On the Capacity of Regular Wireless Networks with Transceiver Multipacket Communication

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Abstract — **In a wireless network with a sophisticated physical layer, the nodes may be capable of simultaneous multiple packet receptions (MPR) and multiple packet transmissions (MPT). Having multiple reception/transmission codes in a CDMA network, or employing directed antenna arrays are some ways of obtaining MPR/T capability. Although, MPR/T is widely considered in the context of communication with a base station in cellular wireless networks, its effect on the performance of peer-to-peer ad hoc networks is unknown. By developing upper and lower bounds, we analyze the effect of physical layer MPR/T capability on the capacity of regular wireless networks. The obtained bounds give the exact capacity value if the nodes do not have MPT but have MPR.**

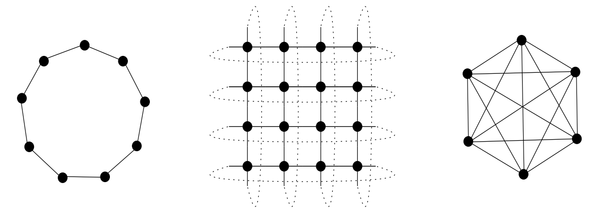
I. Summary of the Results

An ad hoc network is modeled as an undirected graph. In a Manhattan network (Figure 1.b), every node in the network has four neighbors, and the nodes on the edge are connected to the nodes on the other side, like a torus covered by a grid. It is assumed that the time is divided into fixed length slots, and transmission of one packet takes a single slot. The nodes can not transmit and receive at the same time. In each slot a node can transmit at most T packets simultaneously. We assume that a node can distinguish the packets intended for itself among the packets it receives, and decodes only those packets intended for itself. In each slot, a node can correctly receive and decode a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *MPR Matrix* $\mathbf{C} = [C_{n,k}]$, where

$$C_{n,k} = P[k \text{ packets are correctly received} \mid n \text{ packets are transmitted in the neighborhood}].$$

Define $C_n = \sum_{k=1}^{n} kC_{n,k}$ which is the expected number of correctly received packets given n packets are transmitted.

Consider a Manhattan network composed of N nodes. All nodes in the network have the same MPR capability which are described by an MPR matrix $\mathbf{C}$. Every node in the network has an infinite buffer for holding its packets. The network starts operation at time $t = 0$ where buffer of each node is empty. In slot t the node i generates $\beta_i(t)$ packets randomly. $\beta_i(t)$ is a stationary and ergodic process with mean $\mathbb{E}\beta_i(t) = \lambda$, $\forall t, i$. λ is called the *arrival rate*. The destination of a packet generated at a node can be any other node in the network



a) Ring 9 nodes b) Manhattan 4x4 nodes c) Fully connected 6 nodes

Fig. 1: Examples of regular networks

with equal probabilities. The destination of a packet does not change once it is generated. A network protocol is a set of rules governing the transmission decisions of nodes. A protocol is responsible for determining what transmissions are made in each slot (medium access control), and through which path the packets are delivered (routing). The number of packets in the buffer of node i at time t is denoted by $n_i(t)$. A node i is called *stable* if

$$\lim_{\theta\to\infty} \liminf_{t\to\infty} Pr\{n_i(t) < \theta\} = 1. \quad (1)$$

A network is called stable if all nodes in the network are stable.

Theorem 1 *Consider a Manhattan network of N nodes with MPR matrix $\mathbf{C}$ and T-packet transmission capability. Define*

$$\eta_u = \frac{2}{\sqrt{N}} \max_{i=1,\cdots,4T} \frac{TC_i}{i+T}, \qquad \eta_L = \frac{2}{\sqrt{N}} \max_{\substack{i=1,\cdots,4\\ t=1,\cdots,T}} \frac{C_{it}}{1+i}. \quad (2)$$

For an arrival rate $\lambda > \eta_u$, there does not exist any protocol that makes the network stable. For every arrival rate $\lambda < a_N\eta_L$, there exists a protocol that makes the network stable, where a_N is a function of N such that $a_N \to 1$ as $N \to \infty$.

Note that, for $T = 1$ (no MPT) and an arbitrary MPR matrix, the lower and upper bounds coincide. A proof of Theorem 1 is given in [1]. The lower bound is achieved by using some global scheduling medium access and shortest path routing. One can similarly consider the capacity of distributed medium access protocols by extending the slotted ALOHA protocol to MPR/T networks. This is considered for MPR networks [2]. Similar bounds can be obtained for other regular topologies. Finally, we want to note that the problem of node stability (in a network with random arrivals), and the problem of packet scheduling (in a network where packets exist in infinite amount at source nodes) give identical rate regions [1].

References

[1] G. Mergen and L. Tong, "Stability and capacity of wireless networks," to be submitted.

[2] Conference publications at http://www.ee.cornell.edu/∼ltong/.

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