Capacity of Regular Ad Hoc Networks with Multipacket Reception

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Abstract

In this paper, the effects of multipacket reception (MPR) capability on the capacity of wireless networks with regular structures are analyzed. Under uniform traffic and minimum connectivity, the maximum stable packet generation rate for the MPR Manhattan networks is obtained. For a network of size N, it is shown that the capacity is $K_1/\sqrt{N}$, and adding MPR affects the coefficient K_1 of a non-MPR network by no more than 1.6 times. For the same network the stability region of the slotted ALOHA random access protocol with MPR is shown to be $K_2/\sqrt{N}$, for some constant $K_2 < K_1$.

For increased connectivity, it is shown that the capacity can be increased further by adding MPR. In the limiting case, in a fully connected network, MPR increases the capacity linearly. These results indicate that minimum connectivity for MPR networks is not necessarily optimal.

For non-uniform traffic, MPR can contribute much more to particular nodes which have *sink* type of traffic, and the dual concept of multipacket transmission (MPT) is very useful for *source* type of nodes. In other words, MPR and MPT can boost the throughput of the network also by being selectively added to the nodes that bottleneck the performance.

1 Introduction

The physical layer of wireless networks is undergoing tremendous development because of the recent advances in signal processing and multiple antenna systems. In such advanced systems the conventional collision channel assumptions do not hold, and terminals might be capable of receiving multiple packets simultaneously [7]. In spread spectrum, multipacket reception (MPR) can be obtained by assigning multiple codes to a single receiver. In a multiple antenna system proper space-time coding or beamforming would yield MPR.

Although we have a clear understanding of those systems at the physical layer, their effects on other networking layers are not well known. Our objective in this paper is to analyze the effect of physical layer MPR capability to the network capacity and to the

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capacity of the Slotted ALOHA medium access control (MAC) protocol. The effect of MPR to optimal network connectivity is also investigated. As a framework, we consider ad hoc networks with regular topologies which allow for mathematical tractability and provide valuable insights.

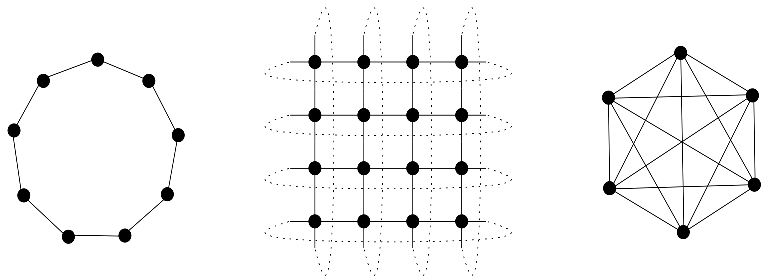
Performance of slotted ALOHA on regular topologies was first analyzed by Silvester and Kleinrock [9] using heuristic arguments. Later, Tsybakov and Bakirov [1] laid the mathematical framework for stability and capacity analysis of multihop ALOHA networks, and analyzed the capacity of slotted ALOHA rigorously by considering a stochastic packet generation model and queueing. Gupta and Kumar [5] have recently analyzed the capacity of random and arbitrary networks by considering a medium access by perfect scheduling. Their results presented the scheduling capacity laws of the network and the trade-off between throughput vs. connectivity. However, although they have developed upper and lower bounds on capacity which are of the same order, for their networks the coefficient of the capacity is not known, which is very important for our purposes. More recently, Grossglauser and Tse [11] considered the effect of node mobility on capacity, and Toumpis and Goldsmith [12]-[13] looked at Shannon theoretic issues in capacity of ad hoc networks.

As a model we take a fixed regular network and a stochastic packet generation process, and define the capacity as the maximum packet generation rate making the network stable with an arbitrary MAC and a routing protocol. We use the same stability definition as the one in [1], and assume the network traffic is uniform. One advantage of a regular topology is that it allows us to compute not only the asymptotic capacity exponent but also its coefficient. Our results indicate that having MPR capability at the physical layer with a fixed connectivity does not change the asymptotic capacity exponent but only its coefficient, so does the slotted ALOHA MAC. The knowledge of the exact capacity helps us to make comparisons between MPR vs. non-MPR cases, and perfect scheduling vs. random access MAC. Our results also show that minimum connectivity is not optimal if the nodes have high MPR capability.

2 The network and the reception model

An ad hoc network is modeled as an undirected graph $G = (V, E)$, where each vertice (node) represents a radio terminal, and edges denote the connections between terminals. In a Manhattan network of N nodes (Figure 1.b), the nodes are placed on a rectangular grid with dimensions $\sqrt{N} \times \sqrt{N}$. Every node in the network has four neighbors, and the nodes on the edge are connected to the nodes on the other side, like a torus covered by a grid. Other regular topologies we will consider in this paper are shown in Figure 1. Note that in our graph model, the signals from those nodes not connected with edges are treated as noise and are modeled probabilistically by the MPR matrix introduced below. It is assumed that the time is divided into fixed length slots, and transmission of one packet takes a single slot. The slot $t \in Z^+$ is defined as the half-open interval $(t-1, t]$. The slots are long enough to accommodate transmission delays. The nodes can not transmit and receive at the same time. In each slot, a node can correctly receive and decode a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *Receiver MPR Matrix* $\mathbf{C}$. The entries of the MPR matrix $\mathbf{C}$ are given as

$$C_{n,k} = P[k \text{ packets are received} \mid n \text{ packets are transmitted in the neighborhood}].$$



a) Ring 9 nodes   b) Manhattan 4x4 nodes   c) Fully connected 6 nodes

Figure 1: Examples of regular networks

The receiver MPR matrix is defined and given as

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{1}$$

This channel model is general enough to model the conventional channel and the capture channel as special cases. The corresponding MPR matrices for the conventional channel, and the capture channel are, respectively,

$$\begin{pmatrix} 0 & 1 & 0 & \dots \\ 1 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & \dots \\ 1-p_2 & p_2 & 0 & \dots \\ 1-p_3 & p_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where p_i denotes the probability of capture given i simultaneous transmissions in the reception neighborhood. We also define

$$C_n \triangleq \sum_{k=1}^{n} k C_{n,k}$$

which is the expected number of correctly received packets given n packets are transmitted.

The MPR matrix $\mathbf{C}$ is a function of a number of parameters. It depends on the ambient noise, the type of modulation used, and the multiuser detection/equalization method receivers apply. In our graph model, the MPR matrix also probabilistically includes the interference coming from transmissions from those nodes which are not directly connected by edges.

3 Capacity of Manhattan networks with MPR

Consider a Manhattan network composed of N nodes. All nodes in the network have the same MPR capability which are described by an MPR matrix $\mathbf{C}$. We will represent the nodes in the network using the set $\mathcal{N} = \{0, \cdots, N-1\}$. Every node in the network has an infinite buffer for holding its packets. The number of packets at node $i \in \mathcal{N}$ at time t is denoted by $n_t^{(i)}$.

The network starts operation at time $t = 0$ where buffer of each node is empty. In slot t the node $i \in \mathcal{N}$ generates $\beta_t^{(i)}$ packets randomly. The statistics of $\beta_t^{(i)}$ are i.i.d. for all nodes for all t, and are given by $Pr\{\beta_t^{\{i\}} = k\} = q_k, \ \sum_{k=0}^{\infty} q_k = 1$. The *packet generation rate* of a node is given by

$$\lambda \triangleq \sum_{k=0}^{\infty} k q_k. \tag{3}$$

The destinations of packets generated at node i are are i.i.d. random variables uniformly distributed among other $N-1$ nodes.

A MAC protocol is a set of rules (or in general a random function) that governs the transmission decisions of nodes. A routing protocol determines through which path the packets are transmitted and are being relayed. For our purposes, the MAC and routing protocols can assume any kind of causal information available for transmission and routing decisions. A packet can be routed arbitrarily or randomly by the nodes on its path.

Definition For a given network of size N, whose MAC and routing protocols are specified, a node i is called *stable* if

$$\lim_{\theta\to\infty} \lim_{t\to\infty} \inf Pr\{n_t^{(i)} < \theta\} = 1. \tag{4}$$

A network is called stable if all nodes in the network are stable.

This definition of stability was previously used by Tsybakov and Bakirov [1] to analyze capacity of slotted ALOHA protocol. See [3] and [2] for a discussion on various kinds of stability and their relations.

3.1 Network stability and capacity

A packet generation rate λ is called *achievable* if there exists a MAC and a routing protocol that makes the network stable. In the following we will show that the set of achievable rates is well defined, in the sense that the achievability does not depend on the packet generation process but only its mean λ. The *capacity of a network* denoted by η is the supremum of all achievable packet generation rates.

A node in the Manhattan network is determined by two coordinates $(x, y) \in \{0, \cdots, \sqrt{N}-1\} \times \{0, \cdots, \sqrt{N}-1\}$. The distance $d\{\cdot\}$ between two nodes (x_0, y_0) and (x_1, y_1) is given by

$$d\{(x_0, y_0), (x_1, y_1)\} \triangleq \min\{\delta x, \sqrt{N} - \delta x\} + \min\{\delta y, \sqrt{N} - \delta y\}, \tag{5}$$

where $\delta x = |x_0 - x_1|$ and $\delta y = |y_0 - y_1|$.

Proposition 3.1 *In a Manhattan network of N nodes, the average distance packets travel using the shortest path is given by*

$$\bar{L} = \begin{cases} \frac{\sqrt{N}}{2}, & \sqrt{N} odd \\ \frac{N\sqrt{N}}{2(N-1)}, & \sqrt{N} even \end{cases} \tag{6}$$

Note that for large N, the $\bar{L}$ is approximately equal to $\sqrt{N}/2$.

Theorem 3.2 (The capacity of MPR Manhattan networks) *For a given Manhattan network of N nodes with MPR matrix* $\mathbf{C}$ *define the capacity as*

$$\eta \triangleq \max_{i=1,\cdots,4} \frac{C_i}{i+1}\frac{1}{\bar{L}}. \tag{7}$$

All rates below capacity are asymptotically achievable. Specifically, for every arrival rate $\lambda < a_N \eta$, there exists a MAC and a routing protocol that makes the network stable, where a_N is a function of N such that $a_N \to 1$ as $N \to \infty$.

Conversely, for an arrival rate $\lambda > \eta$, there does not exist any MAC and routing protocol that makes the network stable.

The proof of the converse of Theorem 3.2 is sketched in Section 4, and in Section 5 the particular schedules which achieve the capacity are given.

Now, we'll see the effect of pure random access to the capacity. For Manhattan topology, the following result is a generalization of [1] to MPR.

Theorem 3.3 (The capacity of MPR Slotted ALOHA) *Define the slotted ALOHA capacity as*

$$\eta_{ALOHA} = \frac{1}{4\bar{L}} \max_{0 \le p \le 1} \sum_{k=1}^{4} \binom{4}{k} p^k (1-p)^{5-k} C_k. \tag{8}$$

A Manhattan network employing shortest path routing and Slotted ALOHA MAC can be stabilized by adjusting the transmission probability p if and only if $\lambda < \eta_{ALOHA}$.

3.2 The implications of the theorems

From the capacity expressions of η and η_{ALOHA} two factors that determine the performance becomes apparent. The capacity theorems 3.2-3.3 show that the scaling law is $O(1/\sqrt{N})$ and the per node capacity of the network goes to 0 regardless of the MPR capability. The main reason behind this pessimistic fact is the uniform traffic pattern which gives average path length of $\bar{L} = O(\sqrt{N})$. In [5] and [6], it's also shown that the capacity of the network goes to 0 as N tends to infinity, and the main reason behind this is the uniform traffic assumption.

The second factor which determines the capacity is the performance of the MAC protocol, namely the (average) number of simultaneously transmitted packets in the network. Adding MPR only affects the coefficient of η or η_{ALOHA}. As a numerical example, take the MPR matrix as $K - perfect - MPR$ which is the case when nodes receive up to K packets without error otherwise collision occurs and nothing is received. This corresponds to

$$C_i = i \cdot 1_{\{1 \le i \le K\}}(i), \tag{9}$$

where $1_{\{\ \}}$ is the indicator function. For $K = 1$ this MPR matrix becomes the collision channel matrix. For $K = 4$ the matrix is the best possible for a Manhattan network since every node has only 4 neighbors. For $K \in \{1, 2, 3, 4\}$ the capacity coefficient $C_K/(K+1)$ of η becomes $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\}$, respectively. Therefore, even having the best MPR channel gives only $\frac{4}{5}/\frac{1}{2} = 1.6$ gain over non-MPR collision channel. In case of slotted ALOHA, for $K \in \{1, 2, 3, 4\}$, the coefficient $\bar{L}\eta_{ALOHA}$ is $\{0.0819, 0.165, 0.226, 0.250\}$, respectively. These figures mean that $\frac{1}{2}N$ transmissions are possible in a network with collision channel while the slotted ALOHA is capable is achieving $0.0819N$ transmissions on the average. Thus, in that case, the best medium access gives about 6 times improvement over ALOHA.

3.3 Other topologies, traffic pattern and connectivity

The assumptions on the Manhattan network in theorems 3.2-3.3 can be modified in various directions. Instead of Manhattan topology we can consider similar topologies like ring and fully connected networks. In a fully connected network it is suprising to observe that increase in MPR capability directly translates to scheduling capacity ($\eta = \max_{i=1,\cdots,N-1} C_i \frac{1}{N}$). Although the scaling law $O(1/N)$ is much worse than that of Manhattan, the capacity can be increased linearly by adding MPR. On the contrary, in a ring network MPR doesn't contribute much to the capacity ($\eta = \max_{i=1,2} \frac{C_i}{i+1} \frac{4}{N}$).

These observations make us believe that the contribution of MPR to the capacity is higher when the network connectivity is higher. Moreover, even in Manhattan network it can be shown that for $12 - perfect - MPR$ minimum connectivity radius is not optimal; there exists a scheduling for 2-hop connected network which yields throughput higher than (7). Note that if the connectivity radius is changed, the MPR matrix of the nodes would also change, and it is hard to make exact comparisons.

It can be observed that the effect of MPR to capacity significantly depends on the type of traffic. In a Manhattan network, the average incoming traffic of a node is the same as the outgoing traffic. However, if we had an asymmetry, and the incoming traffic of a node was higher than the outgoing traffic, the MPR would increase the throughput of that particular node much more. In other words, for *sink* type of nodes (like the up-link of a base station) having MPR capability is very valuable.

4 The converse to Theorem 3.2

In this section, we'll sketch the proof of theorem 3.2. For a given N suppose the MAC and routing protocols are defined. Let $< i, j >$ denote the the j'th packet in i'th node's buffer where $i \in \mathcal{N}$ and at time t, $0 \leq j \leq n_t^{(i)} - 1$. The distance of the packet $< i, j >$ at time t to its destination through the shortest path is denoted by $l_t^{<i,j>}, i \in \mathcal{N}, 0 \leq j \leq n_t^{(i)} - 1$. Let $\mathcal{G}_t$ denote the set of packets generated in slot t, and define

$$G_t \triangleq \sum_{<i,j> \in \mathcal{G}_t} l_t^{<i,j>}.$$

Also let T_t denote the number of transmitted packets in slot t which are correctly received by the intended receivers. We define

$$L_t \triangleq \sum_{i \in \mathcal{N}} \sum_{j=0}^{n_t^{(i)}-1} l_t^{<i,j>},$$

where it can be seen that $L_t \geq \sum_{n=1}^{t} (G_n - T_n)$.

Now we'll state a useful lemma which basically shows that $L_t \to \infty$ almost surely as $t \to \infty$, if the packet generation rate is higher than the capacity.

Lemma 4.1 *For a given Manhattan network of N nodes, with an arbitrary MAC and an arbitrary routing protocol, if $\lambda > \eta$, then for any $\theta > 0$,*

$$Pr\{L_t < \theta\} \to 0 \text{ as } t \to \infty. \tag{10}$$

We omit the proof of Lemma 4.1 because of space limitations. Next, we state an important corollary of the lemma. The corollary follows since $l_t^{<i,j>} \leq \sqrt{N}$ for all i, j, t and $L_t \leq \sum_{i\in\mathcal{N}} \sum_{j=0}^{n_t^{(i)}-1} \sqrt{N} = \sqrt{N} \sum_{i\in\mathcal{N}} n_t^{(i)}$.

Corollary 4.2 *For a given Manhattan network of N nodes, with an arbitrary MAC and an arbitrary routing protocol, if $\lambda > \eta$, then for any $\theta > 0$,*

$$Pr\{\sum_{i\in\mathcal{N}} n_t^{(i)} < \theta\} \to 0 \text{ as } t \to \infty. \tag{11}$$

Using Corollary 4.2 it can be shown that the converse holds.

5 The direct part of Theorem 3.2

Now we'll provide MAC and routing protocols that achieve the capacity asymptotically when $\lambda < \eta$. For routing we'll use a special type of shortest path routing. For MAC we'll provide schedules for transmitters and receivers that optimally utilize MPR.

As a notation, recall that a node in the Manhattan network is determined by two coordinates $(x, y) \in \{0, \cdots, \sqrt{N}-1\} \times \{0, \cdots, \sqrt{N}-1\}$. Furthermore, suppose that each coordinate of (x, y) represents not only a number but also a congruence class in modulo $\sqrt{N}$. For two given nodes (x_0, y_0),(x_1, y_1) the summation $(x_0 + x_1, y_0 + y_1)$ is also defined in modulo $\sqrt{N}$.

Our particular scheduling patterns will be called "κ-MPR scheduling" which is a function of $\kappa \triangleq \arg\max_{i=1,\cdots,4} \frac{C_i}{i+1}$. In κ-MPR scheduling, the network is tiled using the patterns in Figure 2 and their 90°, 180° or 270° rotated versions. The arrows in Figure 2 show the source and the destination of scheduled transmissions. In κ-MPR scheduling, every scheduled receiver receives κ packets intended for him. Each tiling in Figure 2 has the property that one piece of the tile schedules $u_\kappa \times u_\kappa$ nodes where $u_1 = 4, u_2 = 3, u_3 = 4, u_4 = 5$. It is also obvious that by repeating the patterns in Figure 2 side by side, the whole network or a region of it can be tiled. Now, assume $\sqrt{N} > 60$ and define $\sqrt{\tilde{N}} \triangleq 60\lfloor\frac{\sqrt{N}-1}{60}\rfloor$, which is the largest integer less than $\sqrt{N}$ divisible by 60. In our scheduling MAC, in a slot t, a node (x_t, y_t) is assigned as the *first reference point* of the tiling. In a slot t, the set of all reference points for the tiling is

$$\mathcal{T}_t = \{(x_t + au_\kappa, y_t + bu_\kappa) \mid a, b \in \{0, 1, 2, \cdots, \sqrt{\tilde{N}}/u_\kappa - 1\}\},$$

where the addition is in modulo $\sqrt{N}$. When we define the set $\mathcal{T}_t$ and the orientation of the tile, $\tilde{N}$ receivers and transmitters are scheduled, the rest $N - \tilde{N}$ nodes neither transmit nor receive during the slot t. It can also be observed that $\tilde{N}/(\kappa + 1)$ of the scheduled nodes are receivers and $\tilde{N}\kappa/(\kappa + 1)$ nodes are scheduled transmitters.

Our MAC is defined in this way. We use the patterns in Figure 2 and their 90°, 180° or 270° rotated versions. By specifying the first reference point (x_t, y_t) and the orientation of the tiling, we specify the set $\mathcal{T}_t$, all scheduled transmitters and all scheduled receivers in the slot t. In slot t, we specify the first reference point of the tiling as the node $(\tilde{t}, \lfloor\frac{\tilde{t}}{N}\rfloor)$, where $\tilde{t} = \lfloor\frac{t}{4}\rfloor$. We also specify the rotation angle of the tiling as $(t - 4\tilde{t})90°$ clockwise. With these choices for (x_t, y_t) and the rotation angle, every node in the network obtains a fair access to the medium. In every $4N$ slots, our scheduling pattern repeats itself, and any node in the network is given

$$M \triangleq \tilde{N}\kappa/(\kappa + 1) \tag{12}$$

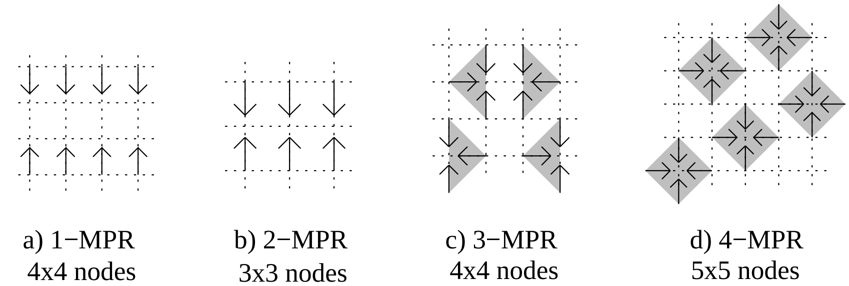


Figure 2: Different scheduling patterns for κ-MPR, $\kappa \in \{1, 2, 3, 4\}$

slots to transmit to each of his four neighbors.

In a slot t, if a node is scheduled to transmit and he holds a packet for his receiving neighbor, he transmits it. Furthermore, assume that if a transmitter doesn't hold a packet for his receiving neighbor, he transmits a dummy packet. A dummy packet does not carry any information, and it is discarded immediately both by the transmitter and the receiver after the transmission. The existence of dummy packets is a convention that simplifies the analysis. With the existence of dummy packets, every scheduled receiver certainly hears κ packets intended for him, and the success of probability of every transmission is $p_\kappa \triangleq \frac{C_\kappa}{\kappa}$.

Now, we'll introduce the routing protocol. We'll assign a fixed route to each packet once it is generated. Our routing protocol applies a type of shortest path routing, however, it depends on if $\sqrt{N}$ is odd or even. In the following, we'll describe the routing for odd $\sqrt{N}$, nevertheless, for even $\sqrt{N}$ a very similar routing can be used. Now, suppose the node $(0,0)$ generates a packet for the node (x,y), where $-\frac{\sqrt{N}-1}{2} \leq x \leq \frac{\sqrt{N}-1}{2}$ and $-\frac{\sqrt{N}-1}{2} \leq y \leq \frac{\sqrt{N}-1}{2}$. Define $\mathrm{sgn}(0) = 0$ and $\mathrm{sgn}(x) = \frac{x}{|x|}$ when $x \neq 0$ If $|x| > |y|$ the route is $R_1 = (0,0) \rightarrow (\mathrm{sgn}(x), 0) \rightarrow \cdots \rightarrow (x, 0) \rightarrow (x, \mathrm{sgn}(y)) \rightarrow \cdots \rightarrow (x, y)$. If $|y| > |x|$ the route is $R_2 = (0,0) \rightarrow (0, \mathrm{sgn}(y)) \rightarrow \cdots \rightarrow (0, y) \rightarrow (\mathrm{sgn}(x), y) \rightarrow \cdots \rightarrow (x, y)$. If $|y| = |x|$ then the node $(0,0)$ chooses either R_1 or R_2 randomly with $1/2$ probabilities. For a packet from node (x_0, y_0) to (x_1, y_1), we assume the node (x_0, y_0) chooses another coordinate axis by taking himself as origin, and assigns a route to his packet by assuming he is node $(0,0)$ and the destination of the packet is $(x_1 - x_0, y_1 - y_0)$. With this, we have defined all routes for networks with odd $\sqrt{N}$.

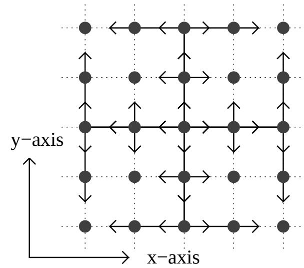


Figure 3: Routes for packets originating from the node at the center

Next, we'll present the idea that makes our proof work: dividing the traffic flow into streams. It is known that from a given origin node packets are generated with equal rates for other $N-1$ nodes in the network. Suppose $\sqrt{N}$ is odd. Assume that a node has $2(N-1)$ separate sub-queues for holding packets that originate from him. From the given source node $(0,0)$ to any other node (x,y) two sub-queues are reserved. Whenever the node $(0,0)$ generates a packet for (x,y), he flips a fair coin and puts this packet randomly into one of two sub-queues reserved for (x,y). Suppose, $-\frac{\sqrt{N}-1}{2} \leq x \leq \frac{\sqrt{N}-1}{2}$, and $-\frac{\sqrt{N}-1}{2} \leq y \leq \frac{\sqrt{N}-1}{2}$. If $|x| \neq |y|$, the packets in these two sub-queues are routed through the same route described above. However, if $|x| = |y|$, then the packets in one sub-queue are routed through the route R_1, and the packets in the other sub-queue are routed through R_2. The packets put in each of these sub-queues is called one *stream*. Besides transmitting the packets generated inside, the node $(0,0)$ also relays packets for his neighbors, *i.e.,* relays other streams. As the node (0,0) divides his traffic into streams, every node in the network applies the same procedure. As a result of uniform traffic and our routing protocol, it can be observed that from every node in the network originates $2(N-1)$ streams, and for a given node $2(N-1)$ streams are destined, and a node relays $2(\bar{L}-1)(N-1)$ streams for other nodes in the network. Thus, a total of $2\bar{L}(N-1)$ streams goes out from each node in the network. In order to accommodate each stream, we assume that every node has $2\bar{L}(N-1)$ sub-queues, one for each stream. It can also observed that for every neighbor the packets are transmitted from $\bar{L}(N-1)/2$ sub-queues. As a notation, for a given node these sub-queues are represented by $Q_k^{up}, Q_k^{down}, Q_k^{left}, Q_k^{right}$, $k \in \{1, 2, \cdots, \frac{\bar{L}(N-1)}{2}\}$, where $Q_k^{\{\cdot\}}$ shows the k'th sub-queue whose packets have to be transmitted in $\{\cdot\}$ direction. Note that in this representation the sequence of the sub-queues is arbitrarily chosen.

Now, we'll specify the servicing discipline at each node, for odd $\sqrt{N}$. Assume the time is divided into non-overlapping and consecutive $frames$ of

$$F = \frac{\bar{L}(N-1)}{2} \cdot 4N = 2\bar{L}(N-1)N$$

slots. In a frame, in the slots between $4N(k-1)$ and $4Nk$, $k \in \{1, 2, \cdots, \frac{\bar{L}(N-1)}{2}\}$, the packets from $Q_k^{up}, Q_k^{down}, Q_k^{left}, Q_k^{right}$ are transmitted. Using this method, a node transmits $M = \tilde{N}\kappa/(\kappa+1)$ packets from each sub-queue in $4N$ slots and in a frame. 7The servicing discipline in the sub-queues is arbitrary (One can assume it is first-come-first-serve). If there does not exist a packet from a specific sub-queue to transmit, the node transmits a dummy packet in that slot (although he might hold some other packets for other streams!). If a node generates a packet or receives a packet to be relayed, he doesn't immediately put it in his transmission sub-queue, the packet waits in the node's waiting buffer without being transmitted until the beginning of the following frame.

The proposed set of protocols essentially convert an interacting queues problem to a series of queues problem which is analyzed in [2]. Because of space limitations, we omit this part of the proof. However, it can be shown that the network is stable when $\lambda < \frac{\tilde{N}}{N}\eta_{ALOHA}$, and $\frac{\tilde{N}}{N} \rightarrow 1$ as $N \rightarrow \infty$.

6 Conclusions and future work

Having multiple transmission codes or having directed antennas would yield the dual concept of multipacket transmission (MPT) which complements the idea of MPR. In a Manhattan network, with MPT, without MPR, it can be shown that a capacity formula which is very similar to (7) applies [8] . Therefore, having MPT alone increases the capacity in a bounded way, like having MPR alone. However, in case of having both MPR and MPT together, it can be observed that the capacity increases linearly with the increase in minimum of the two. Similarly, as a function of traffic, MPT is particularly useful for *source* type of nodes (like the down-link of a base station), as MPR is very useful for sink nodes.

In this paper we looked at the capacity of regular networks with MPR capability. We have shown that MPR alone increases capacity of minimal connected Manhattan network at most 1.6 times, and random access MAC only affects the coefficient of the capacity. The minimal connectivity is not necessarily optimal for MPR networks. MPR is a subject that require further investigation in arbitrary topologies and arbitrary traffic patterns. MPR and MPT can boost the performance of the networks also by being selectively added to the nodes in the congested regions of the network.

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