# Estimation Over Deterministic Multiaccess Channels

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**Abstract**

We study the problem of communicating sensor readings over a Gaussian multiaccess (MAC) channel. We focus on the scenario that each sensor observes a single random variable, and transmits it using certain signaling in a shared channel. The objective is the design of channel waveforms (*i.e.,* signal constellation) to facilitate the estimation of field parameters from the channel output. We propose a new approach named *Histogram-Delivering Multiple Access* (HDMA). In case of symmetric channel gains, it is shown that the HDMA is asymptotically optimal in the limit of large number of sensors. In particular, we show that the HDMA together with a variant of the maximum-likelihood estimator achieves the Cramer-Rao lower bound asymptotically. We then compare the performance of HDMA with other approaches that allocate orthogonal channels to sensors such as TDMA.

# 1 Introduction

## 1.1 Context and Problem Setup

Main functions of wireless sensor networks include sensing of a physical phenomena, and the delivery of the sensed data to a control center. The control center aims to estimate the parameters related to the physical phenomena reliably. Since sensor data are correlated, the efficiency is improved by processing the data locally, and then delivering a compressed

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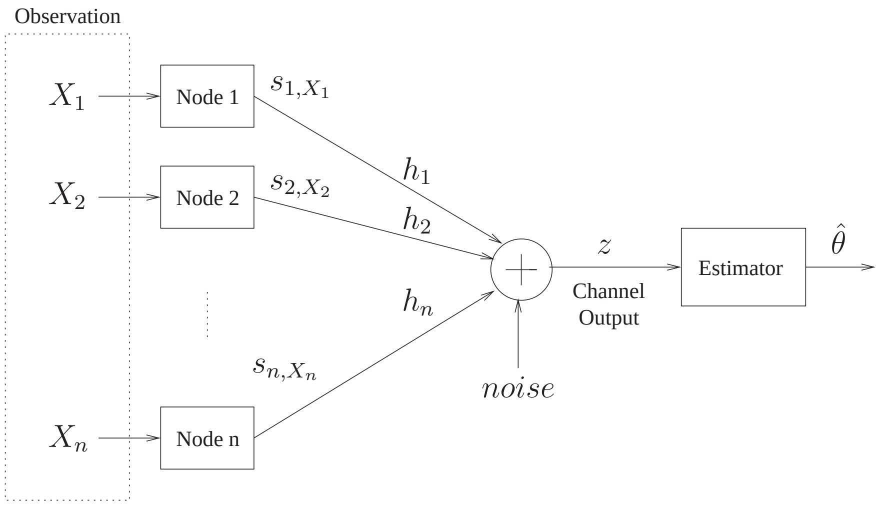


Figure 1: Estimation over MAC setup.

version. Data aggregation can be performed by cluster-heads in a hierarchical sensor network, or by mobile access points [1].

In this work we focus on the multiaccess part of sensor communication. How should the multiaccess (MAC) be designed such that the sensor data is gathered by a cluster-head or an access point most efficiently? The conventional approach mandates the data to be packetized, and then get transmitted according to a MAC protocol. This approach, however, doesn't consider the fact that the sensor data are correlated and that the ultimate objective is the estimation of the field. In this paper we show that significant gains can be realized in estimation quality and in system resource consumption if the physical layer and the MAC are designed jointly for the purpose of estimation.

We consider the case that a group of $n$ sensors observe independent and identically distributed (i.i.d.) data $X_1, \cdots, X_n$ with pdf $p_\theta$ (Fig. 1). The pdfs belong to a family $\{p_\theta : \theta \in \Theta\}$, where $\Theta \subset \mathbb{R}$ is the parameter space, and the objective is to estimate the parameter $\theta$. Each sensor transmits a waveform $s_{i,X_i}$ which depends on the node index $i$ and the observation $X_i$ (energy constraint $||s_{i,X_i}||^2 \leq E$ must be satisfied). The transmitted signals are received through a Gaussian multiaccess channel. The receiver produces an estimate $\hat{\theta}$ of the parameter after reception. The objective is to design the channel waveforms and the estimator such that the mean squared error (MSE) $\mathbb{E}\{(\hat{\theta}-\theta)^2\}$ is minimized. For convenience, it assumed that each $X_i \in \{1, \cdots, k\}$ is discrete[1] with the pdf $p_\theta = (p_\theta(1), \cdots, p_\theta(k))$.

First, consider the ideal scenario that the receiver has access to all $X_i$'s directly. In this case the fundamental limits of estimation are determined by the *Cramer-Rao bound* (CRB) [2]. That is, under regularity conditions on $\{p_\theta : \theta \in \Theta\}$, the MSE of any unbiased

[1] $X_i$ should be viewed as *quantized* data. In this paper, we do not deal with continuous variables, or how the quantization is done.

estimator $\hat{\theta}$ satisfies

$$\mathbb{E}\{(\hat{\theta}-\theta)^2\} \geq \frac{1}{nI(\theta)}, \tag{1}$$

where $I(\theta) = \sum_{i=1}^{k} \frac{(dp_\theta(i)/d\theta)^2}{p_\theta(i)}$ is the *Fisher information*[2] in observation $X_i$. The CRB is not always achievable for finite $n$, but there is a class of estimators, including the Maximum Likelihood (ML) estimator, achieving the CRB asymptotically, *i.e.,*

$$\sqrt{n}(\hat{\theta}-\theta) \xrightarrow{\text{d}} \mathcal{N}(0, \frac{1}{I(\theta)})$$

as $n \to \infty$[3]; such estimators are called *asymptotically efficient.* Of course, the receiver having direct access to $X_i$'s is an idealistic assumption due to the channel noise and the energy constraints, and this performance may not be achievable in the MAC channel.

## 1.2 Histogram-Delivering Multiple Access

In waveform design a crucial observation is that the estimator doesn't need to know the raw data $X_1, \cdots, X_n$ to achieve the best performance. Actually, if the nodes could deliver a *sufficient statistic* with their transmissions, then there is no loss of information. One such sufficient statistic is the *empirical measure*

$$\tilde{p} = \frac{1}{n}(n_1, \cdots, n_k),$$

where $n_j = \sum_{i=1}^{n} 1(X_i = j)$ is the number of nodes that observe $j$[4]. Sufficiency of $\tilde{p}$ motivates us to use the following approach which we call *Histogram-Delivering Multiple Access* (HDMA). let $\delta_1, \cdots, \delta_k$ be $k$ orthonormal waveforms (these can be pulses in time, or tones in frequency). Set $s_{i,X_i} = \sqrt{E}\delta_{X_i}$, *i.e.,* let every node observing $j$ transmit $\delta_j$ with energy $E$. In case of identical channel gains ($h_i = 1$), the received signal contains a noisy version of the histogram of sensor observations. Moreover, the received signal, after appropriate matched filtering, normalized by $\sqrt{E}n$ is equal to the empirical measure plus some noise with power $O(\frac{1}{n^2})$.

In this paper we provide an asymptotic performance analysis of the HDMA. Our main result states that the HDMA together with a variant of the ML estimator is *asymptotically efficient* if the channel gains from different nodes are identical. In other words, the asymptotic performance of HDMA is as if the receiver has access to all $X_i$'s directly. The performance of TDMA (time-divison multiple access) is also analyzed[5], and it is shown that its asymptotic MSE is $\frac{1}{nJ(\theta)}$ for some $J(\theta)$ considerably smaller than $I(\theta)$ at low SNR (the exact value of $J(\theta)$ depends on which modulation is used). Another advantage of HDMA over TDMA is its

[2]It is assumed that $I(\theta) < \infty \Leftrightarrow p_\theta(i) > 0, \forall i$.

[3]Notations $\xrightarrow{\text{d}}$, $\xrightarrow{\text{p}}$ denote the convergence in distribution and convergence in probability, respectively.

[4]$1(\cdot)$ is the *indicator function*, *i.e.,* $1(E) = 1$ if event $E$ happens, and $1(E) = 0$ otherwise

[5]In our channel model, all schemes allocating orthogonal dimensions to different users (*e.g.,* TDMA, FDMA) are mathematically identical and have the same performance.

bandwidth requirement; the HDMA uses $k$ orthogonal dimensions irrespective of the number of users, whereas the bandwidth requirement of TDMA grows linearly with $n$. Finally, the performances of HDMA and TDMA are compared by simulations for Bernoulli and Poisson distributed data. It is observed that the asymptotic analysis provide accurate performance estimates for most of the considered cases even for finite $n$.

### 1.3 Related Work

Estimation over MAC problem has been previously considered in the context of information theory. Gastpar studied the scaling of distortion with respect to the number of sensors for the case that the sensor observations are noisy versions of a Gaussian source [3]. He showed that transmitting uncoded observations in the Gaussian MAC channel gives the best scaling law. Other relevant work includes source compression for detection or estimation under communication rate constraints (*e.g.,* [4]), and joint source-channel coding for the MAC channel (*e.g.,* [5]). The information theoretic approach considers the asymptote that the amount of source data/channel resources is large. On the other hand, our setup models the situation that very many sensors each with *limited* amount of data and *limited* energy access a common channel.

During the preparation of this work, Liu and Sayeed suggested communicating *types* (*i.e.,* the empirical measure) [6], and independently proposed the HDMA scheme for distributed detection [7]. The literature on distributed detection is vast (*e.g.,* [8]). [9] studied quantizer design for distributed estimation.

The organization of the paper is as follows. Section 2 analyzes the asymptotic performance of HDMA in deterministic channels. Section 3 discusses the asymptotic performance of TDMA. Section 4 gives numerical examples, and checks the validity of the asymptotic theory for finite $n$. Section 5 concludes the paper.

## 2 Asymptotic Performance of HDMA

Consider the MAC channel with $n$ nodes shown in Fig. 1. Every node observes a random variable $X_i$ i.i.d. across users with pdf $p_\theta = (p_\theta(1), \cdots, p_\theta(k))$, where $\theta \in \Theta$, $\Theta \subset \mathbb{R}$ is the parameter to be estimated. Node $i$ has an encoder which maps observation $j$ to a discrete-time waveform $s_{i,j} \in \mathbb{C}^\infty$ such that $||s_{i,j}||^2 \leq E$. The received signal at the access point is

$$z = \sum_{i=1}^{n} h_i s_{i,X_i} + v, \tag{2}$$

where each element of $v = (v_1, v_2, \cdots)$ is i.i.d. $\mathcal{CN}(0, \sigma^2)$. An estimate $\hat{\theta}(z)$ of $\theta$ is produced upon observing $z$.

Without loss of generality, assume that $\delta_1, \cdots, \delta_k$ are unit-pulses in time at time instants $1, \cdots, k$. In the HDMA scheme nodes use identical encoders $s_{i,j} = \sqrt{E}\delta_j,\ \forall i, j$. We use the notation $p$ for $p_{\theta_0}$, and $(p_1, \cdots, p_k)$ for $(p_{\theta_0}(1), \cdots, p_{\theta_0}(k))$, where $\theta_0$ is the parameter the data comes from. All vectors, unless transposed, must be understood as column vectors. $(\cdot)'$ denotes derivative with respect to $\theta$. $\mathfrak{Re}(\cdot)$, $\mathfrak{Im}(\cdot)$ denote the real and imaginary parts of a complex number, respectively.

## 2.1 Identical Channels ($h_i = 1$)

When all $h_i$ are unity, the complex part of $z$ only consists of noise, and it does not convey any information. Consider the normalized received signal

$$y := \mathfrak{Re}\left\{\frac{z}{n\sqrt{E}}\right\} = \tilde{p} + \frac{1}{n\sqrt{E}}\mathfrak{Re}\{(v_1, \cdots, v_k)\}.$$

For convenience, let $w = (w_1, \cdots, w_k) := \mathfrak{Re}\{(v_1, \cdots, v_k)\}$. The following lemma gives the asymptotics of $\tilde{p}$ and $y$, which actually turn out to be the same.

**Lemma 1** *$\tilde{p} \xrightarrow{\text{p}} p$ and $\sqrt{n}(\tilde{p} - p) \xrightarrow{\text{d}} \mathcal{N}(0, \Sigma)$ as $n \to \infty$, where*

$$\begin{aligned} \Sigma &= \begin{bmatrix} p_1(1-p_1) & -p_1p_2 & \cdots & -p_1p_k \\ -p_1p_2 & p_2(1-p_2) & \cdots & -p_2p_k \\ \vdots & & & \\ -p_1p_k & & \cdots & p_k(1-p_k) \end{bmatrix} \\ &= Diag(p_1, \cdots, p_k) - pp^T. \end{aligned} \tag{3}$$

*The same convergences hold true for $y$ as well, i.e., $y \xrightarrow{\text{p}} p$ and $\sqrt{n}(y - p) \xrightarrow{\text{d}} \mathcal{N}(0, \Sigma)$ as $n \to \infty$.*

The lemma basically states that $y$ has statistics

$$y \approx \mathcal{N}(p, \frac{1}{n}\Sigma)$$

for large $n$. This property will be instrumental in establishing the asymptotic efficiency of HDMA.

**Proof** It is straightforward to check that the empirical measure has (scaled) multinomial distribution with mean $p$, and covariance $\frac{1}{n}\Sigma$. We have $\tilde{p} \xrightarrow{\text{p}} p$ from the law of large numbers, and $\sqrt{n}(\tilde{p} - p) \xrightarrow{\text{d}} \mathcal{N}(0, \Sigma)$ from the multivariate Central Limit Theorem [10, p. 385]. It follows from Slutky's Theorem [10] that the addition of noise doesn't change this convergence behavior:

$$y = \tilde{p} + \frac{w}{n\sqrt{E}} \xrightarrow{\text{p}} p$$

since $\frac{w}{n\sqrt{E}} \xrightarrow{\text{p}} 0$. And, similarly,

$$\sqrt{n}(y - p) = \sqrt{n}(\tilde{p} - p) + \frac{w}{\sqrt{nE}} \xrightarrow{\text{d}} \mathcal{N}(0, \Sigma).$$

$\blacksquare$

**Lemma 2** *Let $y$ be $\mathcal{N}(p, \frac{1}{n}\Sigma)$ distributed. Then, its pdf is*

$$p(y_1, \cdots, y_k) = \frac{1}{(2\pi/n)^{\frac{k-1}{2}}} exp\left(-\frac{n\sum_{i=1}^{k}\frac{(p_i-y_i)^2}{p_i} + \log\prod_{i=1}^{k} p_i}{2}\right) 1(\sum_{i=1}^{k} y_i = 1) \tag{4}$$

**Proof** Skipped due to space limitations; see [11]. ■

The lemma gives the pdf of $y$ when its distribution is *exactly* equal to $\mathcal{N}(p, \frac{1}{n}\Sigma)$, whereas for our $y$ this is only asymptotically true. In establishing the asymptotic efficiency of HDMA one would like to consider the ML estimator based on $y$. However, the exact likelihood function of $y$ has a complicated form, and the ML based on that doesn't seem tractable. This motivates us to consider the ML based on the asymptotic distribution of $y$, which amounts to maximizing $p(y_1, \cdots, y_k)$ in (4) with respect $p$. However, this is the same as minimizing the exponent. For large $n$, the second term $\log\prod_{i=1}^{k} p_i$ has a negligible effect on the minimization compared to the first one. Therefore, we propose the estimator $\hat{\theta}$ which minimizes

$$M(\theta) := \sum_{i=1}^{k} \frac{(p_\theta(i) - y_i)^2}{p_\theta(i)} \tag{5}$$

with respect to $\theta \in \Theta$. This can be viewed as an asymptotic version of the ML estimator, and as one would expect from the ML, it is asymptotically efficient.

**Theorem 1** *Consider a network with the HDMA scheme and the estimator $\hat{\theta}$ that minimizes $M(\theta)$ with respect to $\theta \in \Theta$. If the family $\{p_\theta : \theta \in \Theta\}$ satisfies certain regularity conditions, then the estimator $\hat{\theta}$ is consistent and is asymptotically efficient, i.e., $\hat{\theta} \xrightarrow{\text{p}} \theta_0$ and*

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{\text{d}} \mathcal{N}(0, \frac{1}{I(\theta_0)}) \tag{6}$$

*as $n \to \infty$.*

**Proof** See [11]. ■

**Remark 1** *Theorem 1 holds irrespective of the value of $E$ as long as $E > 0$. However, the magnitude of $E$ determines the speed of convergence of MSE to the lower bound $\frac{1}{nI(\theta_0)}$. As it will be explored later by simulations, the higher the SNR (i.e., the higher the E), the faster the convergence.*[6]

Next, we will heuristically derive the asymptotic efficiency of $\hat{\theta}$. The distribution family $\{p_\theta : \theta \in \Theta\}$ traces a curve in the $k$ dimensional probability simplex (Fig. 2). Since $M(\theta)$

[6] Actually, Theorem 1 depends on $y$ only through its asymptotic statistics (cf. Lemma 1). Since, the asymptotic statistics of $y$ are independent of the distribution of noise $w = (w_1, w_2, \cdots)$, Theorem 1 holds true irrespective of the statistics of noise *i.e.,* the noise need not even be Gaussian.

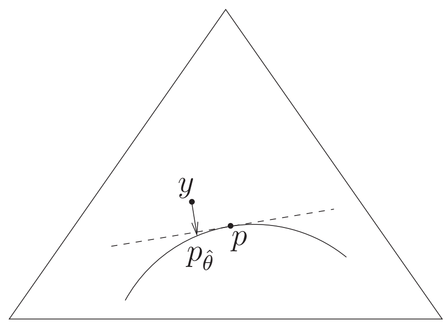


Figure 2: Probability simplex and the parametric family $\{p_\theta : \theta \in \Theta\}$.

in (5) is a form of distance measure, it naturally happens that $p_{\hat\theta} \rightarrow p$ and $e := \hat\theta - \theta_0 \rightarrow 0$ as $y \rightarrow p$. For small perturbations,

$$\theta = \theta_0 + e, \text{ where } |e| << 1,$$

the $p_\theta$ varies almost linearly along the tangent line:

$$p_\theta \approx p + ep' \quad \text{ and } \quad p'_\theta \approx p' + ep'', \tag{7}$$

where $p' := p'_{\theta_0}$, $p'' := p''_{\theta_0}$. This property is very important for us, because $e$ goes to zero as $y \rightarrow p$, and (7) becomes accurate. Instead of minimizing $M(\theta)$, one might as well solve for $\theta$ in $M'(\theta) = 0$.

$$M'(\theta) = \sum_{i=1}^{k} \frac{2p'_\theta(i)(p_\theta(i) - y_i)}{p_\theta(i)} - \frac{p'_\theta(i)(p_\theta(i) - y_i)^2}{p_\theta(i)^2} \approx \sum_{i=1}^{k} \frac{2p'_\theta(i)(p_\theta(i) - y_i)}{p_\theta(i)},$$

where the approximation is because $(p_\theta(i) - y_i)^2$ is much smaller than $(p_\theta(i) - y_i)$ for $p_\theta \approx y$. Substituting (7), one gets

$$M'(\theta) \approx \sum_{i=1}^{k} \frac{2(p'_i + ep''_i)(p_i - y_i + ep'_i)}{p_i + ep'_i} \approx \sum_{i=1}^{k} \frac{2p'_i(p_i - y_i + ep'_i)}{p_i} = 0, \tag{8}$$

where higher order terms with $e^2$ and $e(p_i - y_i)$ are neglected. From the last equation, $e$ is obtained as

$$(\sum_{i=1}^{k} \frac{(p'_i)^2}{p_i})e = \sum_{i=1}^{k} \frac{p'_i(y_i - p_i)}{p_i} \quad \Rightarrow \quad e = \frac{1}{I(\theta_0)} \sum_{i=1}^{k} \frac{p'_i(y_i - p_i)}{p_i}.$$

Since $y - p$ is asymptotically Gaussian, $e$ is asymptotically Gaussian with variance

$$\begin{aligned} \mathbb{E}(e^2) &= \frac{1}{nI^2(\theta_0)} \begin{bmatrix} \frac{p'_1}{p_1} & \cdots & \frac{p'_k}{p_k} \end{bmatrix} \left(\mathrm{Diag}(p) - pp^T\right) \begin{bmatrix} \frac{p'_1}{p_1} & \cdots & \frac{p'_k}{p_k} \end{bmatrix}^T \\ &= \frac{1}{nI(\theta_0)}. \end{aligned}$$

# 3 Asymptotic Performance of TDMA

Packetization is a common practice in communication network design. The conventional layered architecture suggests the data to be mapped into a bit-stream, transmitted using some form of modulation, and then get received without collisions. In this section, the asymptotic performance of such an approach (the TDMA scheme) is considered.

For some $m \in \{1, 2, \cdots\}$, let $s_1, \cdots, s_k$ be vectors in $\mathbb{C}^m$ satisfying $||s_i||^2 \leq 1$ (these are viewed as points in a *constellation*). In TDMA users are allocated non-overlapping time slots of length $m$. Every node uses time shifted versions of the same set of waveforms in its own slot, *i.e.,* node $i$ transmits vector $\sqrt{E} s_{X_i}$ in the $i$'th slot . Notice that the bandwidth requirement of TDMA linearly grows with number of users, whereas the HDMA uses $k$ time units irrespective of the number users. We denote the $i$'th received TDMA packet by

$$z^{(i)} = h_i \sqrt{E} s_{X_i} + v^{(i)}, \tag{9}$$

where $v^{(i)} \sim \mathcal{CN}(0, \sigma^2 I_{m \times m})$.

For simplicity, first let's focus on the case that $h_i = 1$ for all $i$. The random vectors $z^{(1)}, \cdots, z^{(n)}$ are i.i.d. with *Gaussian mixture* density. That is, $z^{(i)}$ is distributed $\mathcal{CN}(\sqrt{E} s_j, \sigma^2 I)$ conditional on $X_i = j$, and the pdf $z^{(i)}$ is

$$f(z^{(i)}) = \sum_{j=1}^{k} p_j \frac{1}{(\pi\sigma^2)^m} \exp\left( -\frac{||z^{(i)} - \sqrt{E} s_j||^2}{\sigma^2} \right). \tag{10}$$

From the CRB, the MSE of any unbiased estimator $\hat{\theta}$ based on $z^{(1)}, \cdots, z^{(n)}$ satisfies

$$\mathbb{E}\{(\hat{\theta} - \theta)\} \geq \frac{1}{nJ(\theta)},$$

where

$$J(\theta) = \mathbb{E}_{z^{(i)}} \left[ \left( \frac{d \log f(z^{(i)})}{d\theta} \right)^2 \right] \tag{11}$$

is the the Fisher information in $z^{(i)}$. Moreover, the ML estimator based on $z^{(1)}, \cdots, z^{(n)}$ achieves the MSE $\frac{1}{nJ(\theta)}$ asymptotically. Hence, the asymptotic performance of TDMA is determined by the Fisher information $J(\theta)$ in each TDMA packet. An important problem is the choice of constellation. One ideally would like to maximize $J(\theta)$ in (11) with respect to $s_1, \cdots, s_k$ to get the best performance. This maximization does not appear tractable in general. For $k = 2$, however, we have the following result (see [11] for a proof).

**Theorem 2** *Let the channel $h_i = 1$ for all $i$. For $k = 2$, the antipodal constellation $s_1 = 1$, $s_2 = -1$ maximizes $J(\theta)$ under the energy constraint $||s_i||^2 \leq 1$.*

For Bernoulli($\theta = 0.8$) distributed data and antipodal constellation, $J(\theta)$ and $I(\theta)$ are plotted in Fig. 3 Notice that the asymptotic MSE of TDMA, $\frac{1}{nJ(\theta)}$, is significantly higher

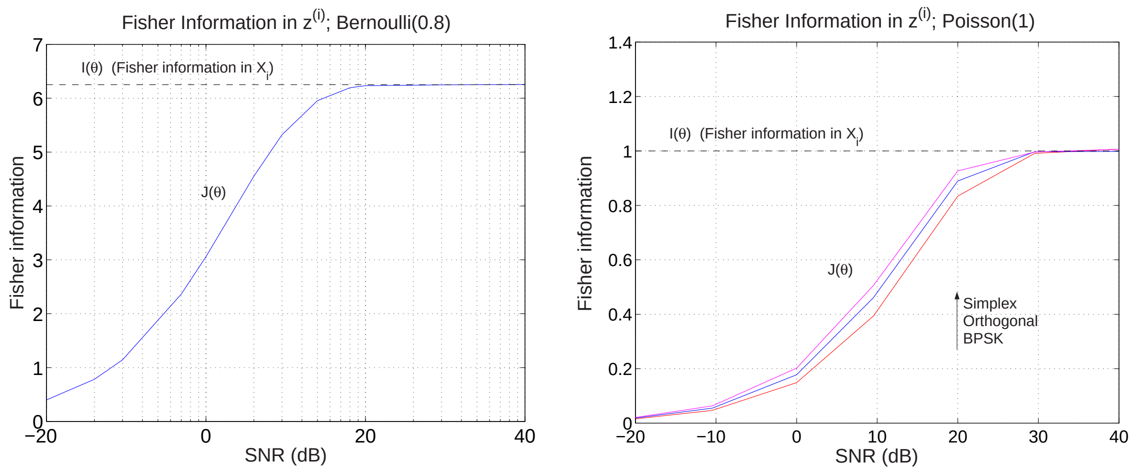


Figure 3: Fisher Information in TDMA packets.

than the asymptotic MSE of HDMA, $\frac{1}{nI(\theta)}$, at low SNR $:= \frac{E}{\sigma^2/2}$. It also seen that in terms of asymptotic MSE the TDMA is as good as HDMA for SNR $\geq$ 20dB.

For $k = 8$, we evaluated $J(\theta)$ by Monte-Carlo integration for three types of constellation: BPSK, orthogonal $s_i$'s and the simplex (orthogonal $s_i$'s translated to have center of mass at the origin, and scaled to satisfy $||s_i|| = 1$). Fig. 3 shows $J(\theta)$ for Poisson (mean$= \theta = 1$) $X_i$ truncated at $k = 8$. The simplex constellation is observed to be marginally better than the other two at all SNR values.

# 4 Numerical Examples

Simulation results Bernoulli Distributed Data ($\theta$ = mean) are given in Fig. 4. The curves are the following:
i) TDMA (antipodal constellation) with the ML estimator based on $z^{(1)}, \cdots, z^{(n)}$.
ii) Direct Access+ML: The hypothetical case that the estimator has access to $X_i$'s directly.
iii) Asymptotic performance (of HDMA): MSE$=\frac{1}{nI(\theta)}$ predicted by our theory.
iv) The HDMA (SNR $= \frac{E}{\sigma^2/2} =$ 0dB) follows the expected asymptotic performance closely, whereas HDMA (SNR$=-10$dB) reaches the asymptotic limit only at large $n$.

# 5 Conclusions

Communication for the purpose of estimation is a central issue in sensor networks. We studied the problem of parameter estimation for the case that a large number of sensors each with limited data and transmission energy access a common channel. We argued that with the use of HDMA significant gains can be realized in estimation quality and in system resource consumption compared to the conventional architecture allocating orthogonal channels to sensors. We characterized the asymptotic performance of HDMA, and observed

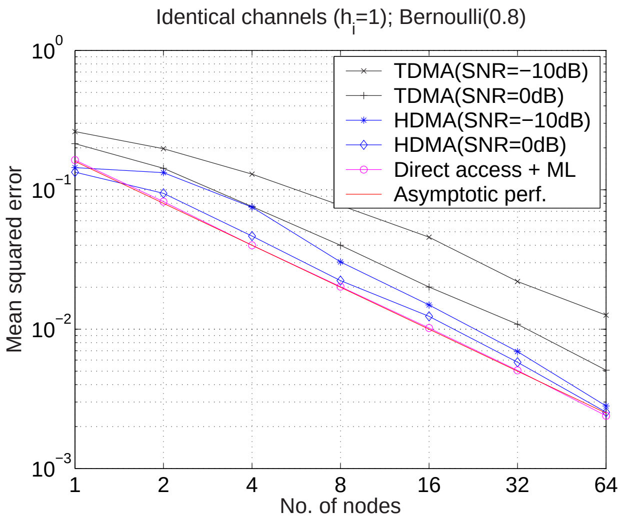


Figure 4: MSE of HDMA, TDMA and direct access.

that this characterization gives reasonably accurate performance estimates even for finite $n$. The results of this paper is extended to fading multiaccess channels in [11].

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