On the Asymptotic Stable Throughput of Opportunistic Random Access

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Abstract — **Asymptotic stable throughput (AST) is the maximum arrival rate a *large* network can support while keeping queues bounded. We characterize the maximum AST of random access with distributed channel state information. The result is applied to CDMA networks with matched filter and linear MMSE (minimum mean square error) receivers. Networks with and without power control are considered. It is shown that power control does not improve AST if the channel support is large enough.**

Keywords: **Medium access, wireless networks, stability, CDMA, distributed channel side information.**

I. Introduction

Medium access (MAC) in cellular wireless networks is traditionally designed based on reservations. That is, first, users contend through a random access channel to gain access into the network. Then, a time slot, a frequency channel or some other resource such as a spreading-code is granted for data communication. Allocated resource belongs to the user as long as he is in the system.

Recently a new paradigm has challenged this architecture. In the so-called *opportunistic communication*, the resources allocated to the users are not fixed, but they are time-varying as a function of the quality of the communication channel. In a fading environment, achievable rate can be improved significantly by exploiting the *good* channel states.

Works on opportunistic communication mainly focuses on centralized implementation. The underlying assumption is that users feedback their channel states to the base station through a control channel. The resource allocation is done by the *base station* considering the channel states of *all* users.

Despite its advantages, centralized MAC places a heavy burden on the control channel whose extensive use reduces the data throughput. To alleviate this problem, several distributed schemes have been proposed [1–4]. Shamai and Telatar [2] considered achievable rates with distributed power control and distributed channel side information. Qin and Berry [3] studied a variant of slotted ALOHA with rate control. They also introduced a distributed *splitting algorithm* that exploits multiuser diversity [4].

In [1], the use of a down-link pilot tone is suggested for distributed implementation. In this architecture the base station continuously broadcasts a pilot-tone and each user estimates his channel via the pilot (the up-link and down-link channel qualities are assumed symmetric). Upon learning his channel state, every user makes a possibly randomized transmission decision. This system is different from the centralized one in that the transmission decisions at different nodes are done *independently* based on the *individual* channel states, not the joint one.

Adireddy and Tong considered a buffered network and studied network stability [1]. They obtained the *stable throughput* in case of symmetric arrivals. They also suggested the use of asymptotic stable throughput (AST) as a performance metric, which is the stable throughput as the number of users go to infinity. This metric is easier to analyze, and it is meaningful for networks with large number of users. They applied their findings to study optimal transmission control in CDMA networks with linear receivers. Later, [5, 6] investigated several aspects of AST with applications to sensor networks.

In this paper we consider the setup in [1], and solve an open problem. Namely, we show that the asymptotic stable throughput shown to be achievable in [1] is actually *optimal*. This result is shown under certain regularity conditions on the reception channel, which are satisfied by CDMA networks. We then extend the setup to include power control, and characterize the AST with power control.

The system model is introduced in the next section. In Section III we discuss the achievability of AST, introduce the regularity conditions, and prove the optimality. We also show that the regularity conditions are satisfied by CDMA networks when the channel distribution has bounded support. We extend the setup to include randomized power control in Section IV, and conclude in Section V.

II. System Model

In the classic slotted ALOHA protocol [7], nodes transmit their backlogged packets according to a certain probability which doesn't change over time. The scheme in [1] can be viewed as a different version of slotted ALOHA where users *adjust* their transmission probability as a function of their channel state. More specifically, consider an up-link with n users. Suppose that the channel gain between m'th user and the access point is $\gamma_m^{(t)} \in \mathbb{R}_+$ in slot t. The user m knows its channel state $\gamma_m^{(t)}$, and uses transmission probability $s(\gamma_m^{(t)})$ to transmit its packets; the function $s(\cdot)$ is called the *transmission control* scheme. A random transmission power can be chosen similarly as a function of the channel state; this will be considered in Section IV.

Some of the transmitted packets may not be received successfully due to interference among users. The successful receptions are determined according to the channel states $\{\gamma_1, \cdots, \gamma_k\}$ of transmitting users (in general, the channel is

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specified by a conditional probability density function [1]). Although our results are applicable in a wider context, in this paper we will be particularly interested in the so-called SINR (signal to interference plus noise ratio) threshold model. The SINR model is considered as a heuristic for CDMA networks with linear receivers. This model is accurate when the signature sequences are random, and the size of the network and the spreading gain are large [8]. For the matched filter receiver, the transmission from user i is successful if

$$\frac{\gamma_i}{\sigma^2 + \frac{1}{L}\sum_{j\neq i}\gamma_j} \geq \beta, \tag{1}$$

for some threshold $\beta > 0$, spreading gain L, and noise power σ^2 . With the linear MMSE receiver, user i is successful if

$$\frac{\gamma_i}{\sigma^2 + \frac{1}{L}\sum_{j\neq i}\frac{\gamma_j\gamma_i}{\gamma_i+\beta\gamma_j}} \geq \beta. \tag{2}$$

The stable throughput of the system is the following. Assume that the $\gamma_m^{(t)}$ for $m = 1, \cdots, n$ and $t \in \mathbb{N}$ are independent and identically distributed with distribution $F(\gamma)$. The *average* probability of transmission for a backlogged user is

$$p_s = \int_0^\infty s(\gamma)dF(\gamma). \tag{3}$$

The *aposteriori* channel state distribution given user transmits is

$$G(\gamma) := \frac{\int_0^\gamma s(\gamma')dF(\gamma')}{p_s} \tag{4}$$

(in the sequel, we will denote this distribution by G_n when we want to emphasize the dependence on n). The maximum stable arrival rate with transmission control $s(\cdot)$ is

$$\lambda_n = \sum_{k=1}^{n}\binom{n}{k}(1-p_s)^{n-k}p_s^k C_k(G) \tag{5}$$

where $C_k(G)$ is the average number of successfully received packets when there are k transmissions each with received power distribution G.[1] If the total arrival rate to the network is less than λ_n, then all queues are stable; otherwise, the queue lengths go to infinity (see [1] for a proof). This result is true under assumptions of symmetric reception probabilities and symmetric i.i.d. arrivals in time and across users.

The quantity in (5) is the expected number of successful receptions in a network with all backlogged nodes. Every node attempts transmission with average probability p_s, and the number of transmitted packets is distributed Binomial(n, p_s). The interesting part of the result is that what matters for the receiver is *not* the actual channel distribution F, but the aposteriori channel distribution G. The maximum stable throughput of the transmission control s is determined only by p_s and G. The function of transmission control is to change the channel distribution from F to G.

The *asymptotic stable throughput* (AST) is the maximum stable throughput as the number of users goes to infinity. This metric is usually easier to analyze and it determines the performance of networks with large number of users. Consider a sequence of transmission controls $s_n(\cdot)$, $n = 1, 2, \cdots$. The AST achieved by $\{s_n\}$ is defined[2] by

$$\lambda_\infty = \liminf_{n\to\infty} \lambda_n.$$

[1]Until Section IV, we assume that the transmissions are with unit power.

[2]We allow population and channel dependent transmission control. [1] also considers AST under population and/or channel *independent* transmission control.

The maximum AST λ_∞^* is the supremum of AST with respect to $\{s_n\}$.

Our objective in this paper is to show that

$$\lambda_\infty^* = \sup_{x, T\ll F} e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T), \tag{6}$$

where the supremum is with respect to $x > 0$, and all distributions T with are absolutely continuous with respect to F (notation $T \ll F$) [9]. The fact that rates less than (6) are achievable is proved in [1]. In the next section we argue that the AST is upper bounded by (6).

III. Maximum Asymptotic Stable Throughput

A. Achievable Rates

The rate

$$f(x, T) := e^{-x}\sum_{k=1}^{\infty}\frac{x^k}{k!}C_k(T) \tag{7}$$

is achievable for $x > 0$ and $T \ll F$. T is called the *target distribution* [1] for reasons that will become apparent. By the Radon-Nikodym Theorem [9], $T \ll F$ implies that there exists a function $t : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying

$$T(\gamma) = \int_0^\gamma t(\gamma')dF(\gamma'). \tag{8}$$

In literature, t is usually called the density function of T with respect to F (similar to the pdf), and is denoted by $\frac{dT}{dF}$. In general t is not bounded, but to obtain intuition about achievability of $f(x, T)$, let's assume that t is bounded. Consider the sequence of transmission controls

$$s_n(\gamma) = \min\left(\frac{x}{n}t(\gamma), 1\right). \tag{9}$$

Since $t(\cdot)$ is bounded, there exists some n_0 for which $s_n(\gamma) = \frac{x}{n}t(\gamma),\ n \geq n_0$. The average probability of transmission is

$$p_{s_n} = \int_0^\infty \frac{x}{n}t(\gamma)dF(\gamma) = \frac{x}{n},$$

for $n \geq n_0$. Similarly, the aposteriori channel distribution becomes $G_n = T$ for $n \geq n_0$.

The crux of the achievability argument is that the p_{s_n} converges to zero, but np_{s_n} converges to x. This implies that the number of transmissions Binomial(n, p_{s_n}) converges to Poisson(x) in a backlogged network. Previously, we have interpreted (5) as an expectation with respect to Binomial random variables. The difference in (7) is that Poisson replaces Binomial, and G_n is replaced by the target distribution T. See [1] for the proof that the control schemes in (9) achieve $f(x, T)$ for general unbounded t.

B. Regularity Conditions and the Converse

Definition We say that the channel satisfies the *regularity conditions* if the following holds for every sequence of transmission controls $\{s_n\}$:

(A1) $C_k(G_n)$ is uniformly bounded

$$\sup_{k,n} C_k(G_n) < \infty. \tag{10}$$

(A2) $\lim_{k\to\infty} C_k(G_n)$ exists for every n, and the sequences $(C_k(G_n), n \geq 1)$ converge to $(\lim_k C_k(G_n), n \geq 1)$ uniformly as $k\to\infty$

The physical interpretation of (A1) is that the base station has bounded reception capability regardless of the received power distribution —this is typical in practical networks. Assumption (A2) is more technical in nature.

The following theorem asserts that (6) is an upper bound to the maximum AST.

Theorem 1 *If the channel satisfies regularity conditions and λ_∞ is achievable, then $\forall\ \epsilon > 0$, there exists $x > 0$, $T \ll F$ such that*

$$\lambda_\infty - \epsilon \leq f(x, T). \tag{11}$$

Proof See the Appendix.

Next, we will argue that matched filter and linear MMSE satisfies the regularity conditions. The following lemma places bounds on the maximum number of successful transmissions. This, in turn, implies that the condition (A1) is satisfied by the matched filter and the linear MMSE receivers.

Lemma 1 *The maximum number of successful transmissions with matched filter is upper bounded by $\lfloor \frac{1}{\beta} L \rfloor + 1$. Similarly, the maximum number of successful transmissions with linear MMSE receiver is upper bounded by $\lfloor \frac{1+\beta}{\beta} L \rfloor + 1$.*

Proof For some integer $l \leq 0$, let powers $\gamma_l \leq \gamma_{l+1} \leq \cdots \leq \gamma_0 \leq \gamma_1 \leq \cdots \leq \gamma_{k-1}$ yield k successful receptions. (1) can be equivalently written as

$$\frac{L}{\beta} \geq \frac{\sigma^2}{\gamma_i} + \sum_{j \neq i} \frac{\gamma_j}{\gamma_i}.$$

User with power γ_0 is successful, therefore,

$$\begin{aligned} \frac{L}{\beta} &\geq \frac{\sigma^2}{\gamma_0} + \sum_{j \neq 0} \frac{\gamma_j}{\gamma_0} \\ &\geq \sum_{j=1}^{k-1} \frac{\gamma_j}{\gamma_0} \\ &\geq k - 1. \end{aligned} \tag{12}$$

This yields the upper bound for the matched filter.

Consider the same setup with the linear MMSE receiver. Observe that (1) is equivalent to

$$L \geq \frac{\beta L \sigma^2}{\gamma_i} + \sum_{j \neq i} \frac{\beta \gamma_j}{\gamma_i + \beta \gamma_j}.$$

Since user with power γ_0 is successful,

$$\begin{aligned} L &\geq \frac{\beta L \sigma^2}{\gamma_0} + \sum_{j \neq 0} \frac{\beta \gamma_j}{\gamma_0 + \beta \gamma_j} \\ &\geq \sum_{j=1}^{k-1} \frac{\beta \gamma_j}{\gamma_0 + \beta \gamma_j} \\ &\geq \sum_{j=1}^{k-1} \frac{\beta \gamma_0}{\gamma_0 + \beta \gamma_0} \\ &= \frac{\beta}{1+\beta}(k-1), \end{aligned}$$

where the last inequality is because $\frac{\beta \gamma_j}{\gamma_0 + \beta \gamma_j}$ is monotonically increasing as a function of γ_j. Hence, we get the upper bound for the linear MMSE receiver. ■

To satisfy the condition (A2) we need one more assumption: The channel gain γ is within an interval $(\gamma_{\min}, \gamma_{\max})$ with probability one, where $\gamma_{min} > 0$ and $\gamma_{max} < \infty$. Here, $\gamma_{\min}$ can be as small as possible given that it is greater than zero. Similarly, $\gamma_{\max}$ can be as large as possible given that it is not infinite.

The last assumption is naturally satisfied in a practical system, but it is not valid for some common channel models. Recall that F is the distribution of the channel gains. We can write the assumption equivalently as $F(\gamma_{\max}) = 1$ and $F(\gamma_{\min}) = 0$. Some common channel models, such as Rayleigh and Rician, do not satisfy these conditions. Nevertheless, we can always truncate the channel distribution at some appropriately small $\gamma_{\min}$ and at some appropriately large $\gamma_{\max}$, and the truncation hardly makes any difference in a practical setting.

Now, let's observe condition (A2) assuming the received power is within interval an $(\gamma_{\min}, \gamma_{\max})$. Consider the matched filter receiver with k transmissions. Eqn. (1) implies

$$\gamma_i \geq \frac{\beta}{L} \sum_{j \neq i} \gamma_j.$$

For large k, the opposite inequality $\gamma_{\max} < \frac{\beta}{L}(k-1)\gamma_{\min}$ holds. This means that even the maximum possible power $\gamma_{\max}$ can not exceed the interference caused by $k-1$ minimum power interferers, and there can not be any success. Therefore, the expected number of successful receptions $C_k(G_n)$ is zero for large k, regardless of $\{s_n\}$ and n. Hence, $(C_k(G_n), n \geq 1)$ converges to its limit $(\lim_k C_k(G_n) = 0, n \geq 1)$ uniformly as $k\to\infty$.

With the linear MMSE receiver, (2) implies

$$\gamma_i \geq \frac{\beta}{L} \sum_{j \neq i} \frac{\gamma_j \gamma_i}{\gamma_i + \beta \gamma_j}.$$

For large k, $\gamma_{\max} < \frac{\beta}{L}(k-1)\frac{\gamma_{\min}\gamma_{\max}}{\gamma_{\max}+\beta\gamma_{\min}}$ holds, and there is no successful transmissions. Similar to the matched filter, (A2) is satisfied because the the expected number of successful receptions $C_k(G_n)$ is zero for large k.

IV. Effect of Randomized Power Control

In this section we will consider transmission power control besides transmission probability control. We will see that the only effect of power control is to enlarge the range of received power distribution.

We will consider an extension of the setup in Section I. $\gamma_m^{(t)} \in \mathbb{R}_+$ denotes the channel gain between m'th user and the access point in slot t. $\gamma_m^{(t)}$ are distributed according to F i.i.d. in time and across users. Given that a user's channel state is γ, he uses a random transmission power $\rho_\gamma \in \mathbb{R}_+$ distributed according to H_γ (before this section, and also in [1], transmissions were assumed to have unit power); the received power at the access point is $\gamma\rho_\gamma$. The transmission probability is given by $s(\gamma, \rho_\gamma)$.[3] The transmission control s and the power control distributions $\mathcal{H} := \{H_\gamma,\ \gamma \in \mathbb{R}_+\}$ are the design parameters.

In our study of the network with power control, we will have the following assumption.

[3]The case that s is only a function of the channel state is naturally included in this setup. This situation is of practical interest, since one would like to make the transmission decisions independent of ρ_γ for simplicity.

(A3) The nodes are limited in peak power, *i.e.,* there exists $\rho_{min} > 0, \rho_{max} < \infty$ such that the random variable ρ_γ is within the interval (ρ_{min}, ρ_{max}), $\forall \gamma$. The transmit powers have pdf h_γ, $\forall \gamma$.

The assumption that the transmit powers having pdf h_γ may seem rather strange. However, it is practically not a restrictive one. For example, if one would like to use a fixed transmit power as a function of γ, then the distribution may have a step, and the pdf does not exist (pdf, in the traditional sense, can not have an impulse [9]). Nonetheless, a step function can always be approximated by a smooth function, and practically one would expect to have a nearly identical performance for approximations close enough.

The achievability results in [1] applies directly to this network. The transmission probability is given by

$$p_s = \int_0^\infty \int_0^\infty s(\gamma, \rho) dH_\gamma(\rho) dF(\gamma). \tag{13}$$

The *aposteriori* received power distribution at the access point is given by

$$G(\gamma) = \frac{1}{p_s} \int_0^\infty \int_0^{\gamma/\gamma'} s(\gamma', \rho) dH_{\gamma'}(\rho) dF(\gamma'). \tag{14}$$

The expression for the maximum achievable rate with a fixed s and $\mathcal{H}$ is same as the one without power control:

$$\lambda_n = \sum_{k=1}^{n} \binom{n}{k} (1-p_s)^{n-k} p_s^k C_k(G). \tag{15}$$

So far we haven't considered a constraint on the average power. If the nodes are bounded in average transmit power, then they may not be able to use every s and $\mathcal{H}$. We will study the AST both with and without average power constraint. Define the AST of a sequence of controls $\{s_n, \mathcal{H}_n\}$ as $\lambda_\infty = \liminf_{n\to\infty} \lambda_n$. The maximum AST *without* average power constraint is $\lambda_\infty^* = \sup \lambda_\infty$, where the supremum is over *all* $\{s_n, \mathcal{H}_n\}$. We define the maximum AST with average power constraint as the previous supremum over all $\{s_n, \mathcal{H}_n\}$ satisfying the average power constraint

$$\int_0^\infty \int_0^\infty \rho s(\gamma, \rho) dH_\gamma(\rho) dF(\gamma) \le \bar{P},$$

for some specified $\bar{P}$, for all n.

Next we will characterize the maximum AST. Let $F_\mathcal{H}$ be the distribution of $\gamma\rho_\gamma$ when the power control distribution is $\mathcal{H}$. Define

$$\mathcal{F} = \{F_\mathcal{H} : \mathcal{H} = \{H_\gamma, \ \gamma \in \mathbb{R}_+\}\}.$$

As a notation, we say that $T \ll \mathcal{F}$, if there exists an $\mathcal{H}$ such that $T \ll F_\mathcal{H}$.

Theorem 2 *The maximum AST with average power constraint is lower bounded by*

$$\lambda_\infty^* = \sup_{x, T \ll \mathcal{F}} e^{-x} \sum_{k=1}^\infty \frac{x^k}{k!} C_k(T). \tag{16}$$

Under regularity conditions,[4] *the maximum AST without power average constraint is upper bounded by the above λ_∞^*. Hence, the maximum AST both with and without average power constraint is equal to λ_∞^*.*

[4] The definition of reguliarity conditions should be modified to include every sequence of transmission controls $\{s_n\}$ and power controls $\{\mathcal{H}_n\}$

Proof For fixed $\mathcal{H}$, the function of transmission control is to shape the received power distribution $F_\mathcal{H}$ to the aposteriori received power distribution G. With the help of power control, all distributions in $\mathcal{F}$ can be used. Furthermore, by transmission control (*cf.* Sec. A)

$$s_n(\gamma, \rho) = \min\left(\frac{x}{n} t(\gamma\rho), 1\right),$$

which is actually a function of $\gamma\rho$, all target distributions $T \ll F_\mathcal{H}$ can be reached asymptotically. This controller gives the AST $f(x, T)$ (eqn. 7) for any $x > 0$ if s_n and $\mathcal{H}$ satisfy the average power constraint. Next, we will observe that this choice of s_n always satisfies the average power constraint for large n. The basic idea is that transmission probability p_s is upper bounded by x/n, and transmit power is upper bounded by ρ_{max}; therefore, the average power constraint is automatically satisfied for large n. More formally, we have

$$\begin{aligned}
&\int_0^\infty \int_0^\infty \rho \min\left(\frac{x}{n} t(\gamma\rho), 1\right) dH_\gamma(\rho) dF(\gamma) \\
&\qquad \le \int_0^\infty \int_0^\infty \rho_{max} \frac{x}{n} t(\gamma\rho) dH_\gamma(\rho) dF(\gamma) \\
&\qquad \overset{(a)}{=} \rho_{max} \frac{x}{n} \int_0^\infty t(\mu) dF_\mathcal{H}(\mu) \\
&\qquad \overset{(b)}{=} \rho_{max} \frac{x}{n} \int_0^\infty dT(\mu) \\
&\qquad \overset{(c)}{=} \rho_{max} \frac{x}{n},
\end{aligned}$$

where (a) follows from a change of variable $\mu = \gamma\rho$, (b) from the definition of Radon-Nikodym derivative, and (c) is because T is a distribution.

The converse proof saying that one can not do any better than λ_∞^* is same as the converse proof for network without power control. ■

Corollary 1 *Let the channel distribution have a pdf f positive within an interval $(\gamma_{min}, \gamma_{max})$, and zero outside.*
(i) The maximum AST can be achieved using the uniform power distribution. That is,

$$\lambda_\infty^* = \sup_{x, T \ll \tilde{F}} e^{-x} \sum_{k=1}^\infty \frac{x^k}{k!} C_k(T), \tag{17}$$

where $\tilde{F}$ is the distribution of $\gamma\rho$ when ρ is uniform in (ρ_{min}, ρ_{max}), and $\gamma \sim F$.
(ii) More generally, (17) still holds if we replace $\tilde{F}$ with any other $F_\mathcal{H}$ with support equal to $(\rho_{min}\gamma_{\min}, \rho_{max}\gamma_{\max})$.

Proof To prove the corollary, we will show that

$$\{T : T \ll F_{\hat{H}}\} = \{T : T \ll \mathcal{F}\}$$

holds if the distribution $F_{\hat{H}}$ has a pdf positive in $(\rho_{min}\gamma_{\min}, \rho_{max}\gamma_{\max})$. From the definition of $\mathcal{F}$, it follows that $\{T : T \ll F_{\hat{H}}\} \subset \{T : T \ll \mathcal{F}\}$. To see the opposite inclusion, we will consider a power control $\mathcal{H}' = \{H'_\gamma\}$, and show that $F_{\mathcal{H}'} \ll F_\mathcal{H}$ —this implies $T \ll F_{\mathcal{H}'} \Rightarrow T \ll F_\mathcal{H}$. To see $F_{\mathcal{H}'} \ll F_\mathcal{H}$, we will argue that $F_{\mathcal{H}'}$ has a pdf with support included in $(\rho_{min}\gamma_{\min}, \rho_{max}\gamma_{\max})$. The support is obvious, and we will focus on showing the existence of pdf. Observe that

$$\begin{aligned} F_{\mathcal{H}'}(\gamma) &= \Pr\{\gamma'\rho_{\gamma'} \le \gamma\} \\ &= \int_0^\infty H'_{\gamma'}(\gamma/\gamma')dF(\gamma') \\ &= \int_0^\infty H'_{\gamma'}(\gamma/\gamma')f(\gamma')d\gamma' \\ &= \int_0^\infty \left[\int_0^{\gamma/\gamma'} h'_{\gamma'}(\rho)d\rho\right] f(\gamma')d\gamma'. \end{aligned} \tag{18}$$

Here, $F_{\mathcal{H}'}$ has a pdf $f_{\mathcal{H}'}(\gamma) = \int_0^\infty \frac{1}{\gamma'} h_{\gamma'}(\gamma/\gamma')f(\gamma')d\gamma'$ (to see this formally, integrate the given $f_{\mathcal{H}'}(\gamma)$ from 0 to some γ, and apply Fubini's Thm. [9] to interchange the integrals —this can be done because the integrand is non-negative). The corollary follows. ■

A. CDMA with Power Control

Thm. 2 (and its corollary) has some interesting consequences. One of them is that if the channel support is already very large, say $(0,\infty)$, then the maximum AST with and without power control are the same. This is the case for Rayleigh or Rician distributed channels.

Note that the regularity conditions for the CDMA model holds only if the channel is confined within an interval $(\gamma_{min}, \gamma_{max})$, where $\gamma_{min} > 0$ and $\gamma_{max} < \infty$. To apply our theory, we have previously argued that we need to truncate the channel at some appropriate $(\gamma_{min}, \gamma_{max})$. The truncation doesn't incur any loss because every receiver has a sensitivity range, and we can always choose γ_{min} and γ_{max} such that the values outside of $(\gamma_{min}, \gamma_{max})$ is not of practical interest. The affect of power control in truncated channels is to enlarge the support to $(\rho_{min}\gamma_{min}, \rho_{max}\gamma_{max})$. However, this enlargement essentially doesn't bring any improvement since the outside of $(\gamma_{min}, \gamma_{max})$ can not be exploited by the receiver anyways. An important conclusion is that adding transmit power control on top of transmission control does not bring any improvement in terms of maximum AST.

V. Conclusions

The asymptotic stable throughput is a performance metric for networks with large number of users. In this paper we have proved that the asymptotic stable throughput shown to be achievable in [1] is also the maximum. We have obtained this result under certain regularity conditions, and showed that CDMA networks with matched filter and linear MMSE receivers satisfy the regularity conditions. For this we have assumed that the received power distribution has support within an interval strictly inside $[0,\infty)$. Whether this constraint can be relaxed is a theoretically interesting open question. Finally, we have studied the AST in networks with power control, and characterized the maximum AST.

Appendix: Proof of Thm. 1

Let λ_∞ be achievable by $\{s_n\}$,

$$\lambda_\infty = \liminf_{n\to\infty} \sum_{k=1}^n \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_n). \tag{19}$$

In the following we will assume that the above lim inf is the actual limit of the sequence; if this is not the case, then the same arguments can be applied for a subsequence $\{n_i,\ i \ge 1\}$ whose limit equals the above lim inf.

Because of (19) there exists n_o such that $\forall\ n > n_0$

$$\lambda_\infty - \epsilon \le \sum_{k=1}^n \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_n). \tag{20}$$

Define $x_n := np_{s_n}$. Two cases are possible: $\liminf_n x_n < \infty$ and $\liminf_n x_n = \infty$. We will analyze each of them separately.

i) $\liminf_n x_n := x < \infty$. Assume a stronger statement, namely $\lim_n x_n = x$; if this is not the case, one can apply the arguments to follow for a subsequence $\{n_j, j \ge 1\}$. We want to show that

$$\left| \sum_{k=1}^n \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_n) - \sum_{k=1}^\infty e^{-x}\frac{x^k}{k!} C_k(G_n) \right| < \epsilon, \tag{21}$$

for n large enough. For this we need the assumption (A1). Using triangle inequality, we see that the left hand side term in (21) is upper bounded by

$$\sup_{k,n} C_k(G_n) \sum_{k=1}^\infty \left| \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k - e^{-x}\frac{x^k}{k!} \right|. \tag{22}$$

Since $np_{s_n} \to x$,

$$f_{n,k} := \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k \to f_k := e^{-x}\frac{x^k}{k!}, \quad \text{as } n\to\infty$$

But, is it true that $\sum_k |f_{n,k} - f_k| \to 0$? The following lemma, which is a special case of the *Schaffe's Theorem* [9], asserts that this is indeed the case.

Lemma 2 *Let $f_{n,k}$ be a non-negative double sequence. Suppose that $f_{n,k} \to f_k$ as $n\to\infty$ for each k. And, assume that $\sum_k f_{n,k} < \infty$, $\sum_k f_k < \infty$ is satisfied. If $\sum_k f_{n,k} \to \sum_k f_k$ holds, then $\sum_k |f_{n,k} - f_k| \to 0$ as $n\to\infty$.*

Invoking assumption (A1) and using Lemma 2, we see that (22) converges to zero as $n\to\infty$. Therefore, (21) holds. Equations (20) and (21) imply

$$\lambda_\infty - 2\epsilon \le \sum_{k=1}^\infty e^{-x}\frac{x^k}{k!} C_k(G_n) \tag{23}$$

for n large enough.

ii) $\liminf_n x_n = \infty$, *i.e.*, $\lim_n x_n = \infty$. Our claim is that $\lim_n \lim_k C_k(G_n)$ exists and is equal to

$$\lambda_\infty = \lim_{n\to\infty} \sum_{k=1}^n \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_n).$$

For this purpose, we will show that

$$\left[\sum_{k=1}^n \binom{n}{k} (1-p_{s_n})^{n-k} p_{s_n}^k C_k(G_n) - \lim_k C_k(G_n) \right] \to 0 \tag{24}$$

as $n\to\infty$. For a moment assume (24), and see why it is enough. Observe that,

$$\sup_x \sum_k e^{-x}\frac{x^k}{k!} C_k(G_n) \ge \lim_k C_k(G_n). \tag{25}$$

This implies,

$$\begin{aligned}\sup_{n,x}\sum_k e^{-x}\frac{x^k}{k!}C_k(G_n) &\geq \lim_n \lim_k C_k(G_n)\\ &= \lambda_\infty,\end{aligned}$$

as required.

To prove (24), we will use (A2). (24) is equivalent to

$$\sum_{k=1}^{n}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k\left[C_k(G_n)-\lim_k C_k(G_n)\right]\rightarrow 0 \quad (26)$$

as $n\rightarrow\infty$. Assumption (A2) implies that there exists k_0 (independent of n!) such that for all $k > k_0$ and for all n, $C_k(G_n) - \lim_k C_k(G_n) < \epsilon$ holds. Therefore,

$$\begin{aligned}&\sum_{k=1}^{n}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k\left[C_k(G_n)-\lim_k C_k(G_n)\right]\\ &\leq 2\left[\sup_{k',n'}C'_k(G_{n'})\right]\sum_{k=1}^{k_0}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k\\ &\qquad\qquad +\epsilon\sum_{k=k_0+1}^{n}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k\\ &\leq 2\left[\sup_{k',n'}C'_k(G_{n'})\right]\sum_{k=1}^{k_0}\binom{n}{k}(1-p_{s_n})^{n-k}p_{s_n}^k+\epsilon \quad (27)\\ &\rightarrow \epsilon, \quad (28)\end{aligned}$$

as $n\rightarrow\infty$. The left term in (27) goes to zero as $n\rightarrow\infty$ since $np_{s_n}\rightarrow\infty$. Since ϵ can be chosen arbitrarily small, (24) follows.

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