Thermal Dynamic Modeling for Home Energy Management: A Case Study

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***Abstract*—The modeling of power system loads is a very important problem since loads are ultimately the driving force behind the entire system. In this report, we consider the problem of thermal dynamics air conditioner load models for residential homes. In particular, we investigate a physical-based load modeling methodology. We propose a method using minimization of the least square error to obtain the essential parameters α, G, and c from the linear differential equation in which the indoor temperature represents the state of energy storage in the house. The essential parameters describe the energy balance for a single house.The data used for this study corresponds to real measurements taken from a house and it was provided by Intel. The data consists of indoor and outdoor temperatures as well as HVAC power consumption for the different compartments of the house. Upon experiments that includes computational simulation, extensive data analysis, it is concluded that 1 day of training samples for parameter prediction performs reasonable well in comparison of 7 and 14 days. The validation of the model is performed using data in two different months. The modeling and prediction mean squared error obtained are below 0.1 degree. These results open the possibility to investigate techniques to incorporate adaptive temperature learning.**

I. Introduction

We consider the problem of thermal dynamic models for home energy management systems. We analyze a single home energy case. The data used for this study corresponds to real measurements of indoor and outdoor temperatures as well as power consumed of the different compartments of the house. The data was taken every 15 minutes. Our purpose is to study the feasibility of a linear thermal model that describes the physical conditions of a house, i.e. the heat and air flow exchange, change in temperature, human presence, climate conditions, etc. We aim to find the optimal method to extract the essential parameters that characterize those physical conditions and use them as inputs for other important Hem systems applications such as control and optimization for price and comfort. Energy consumption has been increasing over the past half century. Even though ultra-low power consuming appliances are being developed, the amount of energy consumption in homes is increasing because of the use of various appliances at peak times. The relationship demand response that leads to the increment of electricity prices lead to investigate methods in order to find the best way to save energy as well as keep the energy prices reasonable. This benefit both the consumer and power generation companies. Demand response manages customer consumption of electricity in response to supply conditions. Demand response is also associated with changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time. It deals with incentive payments designed to induce lower electricity use at time of high wholesale market prices or when system reliability is jeopardized [1]. Consider the case, for instance, during the course of the day, the load that power grids serve varies widely. The consumption at peak demand around 4:00PM is about one and a half times the demand during the night. Consequently, utilities must build enough generation capacity to serve demand during peak periods. Additionally peak power constrains impose large burden on the grid which also increase the demand response. But, if consumers adjust their energy usage in response to conditions on the grid, the result has the potential to create power savings. This process can be achieved using of a home energy management system(HEM) integrated with the grid. A HEM system regulates energy usage within the home. A comprehensive HEM system requires a networking capability to transmit information from the nodes(sensors) to the central control unit. It has the potential to reduce peak energy consumption and has the potential to reduce peak energy consumption and eliminate the need for costly resources to meet peak demand. Ultimately, it can benefit both consumers and utility companies by reducing costs on monthly energy usage and costs. The difficulty in demand response is the challenge of alerting customers of change prices and the transaction costs that results. Thus, home energy management systems combined with a smart grid can provide pricing signals to reap the benefits of demand response. Instead of home owner following the prices, the energy management system follows them and can make adjustment to electricity consumption based on user preference.

The architecture of a HEM system takes into account relevant sensor information such as temperature from the home as well as user preferences in the decision making model[2]. A basic control model would then be able to schedule appliance operations at off peak times to save consumers money based on a real time pricing scheme. Particularly, our study focus on appliances that can be controlled without significantly affecting consumers such as offsetting the timing of an air conditioning system that would maintain an acceptable temperature level, or scheduling the washer/dryer times. In order to construct an accurate physically-based model for a single house, the thermal characteristics of the house are required[3]. This model relates the energy consumed by the air-conditioner, the outside temperature, and the inside temperature. Additionally, The literature also indicates that the model works accurately when it incorporates the effects of weather such as temperature, humidity, solar radiation, wind speed, and the effects of lifestyle and possibly the effects of voltage and frequency

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This work is supported in part by the Intel Fellowship Program, and the National Science Foundation.

fluctuations. This paper addresses the following issues: I) A survey of a detailed model for a single house, II) House physical parameters extraction from the physical model based on multiple linear regression III) Possible unmodeled effects that contribute to errors to the model. III) The feasibility to make the model more adaptive to learn the different physical conditions in the environment for an optimal temperature prediciton

A. Summary of results

The main contribution of this paper is the careful analysis of a linear model to extract the essential parameters that characterize the physical model of a house. We propose the use of a discrete form of the physical model stochastic equation. The essential parameters are extracted using the minimization of the least square errors. We also explore the different possibilities of training data to obtain the optimal parameters since we are unaware of the presence of outliers and the nature of noise. This study shows that using one day of data for training gives better results than using 7 of 14 days of data. The validation is done predicting the temperature for the future next day using real time temperatures in the recursive linear model. The results yields mean square error for modeling and prediction error below 0.1 degree.

B. Related work

The literature on thermal dynamics models dates from 1978. work done on [4] provides an study of the determination of thermal parameters of buildings through two different approaches, deterministic models and methods based on equivalent thermal parameters(ETP) of a building extracted from a physical air conditioner or heater model. While the former are computer applications of heat transfer theory, the latter consist of statistical techniques in order to extract the thermal parameters from a thermal model. In [4], a convenient set of equivalent thermal parameters for residential townhouse is provided by means of a single thermal model. Authors in [5] have proposed an statistical approach to model the dynamics of the electric demand of air conditioners and heaters. This approach requires a diffusion approximation of a high-order hybrid state stochastic system. In [3], a dynamic model of the response of a single residential air conditioner load to weather conditions is developed. This approach uses estimation techniques and a air conditioner load model based upon equations for energy balance and mass balance for the air inside a customer's residence. In [6], an identification algorithm for load models have been proposed exploiting the alternating renewal nature of the thermostat switching process. The author in [7] propose a method for estimation of continuos-time models using the maximum likelihood method and a kalman filter to calculate the likelihood function.

The model adopted in this paper goes back to [3] and the house parameters are obtained using multiple linear regression.

II. Thermal Dynamic Models

A. Physical model

Because of both: the global energy crisis and the necessary improvement of energy efficiency in houses and buildings, simplified models that can represent the physical properties of a house and buildings are desired for diagnosis, control strategy analysis. Our thermal dynamic model is based upon an energy balance. Accurate single customer models are vital to the development of aggregate dynamic load models which can predict response to direct load control actions [3].The total energy content of the air within the house is decomposed into the energy content of the dry air, and the energy content of the water vapor. Conservation of energy requires the energy which contributes to changes in indoor temperature to satisfy

$$\dot{E} = e_1(E_{out} - E_{in}) - swE_{sen} \tag{1}$$

Where $\dot{E}$ is the rate of energy gain by the air volume inside the house, E_{in} is the total energy of the house at the current input temperature, E_{out} is the total energy of the house if it were at the outdoor temperature , e_1 is the percent of indoor air exchanged every hour with the environment, *sw* is a binary switch determining whether the air conditioner is on and off, and E_{sen} is the rate of energy transfer from indoors to outdoors caused by the air conditioning unit. According to [3], the term $e_1(E_{out} - E_{in})$ represents the energy transferred from outdoors to indoors due to the term term difference, and it can be represented by:

$$k_1(x_t^{out} - x_t^{in}) + e_1 V S_{vap}(x_t^{out} H_{out} W_{sat-out} - x_t^{in} H_{in} W_{sat-in}) \tag{2}$$

Where k_1 is the thermal house coefficient for dry effects, x_t^{out} and x_t^{in} are outside and inside temperatures respectively. V is the volume of the house. S_{vap} is the specific heat of water vapor, H_{out} and H_{in} are relative humidities, and W_{sat} are the density of saturated water vapor as a function of temperature. The thermal house coefficient of a house represents the rate of energy gain of the house per hour and degree of temperature difference between outdoors and indoors[3]. The quantity $VS_{vap}(x_t^{out} H_{out} W_{sat-out} - x_t^{in} H_{in} W_{sat-in})$ in equation (2) represents the change in the energy between indoors and outdoors. The term swE_{sen}in equation (1) represents the rate at which the unit removes energy from the house, and corresponds to the sensible capacity of the air conditioning of the unit. Using equation (2) to re-write equation (1)

$$\begin{aligned}\dot{E} \quad = \quad & k_1(x_t^{out} - x_t^{in}) + e_1 V S_{vap}(x_t^{out} H_{out} W_{sat-out} - \\ & x_t^{in} H_{in} W_{sat-in}) - swE_{sen}\end{aligned} \tag{3}$$

Which represent the rate of energy exchange due to the air exchange as a function of the indoor and outdoor temperature, and which is used toward the temperature changes inside the house. In [3] equation (3) is transformed in order to obtain the rate of indoor temperature change as a function of x_t^{out} and x_t^{in}, H_{out} and H_{in}. This was done using:

$$E = x_t^{in} V D_{air} S_{air} + x_t^{in} V H_{in} W_{sat-in} S_{vap} \tag{4}$$

Where D_{air} is the density of the air at a given temperature, S_{air} is the specific heat of the dry air. Equation4 is differentiating respect to time to obtain

$$\dot{E} = \dot{x}_t^{in} V(D_{air} S_{air} + H_{air} W_{sat-in} S_{vap}) \tag{5}$$

Inserting the equation above in equation (3) and rearranging to solve for $\dot{x}_t^{in}$, it is obtained

$$\dot{x}_t^{in} = \frac{k_1(x_t^{out}-x_t^{in})+e_1 V S_{vap}(x_t^{out}H_{out}W_{sat-out}-x_t^{in}H_{in}W_{sat-in}-swE_{sen})}{V(D_{air}S_{air}+H_{in}W_{sat-in}S_{vap}} \tag{6}$$

This is the final equation of the model, which tracks the temperature inside the house as a function of constants and either measurable or estimated variables. Equation 6 can be simplified to

$$d(x_t^{in}) = a(x_t^{out} - x_t^{in})dt + Rp_t dt + \sigma dv_t \tag{7}$$

Where a is the average thermal resistance per thermal capacity of the dwelling in watts/joules, R is the power rating per thermal capacity of the dwelling in watts, and $\sigma v_t'$ is a wiener process with intensity σ. The wiener process accounts for heat gain or heat loss(fluctuating number of people in the residence, and doors and windows being opened and closed, refrigerators and cooking, etc

B. Discrete-time mathematical model

The discrete-time equivalent equation for the continuous physical model in equation 7 is represented:

$$x_{t+1}^{in} = x_t^{in} + \alpha(x_t{}^{out} - x_t^{in}) + Gp_t + c + w_t \tag{8}$$

where
x_{t+1}^{in} is the temperature at time $(t+1)$
x_t^{in} is the indoor temperature at time t
$x_t{}^{out}$ is the outdoor temperature at time t
α, G, c are the essential parameters to estimate
p_t is power consumed from $(t-1)$ to time t
w_t is the noise modeling

III. Real data analysis: modeling and validation

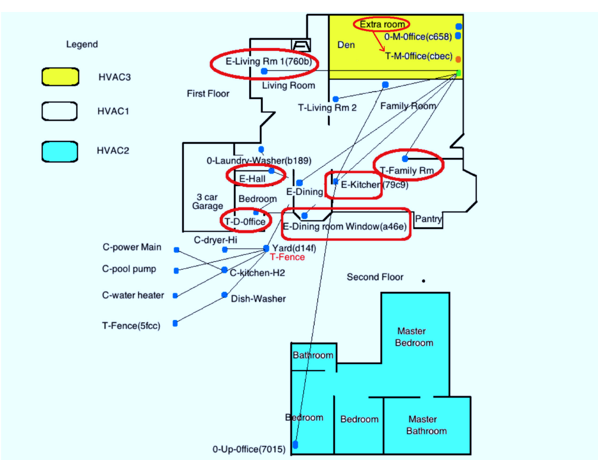

Fig. 1. Layout of a residential home

A. Physical parameters

Given a data set that consists of inside and outside temperatures as well as consumed power from a house, the parameters α, G and$c$ are extracted from the model. These parameters represent the physical properties of a house as explained in section 2. In order to extract an optimally these parameters, we use the minimization of the least square error. Our linear model, however, does not consider the nature of the noise present. Instead, we assume a constant in the equation which represents systematic errors. Once the parameters have been extracted, we use the recursive equation in (8) to validate the model. This task is done using multiple linear regression. In this work, we are not considering the effects of modeling noise w_t.

B. Data characteristics

The data used for simulation in this work has been facilitated by Intel. Intel has equipped a house with environmental and power sensors. The data provided was measured every 15 minutes. The measurements include indoor, outdoor temperature, and HVAC power usage, and corresponds to months from August to November. Figure 1 represents a blueprint of the house where the data was obtained. The house has 2 floors and uses 3 HVACS. HVAC 1 is connected to the main portion of downstairs. This includes .HVAC 2 for the second floor, and HVAC 3 for the extra room, which is a recent addition to the house. The sensor that takes readings for HVAC 1 is located in the hall on the first floor. The one that takes readings for HVAC 2 is located on the second floor, and the one that reads HVAC 3 is located in the extra room as shown in the figure below.

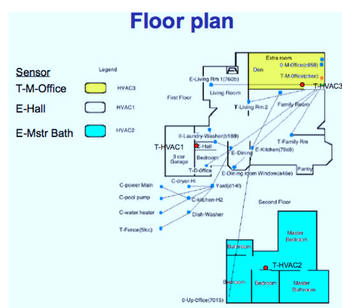

Fig. 2. Sensor position for HVACs

The data facilitates temperatures and HVAC power consumption readings from 7 different compartments of the house. This includes the living room, dining room, kitchen, family room, extra room and 1st floor bedroom. On the second floor, we have data available for the bedroom located on the left side and master bathroom.

C. Regression analysis and parameter estimation

We base our parameters extraction by minimization of the mean square error. Our recursive model equation in 8 can be represented as

$$x_{t+1}^{in} - x_t^{in} = \alpha(x_t{}^{out} - x_t^{in}) + Gp_t + c + w_t \tag{9}$$

Our object is to minimize the objective function:

$$(\hat{c}, \hat{\alpha}, \hat{G}) = \underset{c,\alpha,G}{\operatorname{argmin}} \sum_{t=1}^{n} ||(x_{t+1}^{in} - x_t^{in}) - (c + \alpha(x_t^{out} - x_t^{in}) + Gp_t)||^2 \tag{10}$$

Where n is the number of observations. To obtain $(\hat{c}, \hat{\alpha}, \hat{G})$ To solve the equation above, we use the matrix formulation

$$y = x_{t+1}^{in} - x_t^{in} \tag{11}$$

$$y = \mathbf{X}\theta + \varepsilon \tag{12}$$

Where

$$\mathbf{X} = \begin{bmatrix} 1 & (x_1^{out} - x_1^{in}) & p_1 \\ 1 & (x_2^{out} - x_2^{in}) & p_2 \\ : & : & : \\ 1 & (x_n^{out} - x_n^{in}) & p_n \end{bmatrix}$$

and

$$\theta = \begin{pmatrix} c \\ \alpha \\ G \end{pmatrix}$$

And ε is of the form

$$\varepsilon = \begin{pmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

The matrix $\mathbf{X}$ is a known n x p matrix $(n > p)$ of full rank p. It is also referred as the observation matrix. The objective function in 10 can be represented as

$$J(\theta) = (y - \mathbf{X}\theta)^{\mathbf{T}}(\mathbf{y} - \mathbf{X}\theta) \tag{13}$$

We notice that J is a quadratic function of θ.The the minimization respect to θ is easily accomplished since

$$J(\theta) = y^T\mathbf{X} - 2y^T\mathbf{X}\theta - \theta^T\mathbf{X}^{\mathbf{T}}\mathbf{X}\theta \tag{14}$$

Also note that $y^T\mathbf{X}\theta$ is a scalar. The gradient of (14)

$$\frac{\partial J(\theta)}{\partial \theta} = -2\mathbf{X}^{\mathbf{T}}y + 2\mathbf{X}^{\mathbf{T}}\mathbf{X}\theta \tag{15}$$

Setting the gradient to zero yields

$$\hat{\theta} = (X^TX)^{-1}y \tag{16}$$

Where

$$\hat{\theta} = \begin{pmatrix} \hat{c} \\ \hat{\alpha} \\ \hat{G} \end{pmatrix}$$

Equation (16) is used to obtain the parameters from the available data set. We considered several cases when training data to extract these parameters. In our experiments, data sets corresponding to 1 day, 7 days and 14 days are used for training. We use the mean square error in the fitting model to compare the performance using the the different window lengths. The validation of the model Important emphasis is placed in the start point to start the extraction of parameter. This is mainly because we are unaware of the presence of outliers and the nature of the noise. It is important to address the fact that we are not considering the noise in our model. We don't know its nature, but we analyze the error using autocorrelation, power spectral density, Q-Q plot, and the Kolgomorov-smirnov test to determine if our error has a gaussian nature.

D. *Model validation*

After training data using window data length of 1, 7, and 14 days, and parameter extraction is performed, we validate our model predicting the temperature for the following day and comparing the results against actual data. We use the equation below to obtain the predicted temperature.

$$\hat{x}_{t+1}^{in} = x_t^{in} + \hat{\alpha}(x_t^{out} - x_t^{in}) + \hat{G}p_t + \hat{c} \tag{17}$$

Once the set of predicted temperature is obtained, we obtain the mean square error between the actual temperature from the predicted ones, Ultimately, we compare the modeling errors from the training part with the prediction error obtaining in the validation part. After an exhaustive manipulation of the available data, it was concluded that using one day data (1 day window length) for parameter extraction had better performance than using 7 and 14 days of data for training. We found that using one day data for parameter extraction performs reasonable well that using 7 or 14 days. The validation of the model is performed using data in two different months. The modeling and prediction mean squared error obtained are below 0.1 degree.Our results also include the mean of the parameters and their respective standard deviation. In order to attempt to determine the nature of error, we also plot the autocorrelation and power spectral density of modeling an prediction error. We compare the CDF of our errors against a normal distribution CDF. Finally, we use the kolgomorov-smirnov test.

Room	ε_m	ε_p	$\bar{\alpha} \pm \sigma_\alpha$	$G \pm \sigma_G$	$\bar{c} \pm \sigma_c$
Living Room	0.0103	0.0116	0.0083 ± 0.0046	-0.2076 ± 0.1211	0.0216 ± 0.0552
Family Room	0.0249	0.0282	0.0127 ± 0.0046	-0.1798 ± 0.1608	0.0062 ± 0.0908
Kitchen	0.0247	0.0281	0.0121 ± 0.0048	-0.1997 ± 0.1901	0.0324 ± 0.0894
Dining Room	0.1355	0.1480	0.0209 ± 0.0127	-0.5438 ± 0.1768	-0.0047 ± 0.1201
Office Room	0.0879	0.0956	0.0168 ± 0.0120	-0.5095 ± 0.2337	0.0176 ± 0.0802
Hall	0.0182	0.0212	0.0101 ± 0.0051	-0.3314 ± 0.1787	0.0264 ± 0.0669
Extra Room	0.4122	29.8200	0.0474 ± 0.0388	-5.8284 ± 24.6484	0.0709 ± 0.3141
Ups. Office	0.0184	0.0199	0.0107 ± 0.0048	-0.2990 ± 0.1214	0.0631 ± 0.0521
Ups. bath	0.0372	0.0412	0.0152 ± 0.0070	-0.4435 ± 0.6012	0.0417 ± 0.0820

TABLE I

OVERALL RESULTS USING 1 DAY-WINDOW LENGTH FOR TRAINING. VALIDATION IS DONE FOR 75 DAYS STARTING AT DAY AUGUST 15TH

In the table above, ε_m and ε_p represent the modeling and prediction mean squared error respectively. We also present results for the mean of the parameters for every room. As observed the results for modeling and prediction mean squared error are small for the living room. On the other hand, the results for the extra room is unrealistic, meaning that our model does not work well in this case. One of the reasons could be that this room is not controlled by the thermostat. The owners adjust the temperature mainly at night time. Another important fact we have not consider in our model is the impact of air flow. The living room and the second floor are connected by a cathedral ceiling, and thus this room is coupled with the upstairs. However, a second order model have been consider to address this situation.

Room	T	σ_T	ε_m	ε_p
Living Room	79.1244	1.8073	0.0103	0.0116
Family Room	80.5193	2.5919	0.0249	0.0282
Kitchen	81.8173	2.1409	0.0247	0.0281
Dining Room	79.0615	14.0744	0.1355	0.1480
Office Room	79.8312	9.6645	0.0879	0.0956
Hall	79.2099	1.1447	0.0182	0.0212
Extra Room	83.0398	48.2190	0.4122	29.8200
Ups. Office	78.2068	1.1281	0.0184	0.0199
Ups. bath	77.8422	2.6523	0.0372	0.0412

TABLE II
OVERALL RESULTS USING 1 DAY-WINDOW LENGTH FOR TRAINING. VALIDATION IS DONE FOR 75 DAYS STARTING AT DAY AUGUST 15TH

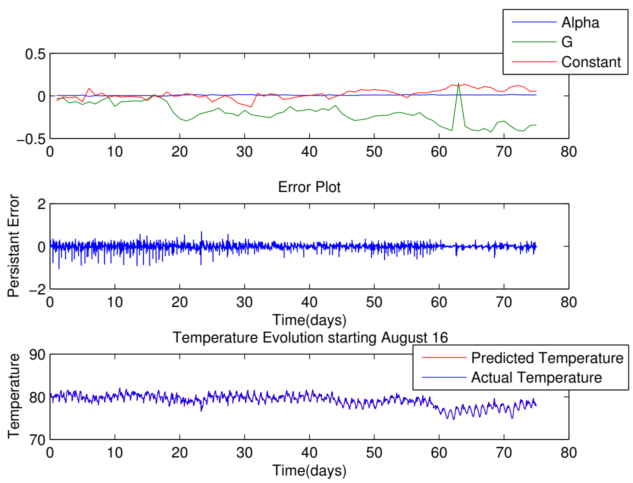

Fig. 3. Parameter evolution, validation error and temperature evolution for living room

In figure (3), we present the parameter evolution for 75 days. We decided to extract parameters starting at day 15 due to some defective data present on the first two weeks(first 15 days of available data). The evolution of parameters seems to vary as time evolve. The middle and bottom part of figure (3) indicate validation error plot as well as predicted temperature and actual temperature.

Figure (4) show the mean square error for modeling and prediction error comparison. This is a way to show the effectiveness of the model. As observed, the MSE of modeling error is smaller that the MSE of prediction error.

Because, we don't know the nature of the noises present in our data, we have used some signal processing techniques to observe modeling and prediction error behavior. In figure 5, we show the autocorrelation of the modeling and prediction error. As observed, neither of them present a flat outline, indicating that we are not dealing with noise of gaussian nature.

To go further, the power spectral density of modeling and prediction error is also presented in figure (6). Clearly, it is observed thew presence of spikes indicating the presence of harmonics in our data.

In the same fashion, figure (7) presents Q-Q plots, in which quantiles of modeling and prediction error are compared against standard normal quantiles. The more our samples follow the reference line would indicate the gaussian nature of the errors. However, the modeling and prediction errors do not follow the reference line fully, giving an indication that they are not of standard gaussian nature.

Figure (8) shows a plot of the CDFs of modeling and prediction error against a CDF of a gaussian distribution. Figure (9) represent 1 day realization, i.e. predicted and actual temperature comparison temperature for day 16 in August. As observed the the predicted temperature follows closely the actual temperature. Ultimately, the Kolgomorov-smirnov is used on both modeling and prediction error. The results rejects the null hypothesis that says both errors belong to a gaussian family distribution.

IV. CONCLUSION

In this paper, we have presented the performance of a linear model to extract the essential parameters that represent the physical characteristics of a single house. Upon exhaustive data manipulation, it has been determined that using one day of data samples gives better results than using 7 and 14 days of data for training. The validation is done using approximately 2 months of data. Our results show that the obtained modeling and prediction mean squared errors are below 0.1 degree. However, more study to address the nature of noise present in the data needs to be done. The model is sensitive to bad sensor data, and depends upon reliable smart sensors and controlled temperature by thermostaht. Further study needs to be done using the model for other data sets from different houses and buildings. Unfortunately, this kind of information is unavailable at this moment. Further, given that MSE of modeling and prediction errors are small, more research on techniques to achieve adaptive temperature learning needs to be done for completeness.

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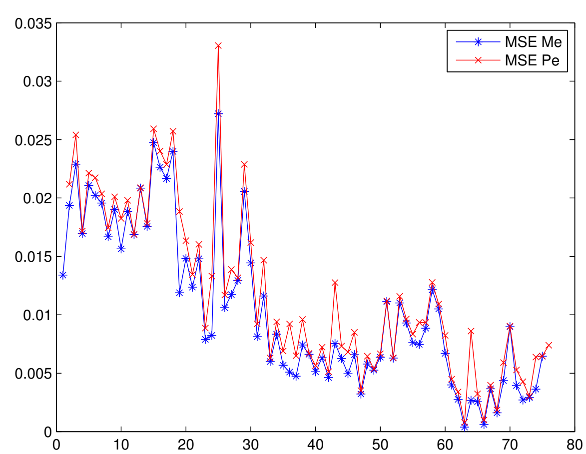

Fig. 4. MSE comparison for modeling and prediction error

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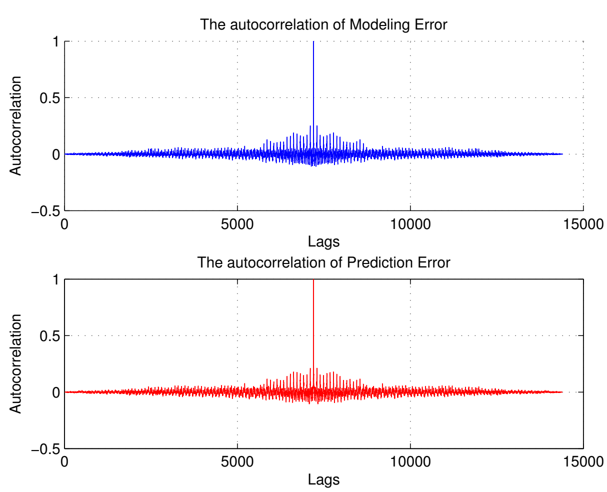

Fig. 5. Autocorrelation for me and pe living room

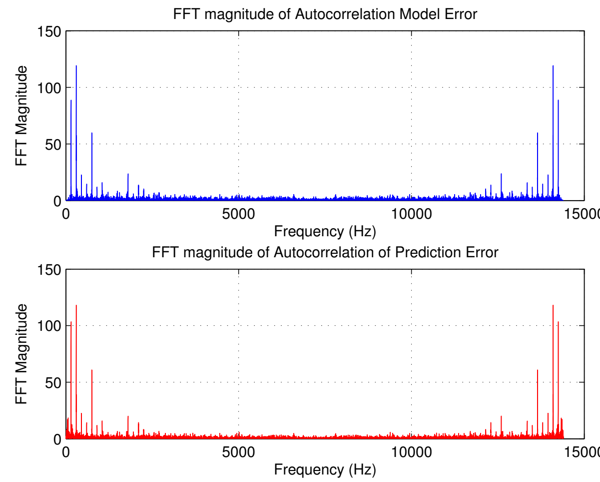

Fig. 6. FFT of Autocorrelation of me and pe

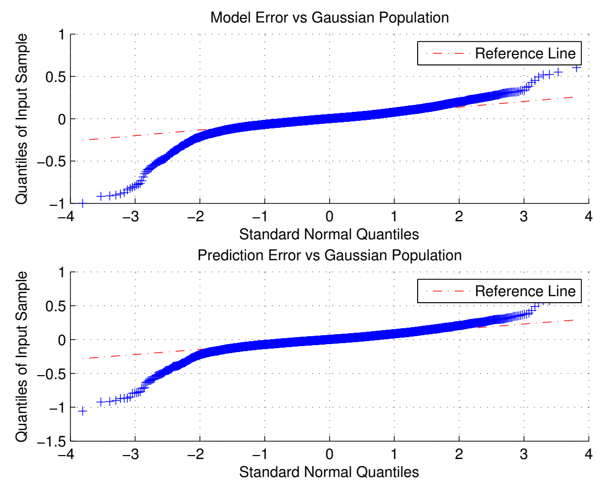

Fig. 7. Q-Q plots

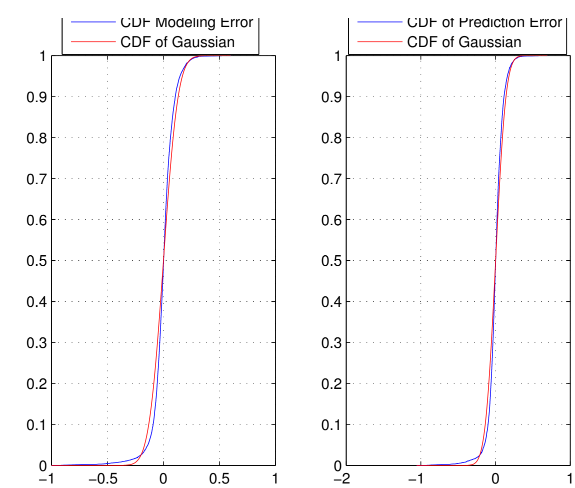

Fig. 8. CDF of me and pe

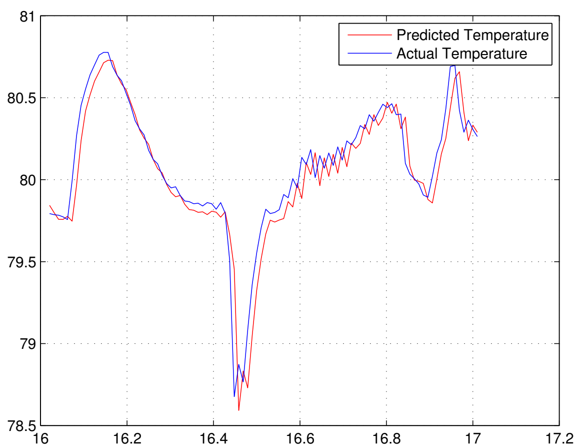

Fig. 9. 1 day realization