RANDOM SCHEDULING MEDIUM ACCESS FOR WIRELESS AD HOC NETWORKS

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ABSTRACT

In this paper, we introduce a novel random scheduling strategy for distributed medium access in spread-spectrum (SS) and multipacket reception networks. The proposed random scheduling (RASC) protocol uses a special method of seed exchange in which the seeds are used for pseudo-random generation. The RASC utilizes the seeds for randomly decomposing the network into independent clusters containing a single receiver. After the decomposition, the network resembles to the up-link of cellular networks whose medium access techniques are well developed. The application of RASC in spread-spectrum networks avoids collisions and provides Quality-of-Service guarantee at the MAC layer. The throughput performance of the protocol is analyzed in fully connected and Manhattan networks using analysis and simulations. While still being a distributed random access protocol, the RASC offers throughput which is much closer to the network capacity compared to the pure random access protocols like slotted ALOHA.

1. INTRODUCTION

The possibility of simultaneously receiving multiple packets is a natural concept in advanced communication systems. The diversity provided by the use of multiple antennas or spread-spectrum codes makes it possible to separate signals coming from different transmitters and allows multipacket reception (MPR) [2].

In cellular networks, the MPR capability is the key for providing high network capacity. Recently, there has been numerous developments about efficiently utilizing MPR in both the physical and medium access layers. Both the information theoretic limits of the cellular systems were investigated, and medium access (MAC) protocols for spread spectrum networks or multiple antenna systems were developed. Although these developments provided a fair understanding of the effects of MPR in cellular networks, the effects of MPR in an ad hoc networks is not yet well understood. Moreover, the problem of efficient medium access for MPR ad hoc networks is yet unsolved.

Recent analyses about the capacity of wireless networks [3, 4] clearly demonstrated that perfect scheduling is the ultimate way to achieve capacity in the MAC layer. However, in a distributed ad hoc network it is impossible to apply perfect packet scheduling, and random access should be used to some extent.

In this paper, we look at the MAC problem for MPR networks, and consider a novel random scheduling strategy that benefits from advantages of both random access and scheduling. In each slot, the RASC protocol pseudo-randomly decomposes the network into independent clusters each of which contain a single receiver and its associated transmitters. The decomposition process provides a connection with the cellular MPR systems, and makes it possible to use the medium access principles developed for the up-link of cellular networks in an ad hoc network. The RASC avoids collisions encountered in spread-spectrum networks [6], and guarantees a certain throughput (*i.e.,* Quality-of-Service) over any link regardless of the network load from other nodes. Furthermore, as a result of the pseudo-random scheduling, the nodes can estimate what will happen in the future time slots, and intelligently schedule the packets generated by delay-sensitive applications.

In Section 2 the MPR model is introduced. In Section 3 the RASC protocol is specified. In Section 4 upper and lower bounds on throughput in an arbitrary topology are derived, and in Section 5 performance of the RASC in fully connected and Manhattan topologies is analyzed.

2. THE RECEPTION MODEL

It is assumed that the time is divided into fixed length slots, and transmission of one packet takes a single slot. The nodes can not transmit and receive at the same time. In each slot, a node can correctly receive and decode a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *Receiver MPR Matrix* $\mathbf{C}$. The entries of the MPR matrix $\mathbf{C}$ are given as

$$C_{n,k} = P[k \text{ packets are received} \mid n \text{ packets are transmitted in the neighborhood}].$$

The receiver MPR matrix is defined and given as

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (1)$$

In fact, in an ad hoc network two kinds of MPR matrices can be defined depending on the channel signaling method. If nodes are listening to every transmission in their neighborhood (like in a common code scheme [6] or in the collision channel) then n in $C_{n,k}$ denotes the number of packets transmitted in the neighborhood of the receiver. In the other case, n in $C_{n,k}$ denotes the number of packets transmitted in the neighborhood of the receiver which are *intended for the receiver* (like in a receiver or transmitter based code selection [6]). We will use the second MPR model since it is better suited for providing higher throughput. It basically assumes that nodes do not try to decode packets which are not intended for them and they know the transmissions they should listen to.

The transmitting nodes adjust their transmission power such that their intended receiver receives the packet with an appropriate power level. Preferably a node receives all packets intended for him with the same power. Note that the interfering packets may have different power levels and the near-far effect is unavoidable in an ad hoc environment, since a transmitter can adjust its power only for one receiver.

Since the MPR matrix $\mathbf{C}$ is a function of a number of parameters, the nodes in a network may have different MPR matrices. The MPR matrix depends on the ambient noise, the type of modulation used, and the multiuser detection/equalization method receivers apply. It also includes the interference coming from transmissions which is not of interest to the receiver.

This channel model is general enough to model the conventional channel and the capture channel as special cases. Also we define a special matrix called *Perfect MPR* which is the case where receivers are strong enough to correctly receive every packet they hear. The corresponding MPR matrices for the conventional channel, the capture channel, and the perfect MPR are, respectively,

$$\begin{pmatrix} 0 & 1 & 0 & \dots \\ 1 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & \dots \\ 1-p_2 & p_2 & 0 & \dots \\ 1-p_3 & p_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0\dots \\ 0 & 0 & 1 & 0\dots \\ 0 & 0 & 0 & 1\dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where p_i denotes the probability of capture given i simultaneous transmissions in the reception neighborhood.

In MPR networks, it can be shown that the probability of a packet success given that n packets are transmitted is $\frac{C_n}{n}$ where

$$C_n \triangleq \sum_{k=1}^{n} kC_{n,k} \quad (3)$$

is the expected number of correctly received packets given n packets are transmitted.

3. RANDOM SCHEDULING PROTOCOL

An ad hoc network is modeled as an undirected graph $G = (V, E)$, where each vertice (node) represents a radio terminal, and edges denote the logical connections between terminals. The links are logical because the nodes also hear transmissions from out of their neighborhood but with very low power that they can not correctly decode. In a receiver's MPR matrix, the transmissions coming from out of his neighborhood are reflected as interference.

The operation of the RASC is briefly illustrated in Figure 1 using a network with 5 nodes. The four phases of the protocol are given as follows: 1.a is the beginning of a slot, 1.b transmitters and receivers are assigned, 1.c transmitters associate with receivers 1.d transmissions take place. In each slot the network goes through these four phases in the given order.

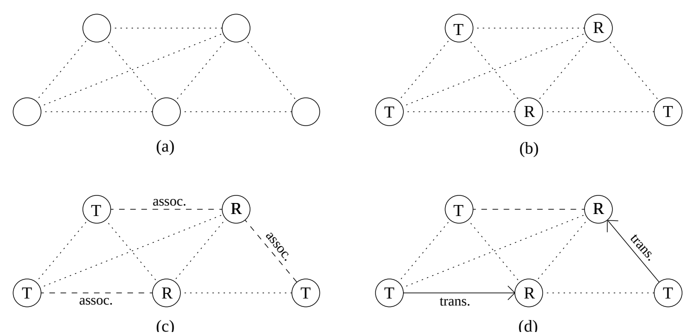

Figure 1: Four phases of the RASC protocol

Every node in the network has a different seed, just as in the SEED-EX protocol [1], and has a sequence of 0's and 1's (*i.e.,* fair coin flips) which are generated pseudo-randomly. Each node knows the topology up to two-hop distance, seeds of his neighbors and seeds of neighbors of neighbors. In every slot a node becomes a transmitter or a receiver with probability 1/2 depending on the state of its random number generator. Since a node knows seed of his neighbors and neighbors of neighbors, he knows if they have become a transmitter or a receiver.

After possible transmitters and receivers are determined, every transmitter in the network looks at the receivers in its neighborhood and choose an associated receiver randomly with uniform probabilities. Since each node has a seed (and a sequence of fair coin flips), he can generate a uniform discrete distribution using his random number generator. A transmitter's association decision is based on his generated uniform distribution. Using this random association method, every node in the network can figure out which neighbors of him are associated with whom since the seeds of nodes up to the two-hop distance are known. In [7] it's shown that any arbitrary discrete distribution can be generated from an infinite number of coin flips. A uniform distribution can also be generated as elaborated more in the appendix.

In the end of association the network is divided into disjoint groups of nodes each of which contain a single receiver and its associated transmitters. There can be also receivers which do not have an associated transmitter, or a transmitter which do not have any receiver to associate with in its neighborhood. Such isolated nodes stay idle in that particular slot, they neither receive nor transmit. A transmitter in a slot can only transmit to his associated receiver, and he uses some form of transmitter or receiver oriented coding such that the transmitted signal can be monitored by the associated receiver. We assume in different groups different set of codes are used which are orthogonal or pseudo-orthogonal, so that the receivers can consider the out of associated neighborhood signals as background noise.

After association, the rest of the problem is very similar to the MAC problem in up-link of cellular networks. Although the network is divided into disjoint groups, a receiver still does not know which nodes in his associated neighborhood hold packets for him (if there are any). Here, there are many different possibilities for the protocol, but we will only consider *perfect local scheduling/polling*. In perfect local scheduling, every receiver controls its associated neighborhood and chooses the ones to transmit by knowing they hold a packet. A receiver can learn the transmitters holding packets for him either by employing a form of RTS/CTS communication or by polling. Such methods provide the highest local throughput if the time spent for polling can be considered negligible comparable to the data packet length.

Given that m associated transmitters are holding packets, it depends on the policy and the MPR matrix of a receiver to choose what to do. If the aim is to maximize throughput a receiver should give

$$\tau_m \triangleq \arg \max_{n=1,\cdots,m} C_n$$

nodes the permission to transmit. If τ_m nodes transmit the average number of packets the receiver receives is

$$\eta_m \triangleq \max_{n=1,\cdots,m} C_n, \tag{4}$$

which is the maximum that can be achieved. Those τ_m nodes out of m can be selected randomly, or by considering Quality-of-Service requirement of the transmitters, or depending on some other factors. In the analysis of the RASC protocol we will assume that given m transmitters hold packets, τ_m users are selected to transmit randomly with uniform probabilities. The receivers receive the packets and after transmissions (at the end of a slot) notify the transmitters if they are correctly decoded.

4. THROUGHPUT ANALYSIS IN A GENERAL NETWORK

In this section, we will determine lower and upper bounds on rates that can be supported by the RASC protocol. Given a network graph G and end-to-end traffic requirement, we suppose the routes are determined and local traffic matrix $\mathbf{R} = [r_{i,j}]$ is computed. The non-negative scalar $r_{i,j}$ denotes the traffic (no. of packets per slot) to be transmitted from node i to node j where node i and j are neighbors in the graph G. We assume the network is fully loaded, i.e., every node holds packets for all its neighbors all the time. A traffic matrix $\mathbf{R}$ is called feasible if there exists a transmitter selection policy of receivers such that the rates $r_{i,j}$ can be achieved with the RASC protocol with probability 1 as time tends to infinity . The problem is to determine whether a given traffic requirement matrix $\mathbf{R}$ is feasible or not. Although the answer of this question yields all the set of achievable rates, it is not clear how it can be answered. Because of this we will provide an upper bound on the feasible set of throughputs by assuming every association yields a transmission, and a lower bound by assuming the receivers choose their transmitters optimally and randomly to maximize the throughput in each slot as explained in Section 3.

Heavy load assumption, and the transmitter selection policy mentioned in Section 3 generates a probability space on the set of events happening in a given slot. Suppose $P\{\cdot\}$ shows probability of an event, and $E\{\cdot\}$ denotes the expectation operator, and $|\{\cdot\}|$ denotes the number of elements in a set. Let $\eta_{i,j}$ denote the throughput on the link from i to j, $\{i \xrightarrow{\text{tx}} j\}$ denote the event of successful transmission from node i to j, $\{i \xrightarrow{\text{assoc}} j\}$ denote the event of association of node i with j, $\{i \text{ tx}, j \text{ rec}\}$ denote the event of i being transmitter and j being receiver. With these definitions the following is clear

$$\{i \xrightarrow{\text{tx}} j\} \subset \{i \xrightarrow{\text{assoc}} j\} \subset \{i \text{ tx}, j \text{ rec}\}. \tag{5}$$

Suppose $\mathcal{N}_i$ denote the set of neighbors of i, and define $N_i \triangleq |\mathcal{N}_i|$ as the number of nodes in the neighborhood of i. The throughput (*i.e.,* the supported rate) on link i, j can be expressed as

$$\begin{aligned}\eta_{i,j} &= P\{i \xrightarrow{\text{tx}} j\} = P\{i \xrightarrow{\text{tx}} j, i \xrightarrow{\text{assoc}} j\} \\ &= P\{i \xrightarrow{\text{tx}} j | i \xrightarrow{\text{assoc}} j\} \; P\{i \xrightarrow{\text{assoc}}\}. \end{aligned} \quad (6)$$

It is obvious that $P\{i \text{ tx}, j \text{ rec}\} = \frac{1}{4}$, and $P\{i \xrightarrow{\text{assoc}} j\}$ can be written as

$$\begin{aligned}P\{i \xrightarrow{\text{assoc}} j\} &= P\{i \text{ tx}, j \text{ rec}\} \; P\{i \xrightarrow{\text{assoc}} j | i \text{ tx}, j \text{ rec}\} \quad (7) \\ &= \frac{1}{4} \sum_{r=0}^{N_i-1} \frac{1}{1+r} \binom{N_i-1}{r} \frac{1}{2^{N_i-1}} \quad (8)\end{aligned}$$

4.1. The upper bound

Recall that $\frac{C_n}{n}$ is the probability of successful transmission of a packet given n packets are transmitted simultaneously. In this case, probability of any transmission is upper bounded by

$$U \triangleq \sup_{n=1,2,\cdots} \frac{C_n}{n}. \quad (9)$$

Which means $P\{i \xrightarrow{\text{tx}} j | i \xrightarrow{\text{assoc}} j\} \leq U$ holds. Using this inequality, (6), and (8), we can upper bound $\eta_{i,j}$ as

$$\eta_{i,j} \leq U \sum_{r=0}^{N_i-1} \frac{1}{1+r} \binom{N_i-1}{r} \frac{1}{2^{N_i+1}}. \quad (10)$$

This upper bound is tight if only if $P\{i \xrightarrow{\text{tx}} j | i \xrightarrow{\text{assoc}} j\} \approx U$, which means once i is associated with j he is highly likely to transmit a packet with probability close to U. This is the case when j doesn't have traffic from its other neighbors and/or the MPR capability is high compared to number of associated neighbors of j. Also in perfect MPR case (where $U = 1$) this bound is achieved.

4.2. The lower bound

Under the full load assumption, the best thing a receiver j can do is choosing the optimal number τ_m of transmitters to transmit given $m = |\{k : k \xrightarrow{\text{assoc}} j\}|$ transmitters are associated with j. In this case the expected number of correctly received packets is η_m (4). With these definitions the following holds

$$P\{i \xrightarrow{\text{tx}} j | i \xrightarrow{\text{assoc}} j\} = E\left\{\frac{\eta_m}{m} \mid i \xrightarrow{\text{assoc}} j\right\} \quad (11)$$

The expectation in (11) is very hard to compute, and we will use a lower bound on (11). $\forall\ m$, $C_1 \leq \max_{n=1,\ldots,m} C_n = \eta_m$ holds, and by Jensen's inequality

$$E\left\{\frac{\eta_m}{m} \mid i \xrightarrow{\text{assoc}} j\right\} \geq \frac{C_1}{E\{m \mid i \xrightarrow{\text{assoc}} j\}} \triangleq L. \quad (12)$$

The quantity L can be computed analytically, but due to the space limitations we leave it to the reader. The lower bound in an implicit form can be expressed as

$$\eta_{i,j} \geq L \sum_{r=0}^{N_i-1} \frac{1}{1+r} \binom{N_i-1}{r} \frac{1}{2^{N_i+1}}. \quad (13)$$

We will give some numerical examples for this lower bound in the following section. This bound is close to the actual throughput only if $C_1 \geq C_i, \forall i$ and the outgoing traffic from each node to every neighbor are the same (*i.e.,* the heavy traffic assumption holds when the network load is high).

5. THROUGHPUT OF SINGLE HOP AND MANHATTAN NETWORKS

In this section, we will present performance of the RASC protocol in MPR networks and compare the results with that of slotted ALOHA. The slotted ALOHA is the only protocol that has an immediate direct extension to the MPR networks among non-MPR protocols such as MACA, CSMA etc. This is why we use it to compare with the RASC. Also slotted ALOHA has a striking similarity with the RASC protocol. In both protocols a node randomly becomes a transmitter with some probability (0.5 in RASC, the retransmission probability p in ALOHA). On the other hand, the RASC has two advantages over ALOHA. In RASC since every transmitter knows which neighbors are receiving, the packets are not lost by transmitting packets to other transmitting neighbors, and secondly, two transmitters do not transmit to the same receiver at the same time (with the help of polling mechanism) and possible collisions are avoided.

Note that in the simulations we will not consider the traffic load of by acknowledgments and polling. The MPR matrix of all nodes in the network is assumed identical, and we will consider two special types of MPR matrices: the collision channel and the perfect MPR. We will also consider two types of network topologies, fully-connected network and the Manhattan network (Figure 2). In a Manhattan network of N nodes, the nodes are placed on a grid with dimensions $\sqrt{N} \times \sqrt{N}$. The nodes on the edge are connected to the nodes on the other side, just like a torus covered by a grid. Capacity results for single hop and Manhattan networks which are obtained in [3] will be used in the comparisons.

We consider the fully connected and Manhattan networks with $N = 100$ nodes, and for both the traffic is uniform. The maximum achievable throughputs with different MAC protocols are tabulated in Table 1 and 2. The unit used in each entry is *packet/slot*. The throughput values in the tables show the maximum achievable throughput on a link from an arbitrary node i to a neighbor j. Those values are the same for any neighboring

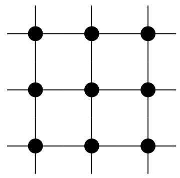

Figure 2: The Manhattan Network

	Collision Channel	Perfect MPR
Capacity	0.125	0.200
η_{ALOHA}	0.044	0.063
η_{RASC}	0.070	0.117
Lbound	0.053	0.053
Ubound	0.117	0.117

Table 1: Performance figures for the Manhattan network

pair. The capacity is obtained by perfect scheduling [3] which is the absolute maximum throughput under uniform load. η_{ALOHA} is the maximum throughput of slotted ALOHA protocol with optimal retransmission probability (obtained analytically). η_{RASC} is the maximum throughput of the RASC protocol which is obtained by evaluating the expectation in (11) through simulations. Lbound and Ubound are the lower and upper bounds obtained in (13) and (10) respectively. As expected, the upper bound is very tight in Perfect MPR case, and the lower bound is quite close to the η_{RASC} for the collision channel.

6. CONCLUSION

For MPR networks, a random scheduling multiple access protocol is proposed. First the MPR model is introduced and random scheduling alternatives are discussed. Then the RASC protocol is stated and its throughput performance is analyzed. The proposed protocol decomposes the network randomly. After the decomposition each receiver controls its associated neighborhood optimally considering the MPR capability. RASC protocol provides high throughput much better than that of ALOHA, and to our knowledge, it is the first distributed MAC protocol which is specifically designed for MPR ad hoc networks with arbitrary topologies and traffic patterns.

	Collision Channel	Perfect MPR
Capacity	$0.500\cdot10^{-2}$	$0.990\cdot10^{-2}$
η_{ALOHA}	$0.161\cdot10^{-2}$	$0.250\cdot10^{-2}$
η_{RASC}	$0.316\cdot10^{-2}$	$0.499\cdot10^{-2}$
Lbound	$0.250\cdot10^{-2}$	$0.250\cdot10^{-2}$
Ubound	$0.500\cdot10^{-2}$	$0.500\cdot10^{-2}$

Table 2: Performance figures for the Single-hop network

7. APPENDIX: GENERATION OF UNIFORM DISTRIBUTIONS

Suppose we are given a random sequence of fair coin flips and we want to generate a random distribution with probabilities $1/N$. We need to define a mapping from coin flips to N events whose probabilities are each $1/N$. If there exists a k such that $N = 2^k$, the required mapping is just to take the first k flips in the sequence and to assign to any N events with any order. For $N \neq 2^k$ there exists an infinite number of such mappings [7] which generates uniform probabilities in about $\log_2 N$ expected number of coin flips. The basic idea is mapping the coin flips to [0,1] interval as considering the flips as the dyadic expansion of a number in [0,1]. An example for $N = 3$ is given in Figure 7.

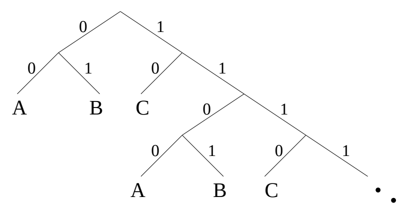

Figure 3: Events A,B,C has probability 1/3 each

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