

# Maximum Asymptotic Stable Throughput of Opportunistic Slotted ALOHA and Applications to CDMA Networks

Gökhan Mergen and Lang Tong

**Abstract**—In this paper we study the maximum asymptotic stable throughput of an opportunistic slotted ALOHA protocol. We provide a characterization of the maximum stable throughput as the number of users in the system goes to infinity. We then apply our findings to CDMA networks with the Signal-to-Interference-Ratio (SIR) threshold model. It is shown that the slotted ALOHA protocol with the power/transmission control rule that equalizes the reception powers achieves  $1 - O(\log N/\sqrt{N})$  channel utilization, which is defined as the throughput divided by the optimal throughput  $N$  achieved by scheduling. This implies that the slotted ALOHA is asymptotically optimal in the sense that its channel utilization converges to 1 as the spreading gain goes to infinity.

**Index Terms**—Medium access, wireless networks, channel-aware ALOHA, stability, CDMA, distributed channel side information.

## I. INTRODUCTION

OPPORTUNISTIC medium access aims to improve the system throughput by exploiting the *multiuser diversity* inherent in wireless systems [1]. Recently, several schemes have been proposed to utilize the multiuser diversity in a distributed way [2]–[5]. Shamai and Telatar [2] considered achievable rates with distributed power control and distributed channel side information. Qin and Berry [3] considered a *channel aware* version of slotted ALOHA with rate control. They also introduced a distributed *splitting algorithm* that exploits multiuser diversity [4].

Adireddy and Tong [5] studied a variant of the slotted ALOHA protocol in which users make randomized transmission decisions based on their individual channel states. They obtained the *maximum stable throughput* in case of symmetric arrivals. For the finite population model, the maximum stable throughput is difficult to optimize. To circumvent this, they suggested the use of asymptotic stable throughput (AST) as a performance metric, which is the limiting stable throughput as the number of users go to infinity. The AST is easier to analyze, and it gives insights into the design and the performance of networks with a large number of users. Later,

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[6], [7] investigated several aspects of AST with applications to sensor networks.

In this paper we solve an open issue from [5]. Namely, we show that the asymptotic stable throughput shown to be achievable in [5] is actually the *optimal* within the class of slotted ALOHA protocols. This result is proved under a certain condition on the reception channel. We then apply our findings to CDMA networks with the SIR threshold model, and analyze the optimal transmission control rules in case of large spreading gain. When the transmission control is designed to equalize the reception powers, it is shown that the slotted ALOHA protocol achieves  $1 - O(\log N/\sqrt{N})$  channel utilization, which is defined as the throughput divided by the optimal throughput  $N$  achieved by scheduling. This implies that the slotted ALOHA is asymptotically optimal in the sense that its channel utilization converges to 1 as the spreading gain goes to infinity.

In the next section, the system model is introduced and the related results from [5] are reviewed. In Section III, we deal with the maximum AST in CDMA networks. Section IV concludes the paper.

## II. SYSTEM MODEL AND KNOWN RESULTS

In the classic slotted ALOHA protocol [8], nodes transmit their backlogged packets according to a certain probability which doesn't change over time. The scheme in [5] can be viewed as a version of slotted ALOHA where users *adjust* their transmission probability as a function of their channel state. More specifically, consider an up-link with  $n$  users. Suppose that the channel gain between  $m$ 'th user and the access point is  $\gamma_m^{(t)} \in \mathbb{R}_+$  in slot  $t$ . The user  $m$  knows its channel state  $\gamma_m^{(t)}$ , and uses transmission probability  $s(\gamma_m^{(t)})$  to transmit its packets; the function  $s(\cdot)$  is called the *transmission control* scheme.

Some of the transmitted packets may not be received successfully due to interference among users. The successful receptions are determined according to the channel states  $\{\gamma_1, \dots, \gamma_k\}$  of transmitting users. In a CDMA network the transmission from user  $i$  becomes successful if

$$\text{SIR}_i := \frac{\gamma_i}{\sigma^2 + \frac{1}{L} \sum_{j \neq i} \gamma_j} \geq \beta, \quad (1)$$

where  $\text{SIR}_i$  is SIR of user  $i$ ,  $\beta > 0$  is some threshold,  $L$  is the spreading gain, and  $\sigma^2$  is the noise power spectral density.<sup>1</sup> The SIR threshold model is a widely considered heuristic for CDMA with the matched filter receiver.

<sup>1</sup>The reader is cautioned that the  $\sigma^2$  defined above is the *noise power spectral density* not the noise power, which is  $L$  times that. This way of defining SIR is consistent with some earlier work *e.g.*, [9].

We use the notation

$$C_k(F) := k \Pr\{\text{SIR}_1 \geq \beta; \gamma_1, \dots, \gamma_k \sim F\}$$

to denote the expected number of successful receptions conditioned on the event that there are  $k$  transmitters and that the channel states are independent and identically distributed (i.i.d.) with distribution  $F$ .

We consider the so-called finite user, infinite buffer model (e.g., [10]). Namely, it is assumed that every user in the network has an infinite buffer to hold his/her packets. The packets arrive randomly to each user according to a stochastic process with mean arrival rate  $\mu/n$  ( $\mu$  = the total arrival rate). A symmetric system is considered, i.e., the number of packet arrivals is i.i.d. across users and across time. Similarly, the channel gains  $\gamma_m^{(t)}$  are i.i.d. with distribution  $F$  for  $m = 1, \dots, n$  and  $t = 1, 2, \dots$ .

#### A. Maximum Stable Throughput

Under a transmission controller  $s(\cdot)$ , the average probability of transmission for a backlogged user is given by

$$p_s = \int_0^\infty s(\gamma) dF(\gamma). \quad (2)$$

The distribution of the channel state conditioned on the event that the user transmits is given by

$$G(\gamma) = \frac{\int_0^\gamma s(\gamma') dF(\gamma')}{p_s}; \quad (3)$$

this is called the *aposteriori* channel state distribution. The following theorem, which is proved in [5], characterizes the maximum stable throughput of the system.

*Theorem 1:* Consider the slotted ALOHA up-link with transmission control  $s(\cdot)$ . If the total arrival rate  $\mu$  is less than

$$\lambda := \sum_{k=1}^n \binom{n}{k} (1 - p_s)^{n-k} p_s^k C_k(G), \quad (4)$$

then the network buffers stay stable (i.e., the queue lengths converge to a proper stationary distribution as the time goes to infinity). However, if  $\mu > \lambda$ , then the buffers become unstable (i.e., they go unbounded) as the time goes to infinity.

The  $\lambda$  is called the *maximum stable throughput* with transmission control  $s(\cdot)$ . This quantity can be viewed as the expected number of successful receptions in a network with all backlogged nodes. In such a network, every node attempts transmission with average probability  $p_s$ , and the number of transmitted packets is distributed Binomial( $n, p_s$ ). Interestingly, what matters for the receiver is *not* the actual channel distribution  $F$ , but the *aposteriori* channel distribution  $G$ ; the maximum stable throughput is determined only by  $p_s$  and  $G$ . The main role of transmission control is to shape the channel distribution from  $F$  to  $G$ .

#### B. A Lower Bound on Maximum AST

Ideally, one would like to maximize  $\lambda$  with respect to the transmission controller  $s(\cdot)$ . This maximization, however, is intractable in general. For this reason we will consider the limiting stable throughput under a sequence of transmission control rules  $\{s_n : n = 1, 2, \dots\}$  as the number of users  $n$  go

to infinity. This metric is usually easier to analyze, and it gives intuition about the design and the performance of networks with large number of users.

Consider a sequence of transmission controls  $s_n(\cdot)$ ,  $n = 1, 2, \dots$ . We use the notations  $G_n$  and  $\lambda_n$  to emphasize the dependence on  $n$ . The *asymptotic stable throughput* (AST) achieved by  $\{s_n\}$  is defined to be

$$\lambda_\infty = \liminf_{n \rightarrow \infty} \lambda_n.$$

The *maximum AST*  $\lambda_\infty^*$  is the supremum of AST with respect to all transmission control sequences  $\{s_n\}$ .

The following result is from [5].

*Theorem 2:* The maximum AST satisfies

$$\lambda_\infty^* \geq \sup_{x, T \ll F} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T), \quad (5)$$

where the supremum is with respect to  $x > 0$ , and all distributions  $T$  which are absolutely continuous<sup>2</sup> with respect to  $F$  (notation  $T \ll F$ ) [11].

Theorem 2 provides a lower bound on the maximum AST. To prove the theorem, it is shown in [5] that the rate

$$f(x, T) := e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T) \quad (6)$$

is achievable for all  $x > 0$  and  $T \ll F$ . The  $T$  is called the *target distribution* for reasons that will become apparent. The  $x$  is the mean of a Poisson random variable that will be obtained as the limit of a sequence of binomial random variables. In the following, we provide a sequence of transmission controls that achieve  $f(x, T)$ , and discuss the intuition behind the achievability result.

By the Radon-Nikodym Theorem [11],  $T \ll F$  implies that there exists a function  $t : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  satisfying

$$T(\gamma) = \int_0^\gamma t(\gamma') dF(\gamma'), \quad (7)$$

where  $t$  is the so-called the density function of  $T$  with respect to  $F$  (similar to the pdf), and is denoted by  $\frac{dT}{dF}$ . In general  $t$  need not be bounded, but for simplicity, let's assume that it is. Consider the sequence of transmission controls

$$s_n(\gamma) = \min\left(\frac{x}{n} t(\gamma), 1\right). \quad (8)$$

Since  $t(\cdot)$  is bounded, there exists some  $n_0$  for which

$$s_n(\cdot) = \frac{x}{n} t(\cdot), \quad \forall n \geq n_0.$$

The average probability of transmission is

$$p_{s_n} = \int_0^\infty \frac{x}{n} t(\gamma) dF(\gamma) = \frac{x}{n},$$

for  $n \geq n_0$ . Similarly, the *aposteriori* channel distribution becomes  $G_n = T$  for  $n \geq n_0$ . In other words, the *aposteriori*

<sup>2</sup>The following examples illustrate the notion of absolute continuity. When  $F$  is a distribution with pdf covering the half real line  $[0, \infty)$  (e.g., the exponential distribution), then the class  $T \ll F$  covers all distributions with pdf over  $[0, \infty)$ . However, if  $F$  is discrete over  $\{0, 1, 2, \dots\}$  (e.g., the geometric distribution), then the class  $T \ll F$  covers all distributions with a probability mass function over  $\{0, 1, \dots\}$ .

channel distribution *converges* to the the target distribution as  $n \rightarrow \infty$ .

Previously, we interpreted (4) as an expectation with respect to a Binomial random variable. The crux of the achievability argument is that the  $p_{s_n}$  converges to zero, but  $np_{s_n}$  converges to  $x$ . This implies that the Binomial( $n, p_{s_n}$ ) random variable converges to Poisson( $x$ ). The difference between (4) and (6) is that Poisson replaces Binomial, and  $G_n$  is replaced by the target distribution  $T$ . In [5] it is shown that for general unbounded  $t(\cdot)$  the control schemes in (8) have the asymptotic stable throughput  $f(x, T)$ .

### III. MAXIMUM ASYMPTOTIC STABLE THROUGHPUT

In this section, we show that the lower bound to the maximum AST given in the previous section is actually tight. This result is shown under the following assumption:

(C1) The channel distribution  $F$  has bounded support contained within some interval  $(\gamma_{\min}, \gamma_{\max})$ , *i.e.*,  $F(\gamma_{\min}) = 0$ ,  $F(\gamma_{\max}) = 1$ , where  $\gamma_{\min} > 0$ ,  $\gamma_{\max} < \infty$ .

This condition is a technical requirement for our proof to work. Note that  $\gamma_{\min}$  can be arbitrarily small as long as it is positive. Similarly,  $\gamma_{\max}$  can be arbitrarily large as long as it is not infinite. Some common channel models such as Rayleigh or Rician do not satisfy (C1). However, the channel distribution can always be truncated to some  $(\gamma_{\min}, \gamma_{\max})$  to get (C1). Intuitively, we expect the performance of the network with the truncated channels to be same as that of the original one as long as  $\gamma_{\min}$  is chosen small, and  $\gamma_{\max}$  is chosen large enough.

*Theorem 3:* Under condition (C1), the maximum AST in a CDMA network satisfies

$$\lambda_{\infty}^* \leq \sup_{x, T \ll F} e^{-x} \sum_{k=1}^{\infty} \frac{x^k}{k!} C_k(T). \quad (9)$$

*Proof:* See Appendix A.  $\square$

*Remark 1:* Theorem 3 combined with Theorem 2 implies that the maximum AST is *exactly* equal to the right hand side of (9). Another corollary of these two theorems is that the control rules in (8) provide the maximum AST.

#### A. Maximum AST with High Reception Capability

Theorem 3 gives a characterization of the maximum AST in terms of a maximization over  $T \ll F$  and  $x > 0$ . In the following we shall consider this maximization after providing a bound on the AST.

*Lemma 1:* The maximum number of successful transmissions in CDMA is upper bounded by

$$N := \left\lfloor L \left( \frac{1}{\beta} - \frac{\sigma^2}{\gamma_{\max}} \right) \right\rfloor + 1. \quad (10)$$

*Proof:* See Appendix B.  $\square$

We call the  $N$  as the *reception capability* of the channel. The motivation behind this concept is that  $N$  provides an upper bound on the throughput of any medium access scheme (whether it is *scheduling* or random access). Moreover, throughput  $N$  can be *achieved* if the base station could fix the number of transmitters to  $N$  each with *fixed* received power

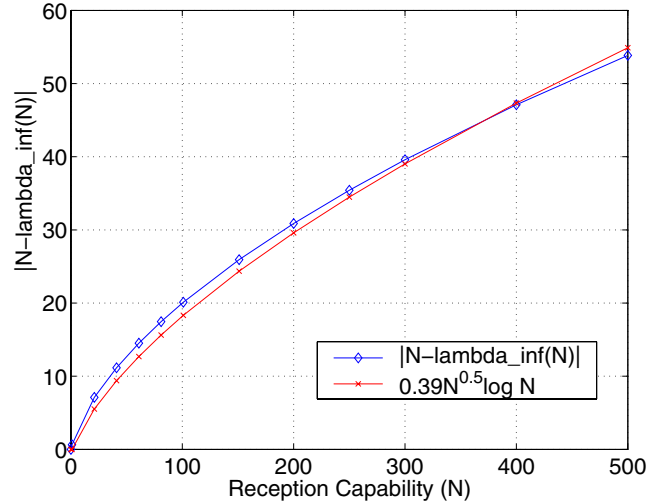


Fig. 1.  $|N - \tilde{\lambda}_{\infty}(N)|$  vs.  $N$ .

$\gamma_{\max}$  (this is easily seen by checking the SIR threshold). Therefore, throughput  $N$  can be viewed as the *scheduling* performance of the system. Our system has random access, and expecting the best performance is too optimistic in general. However, we will see that there exist a target distribution and  $x > 0$  which has AST  $N - O(\sqrt{N} \log N)$ .

The target distribution we will consider is  $T = \delta_{\gamma_{\max}}$ , *i.e.*, a single impulse at power  $\gamma_{\max}$ . In practice, there are two ways to get to this received power distribution. First, the delta distribution can be approximated by a sharp rectangle, *i.e.*,  $T$  can be chosen uniform in  $(\gamma_{\max} - \epsilon, \gamma_{\max})$  for some  $\epsilon > 0$ . For small enough  $\epsilon$ , we expect this system to perform very closely to the system with  $T = \delta_{\gamma_{\max}}$ . A second, more traditional method to get constant received power is via power control. If the nodes can *invert* the channel by controlling their transmit powers, it may be possible to fix the received power to  $\gamma_{\max}$ .

Next, we will compute the AST with the received power distribution  $T = \delta_{\gamma_{\max}}$ . With this choice of target distribution, we have a specific structure for the  $C_k(T)$ :

$$C_k(T) = \begin{cases} k, & k \leq N \\ 0, & \text{oth.} \end{cases} \quad (11)$$

This particular  $C_k(\cdot)$  comes from the fact that when all users are received at the same power, then either all transmissions succeed (if  $k \leq N$ ), or all transmissions fail. The  $N$  determines the reception capability of the channel and can be computed as the maximum  $N$  that satisfies the SIR criterion:

$$\frac{\gamma_{\max}}{\sigma^2 + \frac{N-1}{L} \gamma_{\max}} \geq \beta.$$

Another design parameter is the *mean* of the Poisson( $x$ ) random variable. When the reception capability is high, the idea is to choose the mean such that the probability of transmission failure (*i.e.*,  $\Pr(\text{Poisson}(x) > N)$ ) is small. Yet,  $x$  should be chosen high enough so that the throughput doesn't decrease just because the users do not transmit. It appears that the optimal tradeoff is to choose  $x \approx N - \sqrt{N} \log N$ . This particular value of  $x$  avoids the transmission failures with high probability (failure happens when the number of transmitters exceeds  $N$ ). Moreover, this  $x$  is high enough so

that a majority of nodes transmit, hence the channel resources are used efficiently. In the proof of the following theorem it is shown that the AST with this choice of  $x$  exceeds  $N - 2\sqrt{N} \log N$ .

*Theorem 4:* Let

$$\tilde{\lambda}_\infty(N) = \sup_{x>0} e^{-x} \sum_{k=1}^N \frac{x^k}{k!} k$$

be the asymptotic stable throughput achievable with the delta target distribution. Then,

$$|N - \tilde{\lambda}_\infty(N)| \leq 2\sqrt{N} \log N + 2(e^{1/2+O(1/\sqrt{N})} + 1). \quad (12)$$

*Remark 2:* As mentioned previously, the throughput  $N$  can be achieved if the base station could schedule users each with received power  $\gamma_{\max}$ . The difference  $|N - \tilde{\lambda}_\infty(N)|$  can be viewed as the *loss* due to using random access instead of scheduling. The regime  $N$  is high should be viewed as the regime where the *spreading gain*  $L$  is high. Theorem 4 implies that the *channel utilization*  $\tilde{\lambda}_\infty/N$  with the slotted ALOHA converges to 100% as the spreading gain goes to infinity. In this respect, the slotted ALOHA protocol with the delta target distribution is asymptotically optimal.

*Remark 3:* Theorem 4 gives a somewhat conservative estimate of the difference  $|N - \tilde{\lambda}_\infty(N)|$ . Fig. 1 shows  $|N - \tilde{\lambda}_\infty(N)|$  computed numerically. It appears that the actual difference is  $\approx 0.39\sqrt{N} \log N$ . The figure also approves that the channel utilization loss due to random access decreases as the reception capability grows, *i.e.*, the loss is approximately 20/100 = 20% for  $N = 100$ , whereas for  $N = 500$  it is about 55/500 = 11%.

*Proof:* Define  $\alpha_N := \lceil \sqrt{N} \log N \rceil$ . Let  $X_N$  be a Poisson random variable with mean  $x_N := (N - \alpha_N)$ . We will be interested in the probability

$$p := \Pr\{X_N > N\} + \Pr\{X_N < N - 2\alpha_N\},$$

and show that  $p \rightarrow 0$ . We have the following lower bound

$$\begin{aligned} \tilde{\lambda}_\infty(N) &\geq \sum_{k=N-2\alpha_N}^N e^{-x_N} \frac{x_N^k}{k!} k \\ &\geq (N - 2\alpha_N) \underbrace{\sum_{k=N-2\alpha_N}^N e^{-x_N} \frac{x_N^k}{k!}}_{1-p}. \end{aligned}$$

Therefore,

$$|N - \tilde{\lambda}_\infty(N)| \leq pN + 2(1-p)\alpha_N \leq pN + 2\alpha_N. \quad (13)$$

Next, we will upper bound  $p$  by using the Chernoff inequality

$$\Pr\{X_N > N\} \leq \mathbb{E}[e^{sX_N}]e^{-sN}, \quad \forall s > 0. \quad (14)$$

Chernoff inequality is well known to give tight upper bounds in terms of order; examples can be found in several areas such as error exponents in hypothesis testing, channel coding and large deviations theory. Hence, we expect the following derivation to be order-wise tight. Substituting the moment

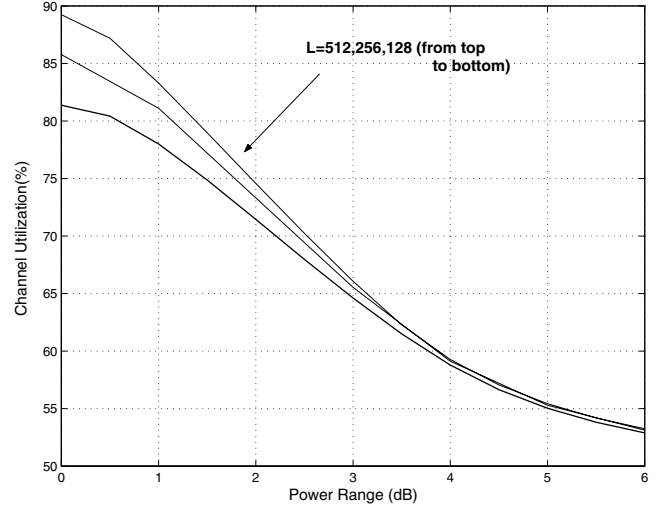


Fig. 2. Channel utilization vs. Power Control Range (PCR).  $\beta = 1$ ,  $\sigma^2 = 0$ . Node powers  $\gamma_i$  are uniformly distributed between  $\gamma_{\max}$  and  $\gamma_{\max}/10^{-\text{PCR}/10}$ . The maximization over  $x$  in AST calculation is computed via exhaustive search.

generating function of Poisson, we get

$$\begin{aligned} \mathbb{E}[e^{sX_N}]e^{-sN} &= e^{(N-\alpha_N)(e^s-1)-sN} \\ &= \exp(N \underbrace{(e^s-1-s)}_{=s^2/2+O(s^3)} - \alpha_N \underbrace{(e^s-1)}_{\geq s}) \\ &\leq \exp(1/2 + O(1/\sqrt{N}) - \log N), \quad s = 1/\sqrt{N} \\ &= \frac{1}{N} e^{1/2+O(1/\sqrt{N})}. \end{aligned}$$

Applying the Chernoff bound to the other term in a similar way, it is easy to obtain

$$p \leq \frac{2}{N} e^{1/2+O(1/\sqrt{N})}. \quad (15)$$

The theorem follows from (13) and (15).  $\square$

### B. Imperfect Power Control

Imperfect power control can have an adverse effect on throughput performance depending on how tight the power is controlled. We demonstrate the effect of imperfect power control on AST in Fig. 2. The channel utilization (*i.e.* the AST divided by  $L/\beta + 1$ ) decreases as the power control range increases. If the power control range can be bounded within 1dB, then the loss looks somewhat small. But, if the power control doesn't do a good job, then the degradation can be significant.

## IV. CONCLUSIONS

The asymptotic stable throughput is a performance metric for networks with a large number of users. In this paper we have proved that the asymptotic stable throughput shown to be achievable in [5] is also the maximum. We have obtained this result under the condition that the channel distribution has support within an interval  $(\gamma_{\min}, \gamma_{\max})$ . Whether this constraint can be relaxed is a theoretically interesting open question.

We have also investigated the maximum AST in a CDMA network with high reception capability  $N$ . It is shown that the channel utilization of slotted ALOHA with *fixed received* power converges to 1 as the reception capability (or, the spreading gain) goes to infinity.

## APPENDIX

### A. Proof of Theorem 3

In this appendix we prove a result (Theorem 5) that is more general than Theorem 3. The advantage of this result is that it is not restricted to the CDMA model in this paper; it applies to the general reception model described in [5] as well. To be able to discuss the general result, we will also need some general conditions (A1), (A2) on the reception channel provided next. After that, we will see that in the CDMA network the condition (C1) naturally implies (A1) and (A2).

**Definition** We say that the channel satisfies the *regularity conditions* if the following holds for every sequence of transmission controls  $\{s_n\}$ :

(A1)  $C_k(G_n)$  is uniformly bounded

$$\sup_{k,n} C_k(G_n) < \infty. \quad (16)$$

(A2)  $\lim_{k \rightarrow \infty} C_k(G_n)$  exists for every  $n$ , and the sequences  $(C_k(G_n), n \geq 1)$  converge to  $(\lim_k C_k(G_n), n \geq 1)$  uniformly as  $k \rightarrow \infty$ .

The physical interpretation of (A1) is that the base station has bounded reception capability regardless of the received power distribution—this is typical in practical networks. For the CDMA model, (A1) directly follows from Lemma 1.

Assumption (A2) is more technical in nature, and for that we will need (C1). To obtain (A2) from (C1), consider the SIR model with  $k$  transmissions. Eqn. (1) implies

$$\gamma_i \geq \frac{\beta}{L} \sum_{j \neq i} \gamma_j.$$

For large  $k$ , the opposite inequality  $\gamma_{\max} < \frac{\beta}{L}(k-1)\gamma_{\min}$  holds. This means that even the maximum possible power  $\gamma_{\max}$  can not exceed the interference caused by  $k-1$  minimum power interferers, and there can not be any success. Therefore, the expected number of successful receptions  $C_k(G_n)$  is zero for large  $k$ , regardless of  $\{s_n\}$  and  $n$ . Hence,  $(C_k(G_n), n \geq 1)$  converges to its limit  $(\lim_k C_k(G_n) = 0, n \geq 1)$  uniformly as  $k \rightarrow \infty$ .

The next theorem is the previously mentioned general result.

**Theorem 5:** If the channel satisfies regularity conditions (A1) and (A2) and  $\lambda_\infty$  is achievable with a sequence  $\{s_n\}$ , then  $\forall \epsilon > 0$ , there exists  $x > 0$ ,  $T \ll F$  such that

$$\lambda_\infty - \epsilon \leq f(x, T). \quad (17)$$

*Proof:* The proof is given in [12] due to space constraints.  $\square$

To obtain (9) from Theorem 5, take the supremum of the right hand side of (17) with respect to  $x > 0$ ,  $T \ll F$ . The resulting inequality holds for all  $\epsilon > 0$ , therefore, it holds for  $\epsilon = 0$ . Take the supremum of the left hand side with respect to  $\{s_n\}$  to obtain (9). This finishes the proof of Theorem 3.

### B. Proof of Lemma 1

For some integer  $l \leq 0$ , let powers  $\gamma_l \leq \gamma_{l+1} \leq \dots \leq \gamma_0 \leq \gamma_1 \leq \dots \leq \gamma_{k-1}$  yield  $k$  successful receptions. Eqn. (1) can be equivalently written as

$$\frac{L}{\beta} \geq \frac{L\sigma^2}{\gamma_i} + \sum_{j \neq i} \frac{\gamma_j}{\gamma_i}.$$

User with power  $\gamma_0$  is successful, therefore,

$$\begin{aligned} \frac{L}{\beta} &\geq \frac{L\sigma^2}{\gamma_0} + \sum_{j=1}^{k-1} \frac{\gamma_j}{\gamma_0} \\ &\geq \frac{L\sigma^2}{\gamma_{\max}} + (k-1). \end{aligned}$$

Leaving the  $k$  alone on the right hand side gives the lemma.

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