RECEIVER CONTROLLED MEDIUM ACCESS IN MULTIHOP AD HOC NETWORKS WITH MULTIPACKET RECEPTION

Gökhan Mergen and Lang Tong School of Electrical and Computer Engineering Cornell University, Ithaca, NY 14853 {mergen,Itong}@ee.cornell.edu

ABSTRACT

The advent of diversity modulation and signal processing has changed the underlying assumption of the conventional collision model that at most one packet can be received at each node of a network. The use of antenna arrays, code division multiple access (CDMA), and space-time processing with multiuser detection makes multipacket reception (MPR) a more realistic model for the physical layer of future wireless networks. In this paper, a multiple access protocol based on receiver controlled transmissions (RCT) is presented for multihop ad hoc networks with MPR nodes. As a hybrid of scheduled and random access MAC, the RCT protocol is capable of providing high throughput at heavy traffic load and short delay in light traffic. Unlike existing global scheduling techniques. RCT requires only coarse coordination among regions of the ad hoc network, and it scales to networks of arbitrary sizes. Capacity analysis and the application of RCT for Manhattan type of multihop networks are presented where scheduled reception interval and the size of local contention are optimized. Performance analysis and simulations are also presented.

1. INTRODUCTION

Medium access control (MAC) is one of the most challenging issues in wireless ad hoc networks. Existing analysis and protocol designs assume the conventional collision model, *i.e.*, each node of the network can receive at most one packet per slot, an assumption no longer valid for nodes employing antenna arrays with sophisticated diversity techniques and multiuser detection. The physical layer of future wireless networks is perhaps more accurately modeled as nodes with multipacket reception (MPR) capability.

MPR introduces greater challenges in network protocol design, and it requires a closer interaction between the physical and MAC layers of the network [6]. The conventional collision model aims to limit the number of transmissions to a single node. A more flexible approach is necessary for networks with MPR nodes so that an optimal number of nodes are enabled. Most conventional MAC do not have a direct extension to MPR networks.

MPR also suggests a shift of responsibility from transmitters to receivers. Specifically, since the transmitter might not be aware of the receiver MPR capability and the traffic directed to the receiver, it is natural that receivers decide how many and which nodes should transmit. Thus it appears that receiver controlled protocols have the inherent advantage in exploiting the MPR property. Even for networks with non-MPR nodes, receiver controlled MAC protocols have many attractive features [3] and [4].

In this paper, we consider the design and analysis of MAC for multihop ad hoc networks with MPR nodes. Our goal is to develop a MAC protocol that achieves high throughput at heavy traffic load and low delay in light traffic. To this end, we propose a Receiver Controlled Transmissions (RCT) protocol that is a hybrid of scheduled and random access protocol. Scheduling is performed coarsely according to transmission periods of multiple slots. Within each transmission period, the MAC is random access and distributed. Similar to adaptive splitting algorithms [9, 5] for networks with base stations, the proposed protocol adaptively adjusts its contention size locally.

To provide a comparison benchmark, we present a capacity analysis of the Manhattan network where an achievable end-to-end throughput is obtained. We then optimize the transmission radii and the transmission period of the RCT protocol to maximize the local throughput. Considering MACs for regular network topologies such as the Manhattan network may appear to be artificial. However, as in the classical paper of Silvester and Kleinrock [8], much insights into effects of node connectivity and network size on throughput can be gained. Furthermore, the Manhattan topology can be considered as an approximation of uniformly distributed nodes in sensor networks.

2. THE RECEPTION MODEL AND NETWORK CAPACITY

We first describe the MPR model, and then present a capacity analysis of Manhattan network. It is assumed that

 $^{^0\}mathrm{This}$ work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564 and the Army Research Office under Grant ARO-DAAB19-00-1-0507.

the time is divided into fixed length slots, and transmission of one packet takes a single slot. Also the slots are long enough to accommodate transmission delays.

Receiver MPR In each slot, a node can correctly receive and decode a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *Receiver MPR Matrix* C. The entries of the MPR matrix C are given as

$$C_{n,k} = P[k \text{ packets are received } | n \text{ packets are transmitted in the neighborhood}].$$

The receiver MPR matrix is defined and given as

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ C_{3,0} & C_{3,1} & C_{3,2} & C_{3,3} \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$
(1)

In general, nodes in a network may have different MPR matrices. In this paper, we assume that all nodes have the same reception capability. Note that MPR matrix C in general is a function of a number of parameters. In particular, it depends on the ambient noise, the type of modulation used, and the multiuser detection/equalization method receivers apply. In an ad hoc environment, it can also be defined to include interference coming from transmissions which is not of interest to the receiver.

This channel model is general enough to include the conventional channel and the capture channel as special cases. The corresponding MPR matrices for the conventional channel and the capture channel, respectively, are

$$\begin{pmatrix} 0 & 1 & 0 & \dots \\ 1 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & \dots \\ 1 - p_2 & p_2 & 0 & \dots \\ 1 - p_3 & p_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (2)$$

where p_i denotes the probability of capture given i simultaneous transmissions in the reception neighborhood.

In MPR networks, it can be shown that the maximum of number of packets that can be received by a node on the average is upper bounded by

$$\eta = \sup_{n=1,2,\cdots} C_n \tag{3}$$

where

$$C_n = \sum_{k=1}^n k C_{n,k} \tag{4}$$

is the expected number of correctly received packets given that n packets are transmitted. We define η as the *capac-ity* of the MPR channel. Throughput (expected number of received packets per slot) of a node can not exceed capacity.

2.1. Capacity Results for the Manhattan network

In the Manhattan network, nodes are placed in the topology of a two dimensional grid. An example Manhattan network is depicted in Figure 1.



Figure 1: The Manhattan Network

In a Manhattan network of N nodes, the nodes are placed on a grid with dimensions $\sqrt{N}\times\sqrt{N}$. The nodes on the edge are connected to the nodes on the other side, just like a torus covered by a grid. In Figure 1, each node has four neighbors, but this need not be always the case. The nodes are capable of adjusting their transmission radius while keeping the links bi-directional.

The performance of slotted ALOHA protocol in Manhattan networks is analyzed by Silvester and Kleinrock [8]. An extension to MPR networks is given in [7]. For a Manhattan network of N nodes with MPR, it is shown that the *end-to-end throughput* of slotted ALOHA can be at most

$$\eta_{ALOHA} = \max_{0 \le p \le 1} \frac{\sqrt{N}}{2} \sum_{k=1}^{4} \binom{4}{k} p^k (1-p)^{5-k} C_k.$$
 (5)

Also the maximum achievable end-to-end throughput is shown to be \sim

$$\eta = \max_{i=1,..,4} \frac{C_i}{i+1} 2\sqrt{N}.$$
 (6)

Both upper bounds assume transmission radius is minimum, and the traffic in the network is uniform, in which the case the average path length that packets travel is close to $\sqrt{N}/2$ hops. The capacity (6) is achievable by global scheduling with shortest path routing, and η_{ALOHA} is obtained under the assumption that every node in the network is backlogged and the transmission probability is p. As a function of N, the order of both bounds are the same indicating that the medium access protocol determines only the coefficient of achievable throughput. Nonetheless, this is quite a difference.

3. THE RCT PROTOCOL

In the RCT protocol, in every slot a number of nodes in the network are scheduled to receive packets. The scheduled

nodes have to select the transmitters in their neighborhood before their reception slot, and to provide feedback after their reception slot, notifying transmitters to release their packets from their buffers if necessary. The control information and feedback is provided between slots, and we do not consider them as a part of traffic.

Scheduling in different topologies can take different forms. It is indeed a hard task to determine fair schedules, and devise distributed algorithms which spatially maximize channel reuse. In the literature, a number of scheduling algorithms/protocols can be found for scheduling in a general network, *e.g.*, [1, 2]. Here we will not deal with the scheduling problem in an arbitrary network. Though RCT protocol is perfectly applicable in a general setting, we will confine ourselves to the scheduling in Manhattan networks whose analysis is analytically tractable.

3.1. Scheduling in Manhattan Networks

For Manhattan networks there are various tilings which cover all the network in a symmetric way. Consider the tiling in Figure 2. If the transmission radius (R) is 1 hop in the network, then each node has four neighbors, and we can select the nodes in the center of shaded regions as scheduled receivers. It is clear that, these receivers have disjoint neighborhoods, and in each neighborhood there are 5 nodes (including the receiver). When we shift the tiling in Figure 2, we see that the network can take 5 different states. We specify the protocol in this way. The network cycles through these 5 states and in each state the nodes in the center of shaded regions control their neighborhood and receive packets intended for them. In each state network stays for L slots, and completion of a cycle takes 5L slots.

It can be shown that these tilings can be drawn for any positive integer R, and the frequency of cycles becomes $(2R^2 + 2R + 1)L$ slots. Note that we haven't fixed values for L and R. As we shall see later, the optimal values for these parameters depend on the traffic load in the network.



Figure 2: Scheduling in Manhattan networks - R = 1



Figure 3: Scheduling in Manhattan networks - R = 2

3.2. Local Random Access

As explained in the previous subsection, RCT protocol applies scheduling to determine the receiving nodes. However, in their reception slots, nodes should choose the transmitters without knowing whether they have packets.

The key to maximizing local throughput is to grant an appropriate subset of users the access to the MPR channel. For the conventional collision channel, this can be accomplished by splitting users in the event of collision [9]. A more flexible approach is necessary for MPR channels because the protocol should allow optimal number of users to transmit. Ideally, ξ —the number that maximizes C_n —users should be allowed to transmit in order to achieve the maximum throughput. Unfortunately, this is not always possible as the number of users holding packets is a random variable not known to the receiver.

For local random access of RCT, we use the same principles developed for cellular networks [5], the protocol operation described below is an extension of the Dynamic Queue Protocol.

Suppose the network connectivity is adjusted (R is selected) and each node has $M = 2R^2 + 2R$ neighbors. Now consider an arbitrary node, Ω , in the network. The scheduled reception period of this node is L slots. After its reception period, the next reception period comes after ML slots. We consider the M neighbors of Ω waiting in a queue in a random order for the transmission of their packets at the beginning of reception period of Ω . We assume each of them holds a packet with probability q intended for Ω . Based on q (traffic load) and the receiver MPR capability (1), Ω determines the size (Q) of the contention class. Then first Q users in the queue are enabled to access the channel in the first slot. At the end of this slot, Ω detects whether this slot is empty or not. If it is empty all Q users are processed and the next Q users are enabled in the next slot. If the slot is not empty and k ($k \ge 0$) packets are successfully received, the sources of these k users are processed; the next Q - k users along with the next k users in the queue are enabled to access the channel in the next slot. This procedure continues for L slots.

We assume the traffic in the network is uniform. To reflect this assumption locally, we assume any neighbor of Ω generates packets, intended for Ω , with probability p in all slots. In the beginning of each reception period of Ω , its neighbors has probability $q = p \frac{\sqrt{N}}{8}$ of holding a packet.

3.3. Local Throughput Analysis

It can be shown that the number of unprocessed users at the beginning of a slot along with the number of packets that will be transmitted in the current slot forms a Markov chain. At the beginning of a slot, the network is in state (j,k) if there are j $(j = 0, \cdots, M)$ unprocessed users, and k $(k = 0, \cdots, \min\{Q, j\})$ packets will be transmitted in the current slot. The transition probability from state (j,k) to state (l,m) is given by

$$p_{(j,k),(l,m)} = \begin{cases} B(m,\min\{N,l\},q_i) \\ (\text{if } k = 0, l = \max\{j - N, 0\}, m = 0, \cdots, \min\{N,l\}) \\ C_{k,j-l}B(m - k + j - l, \min\{j - l, \max\{j - N, 0\}\}, q_i) \\ (\text{if } k = 1, \cdots, \min\{N, j\}, l = j - k, \cdots, j, \\ m = k - (j - l), \cdots, k) \\ 0 \quad (\text{otherwise}) \end{cases}$$
(7)

where $B(\boldsymbol{u},\boldsymbol{U},s)$ denotes the probability of \boldsymbol{u} successes in \boldsymbol{U} independent Binomial trials with success probability s, i.e.,

$$B(u,U,s) = \binom{U}{u} s^u (1-s)^{U-u}.$$
(8)

The initial condition of this Markov chain at the beginning of each reception period is given by

$$P[X_0 = (M, k)] = B(k, Q, q), \ k = 0, \cdots, Q$$
 (9)

where X_0 denotes the initial state of the Markov chain.

Denote the probability of being in state X with a vector

$$\mathbf{X} = [p_{(M,0)}, \cdots, p_{(M,Q)}, p_{(M-1,0)}, \cdots, p_{(1,0)}, p_{(1,1)}, p_{(0,0)}]^t,$$
(10)

where $p_{\left(i,j\right)}$ denotes probability of being in state $\left(i,j\right).$ Define the reception vector as

$$\mathbf{V} = [C_0, C_1, \cdots, C_Q, \ C_0, \cdots, C_{Q-1}, \ \cdots, C_0, C_1, \ C_0]^t,$$
(11)

where C_i is defined in (4). Then we can express the *local* throughput (expected number of received packets per slot per node) as

$$\tau = \frac{1}{(M+1)L} \left(\mathbf{X}_0^t \mathbf{V} + \mathbf{X}_0^t \mathbf{P} \mathbf{V} + \dots + \mathbf{X}_0^t \mathbf{P}^{L-1} \mathbf{V} \right)$$
$$= \frac{1}{(M+1)L} \left(\mathbf{X}_0^t \sum_{i=0}^{L-1} \mathbf{P}^i \mathbf{V} \right)$$
(12)

where **P** is the transition matrix with entries specified in (7). The parameters of the protocol are chosen in a way that maximizes throughput times R (*i.e.*, expected progress per slot) as a function of network load, i.e.,

$$(R^*, L^*, Q^*) = \arg \max_{(R,L,Q)} [R\tau \mid p],$$
 (13)

where (R^*,L^*,Q^*) denotes the optimal values. Note that we need to maximize over three discrete variables. This optimization needs to be done once and can be done off line; after having the optimal values all nodes uses the same (R,L,Q). To convert the local throughput τ (12) to end-to-end network we multiply by N and divide by the average path length $\sqrt{N}/2$, when R=1.

4. SIMULATIONS

In this section, we present simulation results on throughput and delay characteristics of Slotted ALOHA and RCT protocol. We consider an MPR Manhattan network employing CDMA with N = 100 nodes. In this network, each packet is transmitted with a code randomly picked from 3 orthogonal codes. A receiver successfully receives a packet if and only if the code with which the packet is transmitted is used only one of its transmitting neighbors. It can be shown that the capacity (3) of this channel is 4/3, and to achieve this capacity, in each slot, $\xi = 2$ packets should be received at the same time by a receiver.

We set the transmission radius to minimum, R = 1, M = 4, and optimize the performance (12) of RCT protocol as a function of q. The optimal values are easy to express: if $q \leq 0.81$ then $L^* = 1, Q^* = 4$, otherwise $L^* = 2, Q^* = 3$. With optimal L and Q, we find the network throughput for RCT as given in Figure 4. The same figure also gives the throughput performance of Slotted ALOHA under heavy load assumption with retransmission probability q.



Figure 4: Theoretical Performance Characteristics

In the simulation, the throughput performance of RCT and slotted ALOHA protocols are obtained as in Figure 5. Figure 6 presents the delay characteristics. In the simulations, for slotted ALOHA the retransmission probability is selected to be 0.38 that gives the maximum in (5).

It can be seen in Figure 5 that slotted ALOHA carries the load up to packet generation rate about 2.5 packet/slot,

and RCT performs well up to the rate of 4.7 packet/slot. The analytical analysis given in Figure 4 promises rates up to 3 packet/slot with slotted ALOHA and 5 packet/slot with RCT. The analytical values and simulation results are close to each other. One thing that analytical analysis neglects is delay. The delay at high offered load becomes unacceptably high that the network can not exactly reach the throughput values promised by our analysis.

The capacity (6) of this network can be shown to be 8.88 packet/slot. There exist a perfect scheduling mechanism which achieves 8.88 packet/slot throughput, but delay performance of such a scheduling protocol is poor when the traffic if not high. The advantage of RCT protocol is providing low delay for a large range of traffic load, and high throughput.

Note that we have not considered increasing transmission radius more than R = 1. From the analysis and simulations it is observed that R > 1 case is only useful when the network load is low. When load is high, increasing R further increases contention and decreases throughput. However, minimum R is not always the best transmission radius for MPR networks as it is in collision channel [10]. It can be shown that if the nodes had better MPR capability, higher connectivity radius would provide higher throughput.



Figure 5: Throughput vs. Offered Load - N=100, R=1

5. CONCLUSION

Possible advantages of MPR capability at the physical layer are many-fold provided that it is supported with strong higher level network protocols. The proposed RCT protocol scratches the surface of a large number of possibilities and design issues that arise in MPR networks. Non-uniform traffic, arbitrary topologies and, most importantly, nodes with varying level of MPR capability are the crucial factors that make the MAC problem harder and important for the future adaptive networks. We hope the results and insights obtained from RCT on Manhattan networks to be helpful



Figure 6: Delay vs. Offered Load - N=100, R=1

in the design of more advanced protocols for MPR and other spread spectrum type of networks.

6. REFERENCES

- S. Ramanathan and E. L. Lloyd, "Scheduling algorithms for multihop radio networks," *IEEE/ACM Trans. on Networking*, vol. 1, no. 2, pp. 166-177, April 1993.
- [2] A. Ephremides and T. V. Truong, "Scheduling broadcasts in multihop radio networks," *IEEE Trans. on Communications*, vol. 38, no. 4, pp. 456-460, April 1990.
- [3] F. Talucci, M. Gerla, L. Fratta, "MACA-BI a receiver oriented access protocol for wireless multihop networks," *IEEE PIMRC '97*, vol. 2, pp. 435-439, 1997.
- [4] A. Tzamaloukas, J. J. Garcia-Luna-Aceves, "Poll-beforedata multiple access," *IEEE International Conference on Communications*, *ICC '99*, vol. 2, pp. 1207-1211, 1999.
- [5] Q. Zhao and L. Tong, "A Dynamic Queue MAC Protocol for Random Access Channels with Multipacket Reception," *Proceedings of the 34th Asilomar Conf. Sig. Sys. and Computers*, Oct. 2000.
- [6] L. Tong, Q. Zhao, G. Mergen, "Multipacket reception in random access wireless networks: From signal processing to optimal medium access control," *IEEE Communications Magazine*, to appear.
- [7] G. Mergen and L. Tong. "Multipacket Reception in Multihop Ad Hoc Networks". submitted to Allerton Conf. on Communications, Control, and Computing, 2001.
- [8] J.A. Silvester and L. Kleinrock. "On the capacity of multihop slotted ALOHA networks with regular structure". *IEEE Trans. Commun.*, 31(8):974–982, August 1983.
- [9] D. P. Bertsekas and R. Gallager. "Data Networks". Prentice-Hall, 1992.
- [10] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, pp. 388-404, March 2000.