Thus γ_1 can first achieve the threshold γ_1^{FO} as the constraints increase. We also mark the points $\gamma_1^{\mathrm{FO}}(N = 2)$ and $\gamma_1^{\mathrm{FO}}(N = 6)$ in the figure to show the breaking point that when the constraint parameter is bigger than it, the analytical result only serves as an upper bound of the optimal FO-OSA performance. In Fig. 2 and Fig. 3, it is also shown that more channels leads to larger throughput in both PS-MA and FO-OSA policies. It is interesting to note that the throughput curves are non-smooth at the break points. This is because the throughput is obtained as the optimal solution to a linear programming problem. When the constraint changes from tight to loss, the gradient of the throughput to the parameter γ changes abruptly.

When we focus on the curves of the case $N = 6$ in Fig. 2 and Fig. 3, we can compare the performances of the two policies PS-MA and FO-OSA in this case. In both curves of the case $N = 6$, the performances will increase when the constraints increase and are still tight. It is also shown that both of the curves will be saturated when the constraints are loose enough. For PS-MA, it means that all the constraints satisfy $\gamma_i \in (\gamma_i^{\mathrm{MA}}, 1], \forall i$. For FO-OSA, it means no collision constraints. Through computation, we have $\gamma_0^{\mathrm{MA}}(N = 6) = 0.0326$ and $\gamma_1^{\mathrm{MA}}(N = 6) = 0.024$ and thus γ_1 can also first achieve the threshold γ_1^{MA} as the constraints increase. The optimality condition in (23) is $\gamma_i \in (0, \gamma_i^{\mathrm{MA}}], \forall i$. The breaking point $\gamma_1 = \gamma_1^{\mathrm{MA}} = 0.024$ is shown in Fig. 2 , and after we compare the two curves in the figures, we can conclude that under the optimality condition, PS-MA indeed performs the same with FO-OSA, and PS-MA is thus the optimal policy when the constraints is tight.

VI. Conclusion

We present in this paper a simple cognitive access policy (PS-MA) for the single secondary user and N primary user network. We show that, when the collision constraints are tight, PS-MA is optimal, and we provide closed-form expressions on the boundary of interference levels below which PS-MA is optimal.

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