Capacity of Wireless Networks with Bursty Arrivals

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Abstract — **In this paper, we analyze the stability behaviour of wireless networks in case of bursty arrivals. By modelling the channel as a general multipacket reception channel, random reception errors and multipacket receiving nodes are considered. For an arbitrary network with a given stochastic arrival process, the class of static protocols is introduced, and it is shown that every rate inside the stability region can be achieved using such protocols. The introduced protocols admit very simple implementations in the presence of a global scheduler, and this result emphasizes that the dynamic network protocols which are widely studied in the literature and applied in practice do not increase the stability region of the considered wireless networks. We also make a connection with the previously defined notions of network capacity, and show that a network with bursty arrivals with rate λ can be stabilized if and only if the existing packets at the source nodes can be carried away to their destinations with rate λ almost surely.**

I. Introduction

It is well known that an important issue neglected by the classical information theory is the bursty nature of the packet arrivals in real communication systems [1]. Even in multiuser environment, the classical approach essentially assumes that all the information exists in the sender nodes and the objective of a communication network is to deliver the information with a desired rate to the given destination. On the other hand, in real networks the data arrives in bursts, and in many situations, including [3, 4, 5], the connections of random arrivals with the network capacity yet remain unexplored.

As an exception to the described situation, the effect of random arrivals was considered in [6, 7, 8], and the stability of the network under the the slotted ALOHA medium access is analyzed. In a network with random arrivals, stability ensures that every generated packet arrives at its destination in finite time, and it can be considered as the loosest kind of delay constaint. Although, we have stability results in [6, 7, 8] for some particular networks with the ALOHA protocol, it is not known how one can achieve stability in an arbitrary wireless network with arbitrary protocols.

In this paper, we analyze the stability conditions of a wireless network with random arrivals modeled by a stationary and ergodic stochastic process. By using a generalized channel model, we also consider random reception errors and simultaneous multiple packet receptions. Furthermore, we characterize a particular class of protocols which achieve every rate inside the stability region. Our major finding is an equivalent condition for stability in a network with non-bursty arrivals in which the packets are assumed to exist in source nodes in infinite number.

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II. Network Model

In this paper, we consider fixed networks *i.e.,* node mobility is not considered. A network is modeled as an undirected graph $G = (\mathcal{N}, E)$, where vertices $\mathcal{N}$ represent the nodes, and there exists a link $(a, b) \in E$ if node $b \in \mathcal{N}$ is within the transmission range of node $a \in \mathcal{N}$. Graph models can be considered as an approximation of the planar models used in the literature [3, 4, 5]. Assuming the transmission power of nodes are identical, one can obtain the graph corresponding to a planar network by finding out neighboring nodes (Figure 1).

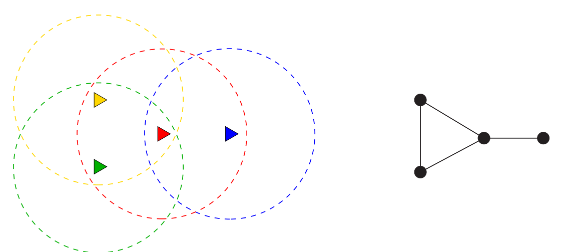

Fig. 1: Nodes and transmission ranges in a planar network are shown on the left. The right figure shows the corresponding graph.

Wireless channels experience strong time-dependent fading and the communication quality is determined by the interfering transmissions and the obstacles in between nodes. Because of these reasons, in general, the connectivity between two wireless nodes can not be represented as a fixed Boolean function. We will consider the reception errors caused by channel imperfectons in a probabilistic way as described in the following section.

II.A. Reception Model

It is assumed that the time is divided into fixed length slots, and transmission of one packet takes a single slot. The nodes can not transmit and receive at the same time. Each node can transmit at most one packet at a time, and simultaneous transmission of the same packet to multiple neighbors (*i.e.,* broadcast) is not allowed. In each slot, a node can correctly receive and decode a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *Receiver MPR Matrix* $\mathbf{C}$. The entries of the MPR matrix $\mathbf{C}$ are given as

$$C_{n,k} = P[k \text{ packets are received} \mid n \text{ packets are transmitted in the neighborhood}].$$

The receiver MPR matrix is defined and given as

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{1}$$

This channel model is general enough to model time-varying fading channels (or in general channels with errors) where fading is i.i.d. from slot to slot and the transmitter does not have channel side information. This channel model also accepts the conventional collision channel as a special case. The corresponding MPR matrices for the channel with errors $\mathbf{C}_p$, the conventional collision channel $\mathbf{C}_1$ are given by

$$\mathbf{C}_p = \begin{pmatrix} 1-p & p & & & \\ 1 & 0 & 0 & & \\ 1 & 0 & 0 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \mathbf{C}_1 = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 0 & & \\ 1 & 0 & 0 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where p is the correct reception probability.

Note that this MPR formulation is quite simplistic and an ideal one. In reality, the reception probabilities of the packets transmitted from different neighbors may be different. The reception probabilities also depend on out-of-neighborhood interference, and since the interference changes from slot to slot, MPR matrix provides only an approximation of the reception probabilities.

II.B. Packet Generation

Consider an ad hoc network composed of N nodes which are represented using the set $\mathcal{N}$. Every node in the network has an infinite buffer for holding its packets. At time t the number of packets at node $s \in \mathcal{N}$ with destination $d \in \mathcal{N}$ is denoted by $n_t^{(s,d)}$. The total number of packets in the buffer of node s is $n_t^{(s)} = \sum_{d \in \mathcal{N}} n_t^{(s,d)}$.

The slot $t \in Z^+ = \{0, 1, 2, \cdots\}$ is defined as the half-open interval $(t, t+1]$. The network starts operation at time $t = 0$ where buffer of each node is empty. In slot $t-1$, node $s \in \mathcal{N}$ generates $\beta_t^{(s,d)}$ packets randomly which are destined for node $d \neq s$. As a convention we define $\beta_t^{(s,d)} = 0, \ \forall t$ when $s = d$. The packets generated in slot $t-1$ are placed in the node buffers at time t, and these packets can be transmitted only in slots $\{t, t+1, \cdots\}$. We suppose $\beta_t^{(s,d)}$ is a strictly stationary and ergodic stochastic process with finite mean. The *packet generation rate matrix* is given as $\lambda = [\lambda_{(s,d)}]_{N \times N}$ such that

$$\lambda_{(s,d)} \triangleq \mathbb{E}\{\beta_t^{(s,d)}\} = \lim_{t \to \infty} t^{-1} \sum_{m=1}^{t} \beta_m^{(s,d)} \quad \text{a.s.} \tag{2}$$

In this paper, we do not consider the problem of multicast *i.e.,* each generated packet is destined for a single node. After a transmission, if a packet is succesfully received by the next node on the route of the packet, it is removed from the transmitter's buffer and put in the receiver's queue. A packet is removed from the network if and only if it is correcly received by its destination.

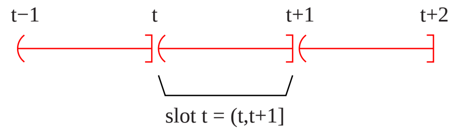

Fig. 2: In the slot t, the transmissions start at time instant $t+$. At time $t+1$, the transmissions end, the correctly transmitted packets are removed from the transmitter nodes' buffers, and are either removed from the network (if the receiver is the final destination of the packet) or placed in the receiver node's buffer. The newly generated packets in slot t are also placed into the buffers at time $t+1$.

II.C. Protocols and Stability

The packets are identified by their destinations, otherwise they are identical. For a given node $a \in \mathcal{N}$, let $V(a) = \{b \mid (a, b) \in E\}$ denote the neighboring nodes of a. Suppose $(a, b, d) \in \mathcal{N}^3$ triple represents a packet transmission from node a to a neighboring node $b \in V(a)$ such that the destination of the packet is $d \in \mathcal{N} \backslash \{a\}$. Define $T = \{(a, b, d) \in \mathcal{N}^3\}$ as the set of all triples, and let $\mathcal{P}(T)$ denote the power set of T.

We define a *network protocol* (or, simply, a protocol) as follows. Suppose $\{U_t \mid t \in Z^+\}$ is a set of i.i.d. uniform[0,1] random variables. A network protocol is specified by a set of deterministic functions $f_t, t \in \{0, 1, \cdots\}$, such that the value of f_t shows the transmissions and receptions which take place in slot t. Namely, f_t is a deterministic function which maps "all information up to time t" to one of the possible transmissions in the set $\mathcal{P}(T)$, *i.e.,*

$$f_t \ : \ (n_i^{(s,d)}, \beta_i^{(s,d)}, U_i \mid (s, d) \in \mathcal{N}^2, \ i = 0, 1, \cdots, t) \longrightarrow \mathcal{P}(T).$$

We also suppose the following is satisfied,

(1) if $(a, b, d) \in f_t(\cdot)$ then $b \in V(a)$ and $d \in \mathcal{N} \backslash \{a\}$.
(2) if $(a, b, d) \in f_t(\cdot)$ then $n_t^{(a,d)} > 0$.
(3) if $(a_i, b_i, d_i) \in f_t(\cdot), i = 1, 2$ and $(a_1, b_1, d_1) \neq (a_2, b_2, d_2)$ then $a_1 \neq a_2$.

A network protocol is a general description of a combination of MAC and routing protocols.

Definition For a network $G = (\mathcal{N}, E)$ whose protocol is specified, node $s \in \mathcal{N}$ is called *substable* if

$$\lim_{\theta \to \infty} \lim_{t \to \infty} \inf Pr\{n_t^{(s)} < \theta\} = 1. \tag{3}$$

A node is called *unstable* if it is not substable. A network is called stable if all nodes in the network are substable; it is called unstable otherwise.

A packet generation rate λ is called *achievable* if there exists a protocol that makes the network stable. The *stability region* of a network is the closure of the set of all achievable rates. In the following we will show that the stability region is well defined, in the sense that the achievability does not depend on the specifics of packet generation process but only its mean λ.

Theorem II.1 *The stability region is well defined. Specifically, for a network $G = (\mathcal{N}, E)$, if a rate λ is in the interior of the stability region, then there exists a protocol which makes the network stable for any packet generation process with rate λ.*

Proof See Section IV.

For some simple networks the stability region can be obtained easily. For instance, in a network with collision channel, if all N nodes are connected to each other (*i.e.,* the network is fully connected), then the stability region can be shown to be

$$\{\lambda_{(s,d)} \mid \sum_{(s,d)\in\mathcal{N}^2} \lambda_{(s,d)} \leq 1,\ \lambda_{(s,d)} \geq 0,\ \lambda_{(s,s)} = 0,\ (s,d) \in \mathcal{N}^2\},$$

and a simple time division multiple access strategy achieves all rates in the interior of this region. However, for more complicated network topologies it may be very hard (or even practically impossible) to find the stability region.

III. A Class of Capacity Achieving Protocols

In a general data network the packet delays significantly depend on the applied routing and medium access protocol [2]. The routing and the associated flow control algorithms choose the routes dynamically in a way that the packets can be sent reliably through the uncongested regions of the network with minimum delay. Similarly, in a wireless environment, the medium access protocol can be designed to aid such kind of congestion control. Although being attractive in terms of packet delay, such protocols make decisions dynamically as a function of the current network state, and they typically require further up-to-date knowledge about the congested regions of the network. Nevertheless, it might be expected that application of such protocols provide larger network capacities.

In this section, we will prove that this expectation is wrong, and as far as the network stability is concerned dynamic protocols does not provide any improvement. In the following, we will retrict our attention to the class of *static* network protocols, and provide an achievable rate region for such protocols. An important advantage of the presented achievability condition is that it only depends on the mean of the arrival process, and hence allows us to prove Theorem II.1. Static protocols are simpler to implement in some ways, and although their delay performance is not necessarily optimal, it will be shown that every rate inside the capacity region can be achieved using static protocols.

III.A. Static Network Protocols

In a static protocol, every packet is assigned a fixed route randomly, once it is generated. For medium access, in each slot, one of the possible transmission settings is applied in an i.i.d. way from slot to slot. Before introducing the routing and medium access schemes precisely, we will give some definitions.

Definition A *routing matrix* $\mathbf{R} = [r_{(a,b,d)}]_{N\times N\times N}$ provides a set of probability measures such that $r_{(a,b,d)} \geq 0$ denotes the probability that the node $b \in V(a)$ is chosen as the next relay node of a packet whose route goes through node a and whose final destination is node d. We suppose the following conditions are satisfied:

(1) $\sum_{b\in V(a)} r_{(a,b,d)} = 1$ for $d \neq a$,

(2) $r_{(a,b,d)} = 0$ if $b \notin V(a)$,

(3) $r_{(a,a,a)} = 1$ and $r_{(a,b,a)} = 0,\ \forall b \in \mathcal{N}\backslash\{a\}$.

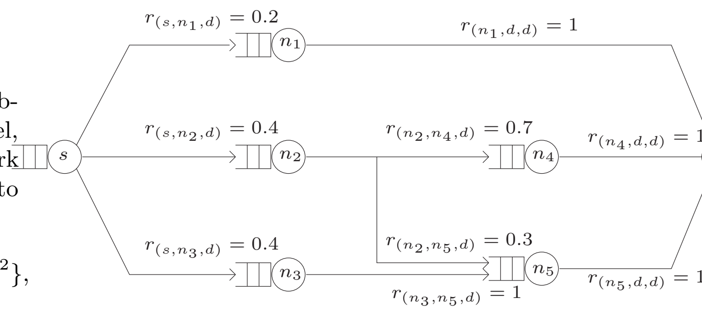

Fig. 3: An example routing pattern for the packet traffic from node s to node d.

(4) For each source destination pair (s,d) there exists a route from s to d with non-zero probability, *i.e.,* there exists $(\varphi_1,\varphi_2,\cdots,\varphi_m) \in \mathcal{N}^m$ such that $r_{(s,\varphi_1,d)}r_{(\varphi_1,\varphi_2,d)}\cdots r_{(\varphi_m,d,d)} > 0$.

(5) Routes do not contain loops, *i.e.,* for all $s,d \in \mathcal{N}$, $s \neq d$, there does not exist $\varphi_1,\varphi_2,\cdots,\varphi_m$ such that $r_{(s,\varphi_1,d)}r_{(\varphi_1,\varphi_2,d)}\cdots r_{(\varphi_m,s,d)} > 0$.

The routes of packets in a static network protocol is determined by a routing matrix $\mathbf{R}$. Now, consider a source destination pair (s,d). Denote the set of all possible routes from s to d with

$$\mathcal{R}^{(s,d)} \triangleq \{(s,\varphi_1,\varphi_2,\cdots,\varphi_m,d) \in \mathcal{N}^{m+2} \mid r_{(s,\varphi_1,d)}r_{(\varphi_1,\varphi_2,d)}\cdots r_{(\varphi_m,d,d)} > 0, m \in \mathcal{Z}^+\}. \quad (4)$$

In a static network protocol, once a packet generated at node s to be send to node d, its route is assigned as one of the possible routes in $\mathcal{R}^{(s,d)}$ such that the route $\varphi \triangleq (s,\varphi_1,\varphi_2,\cdots,\varphi_m,d) \in \mathcal{R}^{(s,d)}$ is selected with probability

$$Pr\{\varphi\} = r_{(s,\varphi_1,d)}r_{(\varphi_1,\varphi_2,d)}\cdots r_{(\varphi_m,d,d)}.$$

As an illustrative example, consider the network in Figure 3. For the source destination pair (s,d), possible routes are given by

$$\mathcal{R}^{(s,d)} = \{(s,n_1,d),(s,n_2,n_4,d),(s,n_2,n_5,d),(s,n_3,n_5,d)\},$$

and the probability of choosing each route is given as $\{0.2, 0.28, 0.12, 0.4\}$, respectively.

For a given routing matrix, define $l_{(a,b,s,d)}$ as the probability of a packet generated at s destined for d to pass through the link from a to b on its route. More explicitly,

$$l_{(a,b,s,d)} \triangleq \sum_{\varphi\in\mathcal{R}^{(s,d)}} Pr\{\varphi\} 1_{\{(s,\varphi_1),(\varphi_1,\varphi_2),\cdots,(\varphi_m,d)\}}(a,b).$$

The packets in a network employing a static network protocol is identified by their routes, all packets with the same route are considered identical. Next, we will introduce the medium access in a static network protocol, given the routing matrix $\mathbf{R}$ and the packet generation rate matrix λ. Suppose the pair $a\xrightarrow{\text{tx}}b$ represents a packet transmission from node a to node b. A *schedule* S is defined as a set of ordered pairs $S = \{a\xrightarrow{\text{tx}}b \mid a \in \mathcal{N}, b \in V(a)\}$ which shows all scheduled transmissions in a slot. As a convention, a transmitter can not be repeated twice in S, since transmitters can only transmit one packet at a time.

The medium access scheme in a static network protocol is specified by a probability measure $\mathbf{M}$ on the set of all possible schedules. At the beginning of each slot one of the possible schedules is selected randomly in an i.i.d. way, and that particular schedule is applied in that slot.

The queueing discipline in each node is as follows. In a slot t, if the schedule S is applied and $a \xrightarrow{\text{tx}} b \in S$, node a flips a random coin and chooses to transmit a packet with route φ with probability

$$Pr\{\text{A packet with route } \varphi \text{ is chosen} \mid a \xrightarrow{\text{tx}} b\} \triangleq \frac{\lambda_{(s,d)} Pr\{\varphi\}}{\sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_{(i,j)} l_{(a,b,i,j)}}, \quad (5)$$

where $(a,b) \in \{(s,\varphi_1),(\varphi_1,\varphi_2),\cdots,(\varphi_m,d)\}$ is satisfied. If node a has a packet with route φ in its buffer, the packet is transmitted, otherwise the node stays in the reception mode. The nodes which are not scheduled as transmitters also stay in the reception mode.

In a static network protocol, the events happening consecutively in each slot can be summarized as follows

- At the beginning of the slot, according to the probability distribution $\mathbf{M}$, a global schedule S is randomly chosen to be applied.
- According to the chosen schedule, every node a which is scheduled to transmit a packet to a node b chooses a route φ randomly with the probabilities given by (5).
- If a scheduled transmitter has a packet with the chosen route in its buffer, it is transmitted during the slot, otherwise the node stays in the reception mode. The nodes which are not specified as transmitters also stay in the reception mode during the slot.
- At the end of the slot, correctly transmitted packets are either removed from the network or moved from transmitter to the receiver's queue. Newly arriving packets are placed in node buffers, a random but fixed route is assigned to each of them according to the routing matrix $\mathbf{R}$.

III.B. A Sufficient Condition for Stability

Next, we'll define the worst-case throughput of a schedule. For this we suppose every node in the network has the same the MPR matrix $\mathbf{C}$. Define

$$C_n \triangleq \sum_{k=1}^{n} k C_{n,k}$$

which is the expected number of correctly received packets given n packets are transmitted. Given S, the number of transmissions in the neighborhood of $b \in \mathcal{N}$ is shown as

$$tx(b) = |\{a \mid a \in V(b),\ c \in \mathcal{N},\ a \xrightarrow{\text{tx}} c \in S\}|.$$

The *throughput matrix* of a schedule S is defined as $\mathbf{T}(S) = [t^S_{(a,b)}]_{N\times N}$ such that

$$t^S_{(a,b)} = \begin{cases} \frac{C_{tx(b)}}{tx(b)} & a \xrightarrow{\text{tx}} b \in S, \text{ and } \nexists\, c \text{ such that } b \xrightarrow{\text{tx}} c \in S \\ 0 & \text{otherwise} \end{cases}$$

In a slot in which the schedule S is applied, the entry (a,b) in the throughput matrix $\mathbf{T}(S)$ gives the probability of successful transmission from a to b given every neighbor of b which are scheduled to transmit indeed transmits.

The throughput matrix of a static network protocol is defined as

$$\mathbf{T} = \mathbb{E}_{\mathbf{M}}\{\mathbf{T}(S)\} = [t_{(a,b)}]_{N\times N},$$

where the expectation is taken over the probability measure on the set of schedules. In a network, in which a static network protocol is applied, given that all scheduled transmitters transmit all the time, the entry (a,b) of the throughput matrix $\mathbf{T}$ shows the probability of a successful transmission from a to b.

Theorem III.1 *Consider a network $G = (\mathcal{N}, E)$ with an arbitrary packet generation process with rate matrix λ. If a static network protocol satisfies the following*

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_{(i,j)} l_{(a,b,i,j)} < t_{(a,b)}, \quad \forall a,b \in \mathcal{N}, \quad (6)$$

then the network is stable under that protocol.

In the proof of the theorem, we show that packet choosing a particular route is stable for every route from any source to any destination. For this we basically specify a transmission scheme (heavy loaded transmissions) and then use techniques developed by Loynes for continuous time $G/G/1$ queueing systems to prove the stability of the suboptimal system. Later, showing that the suboptimal system is stochastically dominating the original system, we will finish the proof.

III.C. Proof of Theorem III.1

Suppose there exists an $(s,d) \in \mathcal{N}^2$ pair such that $\lambda_{(s,d)} > 0$. In the following, we will concentrate on a route $\varphi \triangleq (s,\varphi_1,\varphi_2,\cdots,\varphi_m,d) \in \mathcal{R}^{(s,d)}$ and show that the packets going through the route φ are stable under condition (6).

Now, we will introduce the suboptimal transmission scheme, which will be called *heavy loaded transmissions*. In a slot t, according to the particular schedule S applied, if node a is scheduled to transmit to node b, node a flips a random coin and chooses to transmit a packet on the route φ with probability (5). Previously, we assumed that if the transmitter node doesn't have a packet with route φ then it stays in the reception mode. Here, we will assume that if no such packets exists in the queue of node a, node a transmits a dummy packet which does not contain any information and will be discarded just after the transmission.

In slot t, ω^0_t denotes the number of packets at node s which has chosen the route φ. In slot t, let also $\omega^i_t, i \in \{1,\cdots,m\}$ denote the number of packets at node φ_i which which has chosen the route φ. Let S^0_t represent the number of packets arrived in slot t which has chosen route φ, and $S^i_t, i > 0$ denote the number of packets which is correctly transmitted by node φ_{i-1} to φ_i in slot t. Define $B(p)$ as a binomial random variable such that $Pr\{B(p) = 1\} = p$ and $Pr\{B(p) = 0\} = (1-p)$. S^0_t is a strictly stationary stochastic process and $S^0_t = \sum_{i=1}^{\beta^{(s,d)}_t} X^i_t$ such that $X^i_t, t, i \in \mathcal{Z}^+$ is i.i.d with $X^u_t \overset{d}{=} B(Pr\{\varphi\})$. It can be checked that $\mathbb{E}\{S^0_t\} = \lambda_{(s,d)} Pr\{\varphi\}$ for all t. Define also the event T^u_t showing a successful transmission from S^u_t, $u = \{0,1,\cdots,m\}$. It can be seen that ω^i_t evolves as follows

$$\omega^i_{t+1} = [\omega^i_t - T^i_t]^+ + S^i_{t+1}, \quad (7)$$

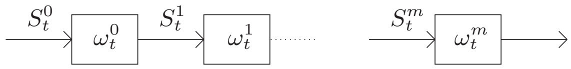

Fig. 4: Series of queues

where

$$S_{t+1}^{i+1} = \omega_t^i - [\omega_t^i - T_t^i]^+ = \omega_t^i - \omega_{t+1}^i + S_{t+1}^i, \quad i > 0. \quad (8)$$

In case of dummy packet transmissions, one important observation is that

$$T_t^i \stackrel{d}{=} B(t_{(a,b)} Pr\{\text{A packet with route } \varphi \text{ is chosen} \mid a \xrightarrow{\text{tx}} b\}), \quad (9)$$

where the random vectors $(T_t^0, T_t^1, \cdots, T_t^m)$ are i.i.d. for different t, while for a fixed t, $T_t^0, T_t^1, \cdots, T_t^m$ are dependent in general. Furthermore, equation (9) and condition (6) implies that, for any $(a,b) \in \{(s,\varphi_1), (\varphi_1,\varphi_2), \cdots, (\varphi_m, d)\}$,

$$\begin{aligned} \mathbb{E}\{S_t^0\} &= \lambda_{(s,d)} Pr\{\varphi\} \\ &= \frac{\lambda_{(s,d)} Pr\{\varphi\}}{\sum_{i=1}^N \sum_{j=1}^N \lambda_{(i,j)} l_{(a,b,i,j)}} \sum_{i=1}^N \sum_{j=1}^N \lambda_{(i,j)} l_{(a,b,i,j)} \\ &< Pr\{\text{A packet with route } \varphi \text{ is chosen} \mid a \xrightarrow{\text{tx}} b\}\ t_{(a,b)} \\ &= \mathbb{E}\{T_t^i\}, \quad i = \{0, 1, \cdots, m\}. \end{aligned} \quad (10)$$

Let's make a change of variables, define

$$\tilde{\omega}_t^i \triangleq \omega_t^i - S_t^i, \quad (11)$$

in which case

$$\begin{aligned} \tilde{\omega}_{t+1}^i &= [\tilde{\omega}_t^i + S_t^i - T_t]^+, \quad (12) \\ S_{t+1}^{i+1} &= \tilde{\omega}_t^i - \tilde{\omega}_{t+1}^i + S_t^i \quad (13) \end{aligned}$$

follow from (7) and (8). We will also define $U_t^i = S_t^i - T_t^i$, and consequently

$$\tilde{\omega}_t^i = [\tilde{\omega}_t^i + U_t^i]. \quad (14)$$

Using induction on (14), it can be observed that

$$\tilde{\omega}_t^i = [\sup_{1 \le r \le t-1} \sum_{r}^{t-1} U_k^i]^+. \quad (15)$$

As a result of (10), $\mathbb{E}\{U_t^0\} < 0$ follows, and we can establish the stability of $\tilde{\omega}_t^0$ using the following lemma.

Lemma III.2 (Loynes, Lemma 1) *Let the random variables ω_t $(t \ge 1)$, be related by the transformation*

$$\omega_{t+1} = f(\omega_t, U_t), \quad (16)$$

where $\{U_t \mid -\infty < t < \infty\}$ is a stationary sequence and $\omega_1 = 0$. Suppose in addition that $f(x,y)$ is monotonic increasing and continuous from the left in x, and non-negative. Then there exists a stationary sequence of random variables $\{M_t \mid -\infty < t < \infty\}$, satisfying

$$M_{t+1} = f(M_t, U_t), \quad (17)$$

such that the distribution function of ω_t tends monotonically to that of M_0 as t tends to ∞. Furthermore, if $\hat{\omega}_1$ is another sequence satifying (16) such that $\hat{\omega}_1 > 0$, then $\hat{\omega}_t \ge \omega_t,\ \forall t$.

Returning to the particular problem at hand, we see (by setting $f(x,y) = [x+y]^+$) that the transformation of $\tilde{\omega}_t^0$ defined by (15) is of just the type considered in the lemma. It follows that there exists a stationary sequence $\{M_t \mid -\infty < t < \infty\}$ satifying

$$M_{t+1} = [M_t + U_t^0]^+,$$

with the minimality property already described, and that the distribution function of $\{\tilde{\omega}_t^0\}$ tends monotonically downward to the distribution of M_0. Using (15), we see that

$$M_t = [\sup_{r \ge 1} \sum_{k=1}^r U_{t-k}^0]^+.$$

Thus M_0 is finite if and only if $\sup \sum_1^r U_{-k}^0 < \infty$, which in turn true if and only if $\limsup \sum_1^r U_{-k}^0 < \infty$. Using these observations, we state the following lemma which is an immediate extension of the arguments in section 2.32 and Theorem 7 of Loynes [9].

Lemma III.3 *In case of dummy packet transmissions, $(S_t^0, T_t^0, T_t^1, \cdots, T_t^m)$ forms a strictly stationary stochastic process, and as a result of $\mathbb{E}\{U_t^i\} = \mathbb{E}\{S_t^0\} - \mathbb{E}\{T_t^i\} < 0$, for all $i \ge 0$, $\tilde{\omega}_t^i$ converges in distribution to a proper random variable as $t \to \infty$.*

Lemma III.4 *The following is satisfied for the number waiting packets in the queues for $i = 0, 1, \cdots, m$:*

$$\frac{\tilde{\omega}_t^i}{t} \to 0 \text{ as } t \to \infty, \text{ almost surely.}$$

Therefore, the average number of correcly transmitted packets to the final destination converges to the arrival rate with probability one.

As a result of the previous lemmas, the stability of the network with dummy transmissions can be easily established. Stability of $\tilde{\omega}_t^i$ implies stability of ω_t^i. Moreover, stability is preserved when we add stable sequences componentwise. Therefore, in case of dummy packet transmissions, we can observe that all nodes in the network are stable given each stream is stable.

Next, we'll state that the network is stable without dummy packet transmissions.

Lemma III.5 (Stochastic Dominance) *The queue lengths with dummy packet transmissions is stochastically larger than the queue lengths in a network without dummy packet transmissions. Therefore, stability of the network with dummy packet transmissions implies stability of the network.*

IV. The Capacity Region is Well Defined

In this section, we will prove that if an achievable rate is in the interior of the capacity region, then there exists a static network protocol which achives the same rate. First, we will state a rather technical lemma, then prove Theorem II.1.

Lemma IV.1 *If the network is stable with a network protocol, then $\forall s \in \mathcal{N}$,*

$$\frac{n_t^{(s)}}{t} \xrightarrow{\mathcal{P}} 0, \frac{\mathbb{E}\{n_t^{(s)}\}}{t} \to 0, \text{ as } t \to \infty. \quad (19)$$

Here $\xrightarrow{\mathcal{P}}$ denotes convergence in probability.

Proof *(Theorem II.1)* Suppose for a given network, a rate matrix $\lambda = [\lambda_{(s,d)}]_{N\times N}$ is in the interior of the capacity region. Then there exists a $\epsilon > 0$ such that a network protocol which achieves $\lambda + \epsilon 1_{N\times N}$. Note that $1_{N\times N}$ is a $N \times N$ matrix with all non-diagonal entries 1 and the rest 0.

In order to prove such a result, we will contruct a static protocol which achieves the same rates regardless of the specifics The measure on schedules which will be used for the medium access is easy to describe: Pick a random number from 1 to n uniformly, and use a schedule which is applied in that particular slot with corresponding probabilities. The rest is to be written!

V. An Equivalent Formulation of the Capacity Region

In this section, we look at a more classical network model in which the packets exists in all network nodes, and the objective is to find the rates with which packets can be sent to their destinations with probability one. Our major contribution is to make a connection between two different approaches by showing that the existing packets can be sent with rate λ if and only if a network with bursty arrivals with rate λ can be stabilized.

V.A. A Network Model with Existing Packets

Suppose that the network is again modeled as a graph, and the transmissions are governed by an MPR matrix. However, there is no random arrivals and we suppose there exists an infinite number of packets at any node of the network to be sent to any other node in the network. For this network model, we can define a network protocol similarly (the only difference is that in this case $f_t, t = 1, 2, \cdots$ is not a function of $\beta_t^{(s,d)}$). At time t, $W_{(s,d)}(t)$ denotes the number of packets correctly received by node d whose source is node s. In accordance with the definition in[Gupta, Tse], a rate λ is called *feasible* if there exists a network protocol such that

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} W_{(s,d)}(t) \geq \lambda_{(s,d)},\ \forall (s,d) \in \mathcal{N}^2, \qquad (20)$$

is satisfied with probability one. Using these definitions, we can state the following theorem.

Theorem V.1 *The closure of the set of all feasible rates in a network with existing packets is exactly the same as the capacity region of the same network with bursty arrivals.*

V.B. Proof of Theorem V.1

If a rate λ is inside the capacity region, then there exists a static protocol achieving λ. When the source, introduces the packets according an i.i.d. Bernoulli process to the network with probability generation probabilities given by λ, then according to Lemma III.4 the total number of nodes in the buffers tend to 0, and hence the packets are transmitted with rate λ almost surely, which means λ is feasible.

Conversely, suppose rate λ is feasible. Since $W_{(s,d)}(t)$ is non-negative, the following holds as a result of Fatou's lemma, $\forall\ (s,d) \in \mathcal{N}^2$

$$\liminf_{T\to\infty}\ \mathbb{E}\{\frac{1}{T}\sum_{t=1}^{T} W_{(s,d)}(t)\} \geq \mathbb{E}\{\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} W_{(s,d)}(t)\} \geq \lambda_{(s,d)}, \qquad (21)$$

Equation (21) allows us to use the protocol construction given in section IV which basically says that in case of (21), there exists a protocols achieving rates aribitrarily close to λ. Therefore, feasibility implies achievability. ∎

References

[1] A. Ephremides and B. Hajek, "Information theory and communication networks: an unconsummated union," *IEEE Trans. on Inf. Theory,* vol. 44, no. 6, pp. 2416-2434, Oct. 1998.

[2] D. P. Bertsekas and R. Gallager. "Data Networks". *Prentice-Hall*, 1992.

[3] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. on Inf. Theory*, vol. 46, pp. 388-404, March 2000.

[4] M. Grossglauser and D. Tse, "Mobility Increases the capacity of wireless ad hoc networks," *IEEE Infocom,* 2001.

[5] S. Toumpis and A. J. Goldsmith, "Some Capacity Results for Ad Hoc Networks," *Allerton Conference on Communication, Control and Computing*, 2000.

[6] B. S. Tsybakov and V. L. Bakirov, " Packet transmission in radio networks," *Problemy Peredachi Informatsii,* vol. 21, no. 1, pp. 80-101, 1985.

[7] V. Anantharam, "The stability region of the finite-user slotted ALOHA protocol," *IEEE Trans. on Inf. Theory,* vol. 37, no. 3, pp. 535-540, May 1991.

[8] Luo Wei and A. Ephremides, "Stability of N interacting queues in random-access systems," *IEEE Trans. on Inf. Theory,* vol. 45, no. 5, pp. 1579-1587, July 1999.

[9] R. M. Loynes, "The stability of a queue with non-independent inter-arrival and service times," *Proc. Camb. Philos. Soc.*, vol. 58, pp. 497-520, 1962.