A Characterization of Delay Performance of Cognitive Medium Access

Shanshan Wang, Junshan Zhang, and Lang Tong

***Abstract*—We consider a cognitive radio network where multiple secondary users (SUs) contend for spectrum usage, using random access, over available primary user (PU) channels. Our focus is on SUs' queueing delay performance, for which a systematic understanding is lacking. We take a fluid queue approximation approach to study the steady-state delay performance of SUs, for cases with a single PU channel and multiple PU channels. Using stochastic fluid models, we represent the queue dynamics as Poisson driven stochastic differential equations, and characterize the moments of the SUs' queue lengths accordingly. Since in practical systems, an SU would have no knowledge of other users' activities, its contention probability has to be set based on local information. With this observation, we develop adaptive algorithms to find the optimal contention probability that minimizes the mean queue lengths. Moreover, we study the impact of multiple channels and multiple interfaces on SUs' delay performance. As expected, the use of multiple channels and/or multiple interfaces leads to significant delay reduction. Finally, we consider packet generation control to meet the delay requirements for SUs, and develop randomized and queue-length-based control mechanisms accordingly.**

***Index Terms*—Delay analysis, fluid approximation, cognitive radio networks.**

I. INTRODUCTION

IN a hierarchical overlay cognitive network [1], a secondary user (SU) communicates opportunistically by exploiting spectrum "white space" left temporarily by primary users (PUs). As a result, transmissions of an SU is limited by the stochastic nature of PUs. An SU hoping to run certain applications (e.g. VoIP or streaming) would like to know what kind of rate and delay a secondary network can provide. In the same token, an owner of a secondary network would like to attract potential users by advertising a certain level of quality of service (QoS) assurance.

Characterizing the delay of a cognitive network is challenging. Specifically, the delay of an SU is affected by not only its own buffer and traffic properties, but also PUs' traffic characteristics, other competing SUs, and access policy of SUs. These interacting factors make delay analysis often analytically intractable, and only a limited number of results have been reported in the literature (see e.g., [2]–[4]).

We analyze in this paper the delay performance in a cognitive radio network, where SUs contend for channels using an Aloha-based random access policy. In particular, an SU senses a channel owned by a PU and transmits only if the PU channel is idle. We model the PU's traffic generation as an ON-OFF process where the PU generates data only during the ON periods. For SUs, we assume that they generate data packets in each slot according to a Poisson distribution. Based on stochastic fluid queue theory, we model the system dynamics by using *Poisson driven stochastic differential equations* (PDSDE), and analyze the steady state queue lengths of SUs accordingly. To facilitate tractability, we focus on the light traffic regime where the traffic intensity is low, as is often the case for delay analysis of buffered Aloha, e.g., [5], [6]. We consider the homogeneous case where the arrival rates of SUs are the same, and characterize the moments of the random queue lengths of SUs, for cases with a single PU channel (SCH) and multiple PU channels (MCH). Clearly, these moments provide critical statistical information about SUs' queueing length distribution. We also examine the impact of the PU traffic on SUs' queue lengths and the gain of using multiple PU channels. Adaptive algorithms, based on local information only, are developed to find the optimal contention probabilities that achieve the minimum mean queue lengths.

Next, we explore the gain of using two interfaces per SU, i.e., each SU is equipped with two interfaces (radios). Accordingly, each SU can sense two channels at a time and thus transmit on up to two channels, as long as the PU channels are idle and no contention collisions occur. Our analysis and numerical examples corroborate the intuition that the usage of two interfaces can greatly improve the delay performance by decreasing the mean queue lengths of SUs.

Furthermore, it is of equal importance to consider the scenario where stringent delay requirements are imposed on the SUs. In such a scenario there exists a maximum amount of traffic accommodable, necessitating traffic control. In this study, we consider packet generation control. As representative approaches, we develop two control mechanisms, one randomized and the other based on the queue lengths of the SUs.

The approach adopted in this paper originates from the early work of Liu and Gong who studied the delay performance of priority queues using fluid models [7]. Given the access structure of a hierarchical cognitive network, the problem of queueing analysis indeed resembles that of the priority queue problem. There are, however, nontrivial differences arising

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from cognitive radio specific applications. In particular, the problem considered in [7] arises from centralized scheduling of high and low priority queues, whereas, in this paper, we consider multiple SUs competing for transmission opportunities by random access. This random access to the PU channels gives rise to the coupling across SUs' queue dynamics, which was not the case in [7] since only one low priority flow was considered there. In addition, in contrast to [7] where the single low priority flow receives a constant service rate whenever the buffer of the high priority flows is empty, in our study, SUs receive randomly arrived packets. As a result, the number of backlogged SUs is time-varying, and the service rate is random. Besides the work in [7], the delay performance of a multi-hop wireless ad hoc network was studied in [8], where *diffusion approximation* was used to characterize the average end-to-end delay. In [9], WLANs with access points connecting a fixed number of users in the presence of HTTP traffic was considered. A processor sharing queue with state-dependent service rate was used to model the system and analyze the mean session delay. In [10], queueing delay at nodes in an IEEE 802.11 MAC-based network was analyzed, where each node was modeled as a discrete time $G/G/1$ queue. Delay analysis for buffered Aloha was also studied (see [5], [6], [11]–[13] and references therein). In [11] and [13], the approach named "tagged user" was adopted. Specifically, the interfering users/nodes were modeled as "independent" queues in the sense that the analysis was conducted on one particular user, named the "tagged user," while the interference across users was incorporated into the characterization of the service time distribution of this tagged user. Another approach utilizing Markov chains with reduced state space to approximate delay analysis can be found in [5], [6], [12]. Two Markov chains, one for the queueing dynamics at one user, and the other for the system status (i.e., the number of busy users, and/or the identities of the users (empty, busy or blocked)), were employed for characterizing the steady state distributions of the system as well as the delay. It is worth noting that the approximation worked well only for the light traffic regime, as has been pointed out in [5] and [6]. In [3] and [4], a large deviation approach was used to analyze delay characteristics of SUs. Inner and outer bounds on the large deviation rate region were obtained in [4] for a set of SUs with orthogonal sharing of spectrum opportunities. The present paper is an extension of its earlier conference contribution [2] with additional theoretic results, completed proof, and further simulation study.

We have a few more words on fluid models. Fluid approximation is a widely used tool for performance analysis in many fields, including communication networks and control techniques [14], [15]. It can provide a good approximation to the original systems by converting the discrete packets into a continuous fluid and offers greater tractability in analyzing the system performance. We should note that along a different avenue, the *deterministic* fluid model has been developed to analyze queueing systems, where microscopic fluctuations in the original systems are replaced by their mean values (see, e.g.,[16], [17]). For a given random process $G(t)$, the resulted *fluid scale process*, obtained by using the *Functional Law of Large Numbers*, is defined as $\tilde{g}^{\beta}(t) = G(\beta t)/\beta$, i.e., the time and space are scaled by the same factor β for β being large. This deterministic model leads to the application of *ordinary differential equations* (ODE), which is in contrast to the stochastic differential equations we shall use in our context.

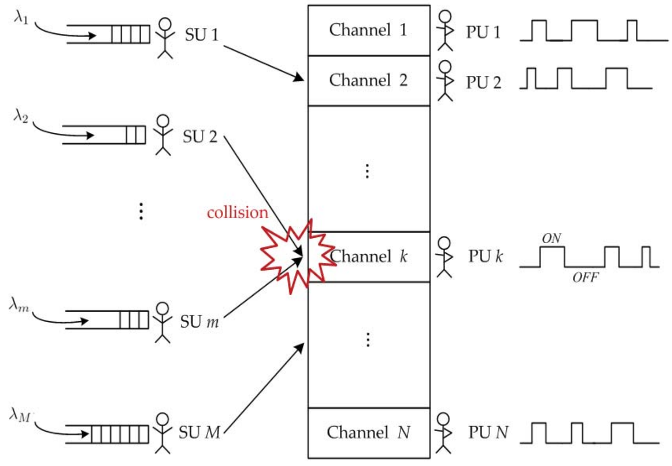


Fig. 1. A cognitive radio network with multiple PUs and SUs.

The rest of the paper is organized as follows. In Section II, we introduce the system model. Fluid flow approximation and PDSDE-based analysis on the single PU channel case are given in Section III. Section IV studies the case with multiple channels, where a variant model considering two interfaces per SU is analyzed in Section IV-D. Packet generation control for SUs under delay requirements is considered in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

Consider a time-slotted (with slot duration normalized to be 1) cognitive radio network with N PU channels and M SUs, where SUs contend for the channels using distributed random access policies when the PUs are inactive, as illustrated in Fig. 1. This model is of interest to many practical scenarios. For example, in a sensor network equipped with cognitive radios, sensors send out measurement data of the environment sporadically and opportunistically over "empty" PU channels.

Without loss of generality, we associate one PU with one channel (one can use a virtual PU to represent the PU activity). The data generation of the PU on channel j can be represented as a continuous-time ON-OFF process $x_j(t), j = 1, 2, \cdots, N$, i.e., when $x_j(t) = 1$ (ON periods), the PU generates data traffic at rate r_j; otherwise, no data is generated. The transmission rate on each channel is normalized to be 1. We are interested in the case where $r_j > 1$ during the ON periods (the case with $r_j \leq 1$ is trivial since the PUs' buffers are always empty). Let $A_{l,j}$ and $S_{l,j}$ denote the lth active and silent period of $x_j(t)$ respectively. We assume that[1] $\{A_{l,j}\}$ are $i.i.d.$ and follow an exponential distribution with $E[A_{l,j}] = 1/\mu_{H_j}$, and that $\{S_{l,j}\}$ are independent from $\{A_{l,j}\}$ and follow an exponential

[1]This continuous-time Markovian model is widely used in the literature to model the primary user's traffic (see, e.g., [3], [18]).

distribution with $E[S_{l,j}] = 1/\lambda_{H_j}$. It is worth noting that since PU's ON/OFF periods are typically much larger than the duration of one slot, we here neglect the edge effect where collisions between PUs and SUs occur when PUs transit from OFF to ON. That is, the probability that PUs generate new data during the middle of a slot and therefore preempt the transmission of SUs is negligible.

We assume that in each slot, each SU generates data packets according to a Poisson distribution with rate λ. In an overlay cognitive radio network, PUs have strict priority over SUs; SUs can transmit only if the channels are unoccupied by PUs. The channel access process is outlined as follows: each SU with backlogged data chooses a channel independently and uniformly at a time to probe. If the channel is sensed to be unoccupied, it contends for the channel with probability p. If the contention is successful (i.e., no other SUs are contending on the channel at the same time), the user then transmits its backlogged data. In fact, this simple random access policy turns to be throughput optimal for small p and when there is only one SU [18]. Note that in practical scenarios, an SU would not have the knowledge of how many backlogged SUs there are, and accordingly we set the contention probability p to be oblivious of backlogged SUs.

For notational convenience, let $H_j(t)$ and $L_i(t)$ denote the queue lengths corresponding to PU j and SU i at time t, respectively, and P_{I_j} be the probability that PU j is idle, i.e., $P_{I_j} = \Pr(H_j(t) = 0)$. In the following, we shall focus on characterizing the queue lengths of SUs. For better reference, we summarize the main notation used in the paper in Table I.

III. Multiple SUs Meet Single PU

A. *Sample Path Description Using Poisson Driven Stochastic Differential Equations*

We first consider the case with a single PU channel. For notational convenience, we drop the subscript j related to the PU parameters. In order to guarantee system stability, we enforce that

$$\lambda < \min\left\{\frac{1}{M}\left(1 - \frac{r\lambda_H/\mu_H}{\lambda_H/\mu_H + 1}\right), \frac{1}{eM}P_I\right\}. \qquad (1)$$

It is worth mentioning that the second term in (1) was established using the idea of "dominant systems," which has been used in characterizing the stability region of interacting queues in random access systems (e.g.,[19], [20]). In our context, the "dominant system" is a system where an SU continues to probe the PU channel regardless of its buffer state (empty or backlogged). Accordingly, the stability region for this system is given by $\lambda < P_I p(1-p)^{M-1}$. Based on [19] and [20], the original system is stable if the dominant system is stable. In other words, the stability region obtained through the dominant system serves as an inner bound to that of the original system.

The queue dynamics of SU i can be written as

$$L_i(d+1) = [L_i(d) + U_i(d) - V_i(d)]^+,$$

where $U_i(d)$ and $V_i(d)$ stand for the arrivals and departures to/from SU i's queue during slot d.

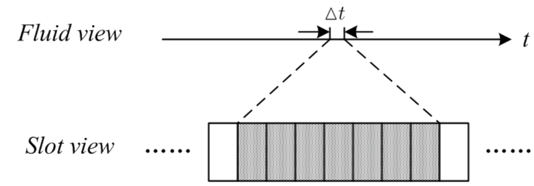


Fig. 2. Fluid approximation of a slotted system.

To facilitate analysis, in the following, we take a macroscopic view on the queue evolution of SUs across multiple slots and use continuous approximation to characterize the dynamics in SUs' activities (as illustrated in Fig. 2). Let $\zeta_i(t)$ be the indicator random variable for the contention of SU i at time t (i.e., when it contends, $\zeta_i(t) = 1$; otherwise $\zeta_i(t) = 0$). The following stochastic differential equation is thus obtained:

$$dL_i(t) = dN_i(t) - (1 - \mathcal{I}_{H(t)})\zeta_i(t)\mathcal{I}_{L_i(t)} \times \prod_{k\in\{1,\ldots,M\}\setminus\{i\}} \left[1 - \mathcal{I}_{L_k(t)}\zeta_k(t)\right] dt, \qquad (2)$$

where $\{N_i(t)\}$ are a set of Poisson counters with rate λ; and $\mathcal{I}_{f(t)}$ stands for the indicator function $\mathbf{1}(f(t) > 0)$.

Furthermore, it is clear that for the PU, its dynamics can be characterized as follows:

$$dH(t) = rx(t)dt - \mathcal{I}_{H(t)}dt. \qquad (3)$$

Observe that (2) forms a set of Poisson driven stochastic differential equations (PDSDE) [21], [22]. Simply put, in a PDSDE, Poisson processes are the driving sources capturing the system dynamics, and this is in contrast to the conventional SDE where the Brownian Motion is used to describe the dynamics in the trajectory of a stochastic differential equation. In general, a PDSDE can be given as

$$z(t) = z(0) + \int_0^t f(z(\sigma),\sigma)d\sigma + \int_0^t g(z(\sigma),\sigma)dN_\sigma, \qquad (4)$$

where N_σ is a Poisson counter. For the sake of completeness, we restate the definition of the solution to the above PDSDE [21].

Definition 1: A function $z(\cdot)$ is a *solution to* (4)*, in the Itô's sense*, if on an interval where N_σ is constant, z satisfies $\dot{z} = f(z,t)$ and if N_σ jumps at t_1, z behaves in a neighborhood of t according to the rule

$$\lim_{\substack{t\to t_1\\ t>t_1}} z(t) = g(\lim_{\substack{t\to t_1\\ t<t_1}} z(t), t_1) + \lim_{\substack{t\to t_1\\ t<t_1}} z(t),$$

and $z(\cdot)$ is taken to be continuous from the left. When this definition is adopted, we can rewrite (4) as

$$dz(t) = f(z,t)dt + g(z,t)dN_\sigma(t).$$

Based on the properties of PDSDE [21], it can be shown that for $n \geq 2$,

$$dL_i^n(t) = nL_i^{n-1}(t)dL_i(t) + \sum_{k=2}^{n}\binom{n}{k}L_i^{n-k}(t)dN_i(t).$$

It follows that the moments of $L_i(t)$ in the steady state satisfy the following recursive equation[2]:

$$nE[L_i^{n-1}F] - \sum_{k=1}^{n} \binom{n}{k} E[L_i^{n-k}]\lambda = 0, \tag{5}$$

where

$$F = (1 - \mathcal{I}_{H(t)})\zeta_i(t)\mathcal{I}_{L_i(t)} \prod_{k \in \{1,\ldots,M\}\setminus\{i\}} \left[1 - \mathcal{I}_{L_k(t)}\zeta_k(t)\right].$$

B. Moments of SU Queue Lengths

We now start to study in more detail the moments of the queue lengths of SUs based on the above PDSDEs. Recall that SUs can access the channel only when the buffer of the PU is empty. With this observation, we first examine the idle period of the PU P_I. Note that the PU generates data at rate r only during an ON-period, and that the buffer is depleted at rate 1 as long as the queue is nonempty. The sample path description of the PU traffic then satisfies the following PDSDE:

$$\begin{aligned} dx(t) &= (1 - x(t))dN_{H_1}(t) - x(t)dN_{H_2}(t), \\ dH(t) &= rx(t)dt - \mathcal{I}_{H(t)}dt, \end{aligned} \tag{6}$$

where $N_{H_1}(t)$ and $N_{H_2}(t)$ are a pair of Poisson counters driving $x(t)$, with rate λ_H and μ_H respectively. It is not difficult to show that in the steady state

$$P_I = 1 - \frac{r\lambda_H}{\lambda_H + \mu_H}. \tag{7}$$

We then start characterizing the moments of the SU queue lengths. Based on (2), we observe that the M SU queues interact with each other through channel contention. In other words, besides the impact from PU activities, the service time of one SU also depends on other SUs' activities, and it turns out to be a quantity that follows a general distribution which is difficult to determine.

For ease of exposition, we shall focus on the light traffic regime and approximate the SU activities as if they were "weakly coupled" in the sense that the event that one SU is idle (i.e., with no backlogged data) is independent from other SUs being idle. Similar approximations to "decouple" the interacting queues have been made in [11] and [13], among other works. According to the homogeneity assumption, this *idle probability* would be the same across all SUs. Let p_0 be this probability. It is clear that the number of backlogged SUs follows a Binomial distribution with its probability mass function given by

$$P_m = \binom{M}{m}(1 - p_0)^m p_0^{M-m}, \tag{8}$$

where p_0 can be shown to satisfy [13] $p_0 = 1 - \rho$, with $\rho = \frac{\lambda}{\mu}$ and μ being the mean service rate. In the case with a single PU channel, μ can be calculated as

$$\mu = \frac{1}{M}\sum_{m=1}^{M} mp(1-p)^{m-1}P_I P_m = pP_I(1-p_0)(1-p+pp_0)^{M-1},$$

where the characterization is done under the homogeneity assumption and is conditional on the number of backlogged SUs in the system. It follows that

$$p_0 = 1 - \frac{\lambda}{pP_I(1 - p_0)(1 - p + pp_0)^{M-1}}. \tag{9}$$

Now with all related parameters being characterized, we are in a position to calculate the moments of the queue lengths for SUs. Based on (2) and (5), the first two moments of SU i's queue length can be derived as

$$E[L_i] = \frac{\lambda}{-2\lambda + 2\alpha_S}, \quad E[L_i^2] = \frac{\lambda(\lambda + 2\alpha_S)}{6(\lambda - \alpha_S)^2}, \tag{10}$$

where α_S is given by

$$\alpha_S = \sum_{m=1}^{M} p(1 - p)^{m-1}P_m P_I. \tag{11}$$

C. Adaptive Algorithm for Optimal Contention Probability

The analysis above indicates that the contention probability, p, and the idle probability of one SU, p_0, are two key parameters to the characterization of the mean queue lengths of SUs, and thus the delay performance. Intuitively speaking, when p is very small (approaching 0), SUs contend for the channel sporadically, and p_0 is small. On the other hand, when p is very large (approaching 1), all SUs with backlogs contend for the channel almost always, leading to a high contention collision among SUs, which makes the queue lengths increase. It is thus indicated that there exists an optimal value of p, which minimizes the mean queue lengths.

We note that (9) formulates a fixed point equation[3] for the idle probability p_0, and p_0 is in itself an implicit function of the contention probability p, i.e., p is the argument of p_0. Therefore, we obtain the optimal value of p by taking derivative with respect to p on both sides of (9) and setting $dp_0/dp = 0$. After some straightforward calculation, we obtain

$$p = \min\left\{\frac{1}{M(1 - p_0)}, 1\right\}. \tag{12}$$

Intuitively speaking, $M(1 - p_0)$ corresponds to the average number of backlogged users who would contend for channel access. Recall that p_0 is the probability that one SU's queue is empty. Accordingly, stochastic approximation algorithms, based on *local* information only, can be readily developed to find the optimal contention probability[4]. For simplicity, rewrite $\Phi_i(t) \triangleq 1 - \mathcal{I}_{L_i(t)}$. We note that the adaptation of p is based on the update of p_0. It follows that we can devise the following adaptive algorithms to obtain the optimal p. First, we use stochastic approximation to update p_0 as:

$$p_0(t + 1) = \left(1 - \frac{1}{t + 1}\right)p_0(t) + \frac{1}{t + 1}\Phi_i(t + 1). \tag{13}$$

Based on this adaptation, we next derive the adaptive algo-

[2]We drop the time index t as the meaning is clear.

[3]See Lemma 6.1 in Appendix for the proof of the uniqueness on the solution to the fixed-point equation.

[4]Note that the SUs are statistically identical and will adopt the same update procedure.

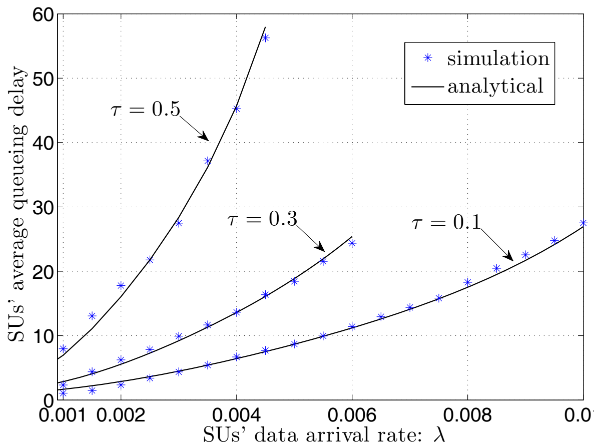


Fig. 3. Average queueing delay of SUs for the case with a single PU channel.

rithm for achieving the optimal p as[5]:

$$p(t+1) = p(t) + \frac{1 - Mp(t)(1-\Phi_i(t+1))}{t + Mp(t)(1-\Phi_i(t+1))}p(t). \quad (14)$$

The convergence of (13) and (14) can be shown by using the standard arguments from stochastic approximation [23].

We now illustrate by numerical examples the above results where the contention probability is set to be the optimal value given by (12), and SU's average queueing delay is transformed from $E[L_i]$ via Little's Law [24]. We also compare the analytical results obtained from the above fluid approximation with the Monte Carlo simulation studies of the underlying system. As shown in Fig. 3 (with $M = 20$ and $r = 1.2$), the queueing delay increases with the arrival rate of SUs, and as the *duty cycle* of $x(t)$, defined as $\tau \equiv \frac{\lambda_H}{\lambda_H + \mu_H}$, increases, the average queueing delay of SUs increases, indicating the impact of the PU traffic on SUs' delay performance. In addition, the simulation and analytical results are shown to match with each other closely.

IV. Multiple SUs Meet Multiple PUs

We next consider the case where there are multiple PU channels, and examine the performance gain therein.

A. Sample Path Description Using Poisson Driven Stochastic Differential Equations

In this case, to keep the system stable, we enforce that

$$\lambda < \min\left\{\frac{1}{M}\left(N - \sum_{j=1}^{N}\frac{r_j\lambda_{H_j}/\mu_{H_j}}{\lambda_{H_j}/\mu_{H_j}+1}\right), \frac{N}{eM}P_I\right\}, \quad (15)$$

where, again, the second term was obtained along the same line as in the previous section using the idea of "dominant system."

Recall that each SU with backlogged data independently chooses one of the N PU channels uniformly at random. Let $\xi_{ij}(t)$ be an indicator random variable denoting that SU i chooses channel j at time t. As in the single PU channel case, we do continuous approximation when characterizing the dynamics of SU activities. The system dynamics can then be written as: for $j = 1, 2, \cdots, N$,

$$dH_j(t) = r_j x_j(t)dt - \mathcal{I}_{H_j(t)}dt, \quad (16)$$

and for $i = 1, 2, \cdots, M$,

$$dL_i(t) = dN_i(t) - \sum_{j=1}^{N}(1-\mathcal{I}_{H_j(t)})\mathcal{I}_{L_i(t)}\xi_{ij}(t)\zeta_i(t) \times \prod_{k\in\{1,\ldots,M\}\backslash\{i\}}\Big[1-\mathcal{I}_{L_k(t)}\xi_{kj}\zeta_k(t)\Big]dt. \quad (17)$$

Again, the coupling across SUs is observed in (17). We next carry out analysis on the moments of SUs' queue lengths by focusing on the light traffic regime as before.

B. Moments of SU Queue Lengths

Along the same line as in the single PU channel case, we first characterize the idle period of PUs. The sample path description for PU j is given by the following PDSDE:

$$\begin{aligned} dx_j(t) &= (1-x_j(t))dN_{H_1}^{(j)}(t) - x_j(t)dN_{H_2}^{(j)}(t), \\ dH_j(t) &= r_j x_j(t)dt - \mathcal{I}_{H_j(t)}dt, \end{aligned} \quad (18)$$

where $N_{H_1}^{(j)}(t)$ (with rate λ_{H_j}) and $N_{H_2}^{(j)}(t)$ (with rate μ_{H_j}) are a pair of Poisson counters driving $x_j(t)$. For better tractability, we consider the case where the PU channels are *i.i.d.* It follows that $P_{I_j} = P_{I_{j'}}, \forall\, j \neq j'$. Denote by $P_I = P_{I_j}$ for simplicity. It is easy to show that in the steady state, P_I can be calculated as is given in (7).

Next, we turn our attention to study the moments of SUs' queue lengths. Applying the PDSDE tools, we obtain for SU i,

$$E[L_i] = \frac{\lambda}{-2\lambda + 2\alpha_{\mathcal{M}}}, \quad E[L_i^2] = \frac{\lambda(\lambda + 2\alpha_{\mathcal{M}})}{6(\lambda - \alpha_{\mathcal{M}})^2}, \quad (19)$$

where

$$\begin{aligned} \alpha_{\mathcal{M}} &= \sum_{l=1}^{N}\sum_{m=1}^{M}\sum_{k=0}^{m-1}\binom{m-1}{k}p(1-p)^k\left(\frac{1}{N}\right)^{k+1}\left(1-\frac{1}{N}\right)^{m-(k+1)}P_m P_I \\ &= \sum_{m=1}^{M} pP_I\left(1-\frac{p}{N}\right)^{m-1}P_m, \end{aligned} \quad (20)$$

with P_m being given by (8). Furthermore, the mean service rate μ in this case can be calculated as:

$$\begin{aligned} \mu &= \frac{1}{M}\sum_{m=1}^{M}mp\sum_{j=1}^{N}\frac{1}{N}\sum_{k=0}^{m-1}\binom{m-1}{k} \\ &\quad \times\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{(m-1)-k}(1-p)^k P_I P_m \\ &= pP_I(1-p_0)\left(1-\frac{p}{N}+\frac{pp_0}{N}\right)^{M-1}. \end{aligned}$$

It follows that

$$p_0 = 1 - \frac{\lambda}{pP_I(1-p_0)\left(1-\frac{p}{N}+\frac{pp_0}{N}\right)^{M-1}}. \quad (21)$$

[5]We assume that the number of SUs, M, is given and known to all SUs as a system parameter, and so is the number of PU channels, N, which will be used in the subsequent sections.

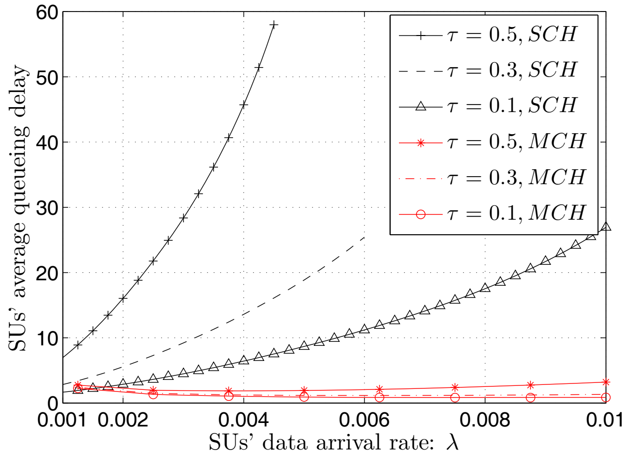


Fig. 4. Comparison of average queueing delays, for cases with a single PU channel and with multiple PU channels.

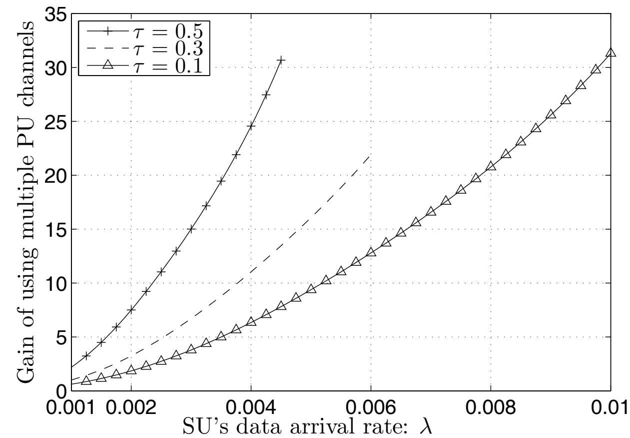


Fig. 5. Gain of using multiple PU channels.

The characterization of P_m and $E[L_i]$ then follows.

C. Adaptive Algorithm for Optimal Contention Probability

Similar to the single PU channel case, taking derivative with respect to p and setting $dp_0/dp = 0$ yields that

$$p = \min\left\{\frac{N}{M(1-p_0)}, 1\right\}. \tag{22}$$

It is not difficult to see that $M(1-p_0)/N$ is the average number of backlogged SUs per PU channel. Based on (22), similar adaptive algorithms for obtaining optimal p can be developed as in the single PU channel case.

Meanwhile, we note that $p = \frac{N}{M(1-p_0)}$ holds when $N < M(1-p_0)$. In fact, this is the regime of interest when we characterize the gain of using multiple PU channels. Here we present numerical examples to illustrate the above analysis. The contention probabilities are set to be their optimal values. As illustrated in Fig. 4 (with $N = 5, M = 20$ and $r = 1.2$), the mean queue lengths of SUs decrease significantly when multiple PU channels are present, pointing to a *multi-channel gain* therein. An illustration of such a gain was depicted in Fig. 5, where the gain was defined as the ratio $E[L_i]^{(\mathcal{S})}/E[L_i]^{(\mathcal{M})}$, with the superscripts $\mathcal{S}$ and $\mathcal{M}$ denoting the cases with a single PU channel and multiple PU channels, respectively. It can be seen that as the arrival rate of SUs increases, or the duty cycle of PUs increases, the multi-channel gain increases as well.

D. Power of Two Interfaces

Intrigued by the celebrated results in [25] and [26], in this section, we explore the impact of using two interfaces (radios) by each SU on the delay performance in a cognitive radio network.

In this new setting, each SU is equipped with two interfaces (this can be readily generalized to cases with more radios), and randomly chooses two channels independently and uniformly at a time. If the chosen PU channels (denoted as $c_1(t)$ and $c_2(t)$ for SU i) are unoccupied, the SU contends for each of them with probability p. If no collisions occur, it starts transmission of different packets on the channels. Clearly, in this case, each SU can access up to two channels for transmission at a time, thus decreasing the delay.

The queueing dynamics for the SUs, after fluid approximation, are updated as: for $i = 1, 2, \cdots, M$,

$$\begin{aligned} dL_i(t) &= dN_i(t) - \sum_{\{c_1,c_2\}} \xi_{ic_1}(t)\xi_{ic_2}(t)\mathcal{I}_{L_i(t)}\zeta_i(t) \\ &\qquad \times\Big(D_1 + D_2\Big)dt, \end{aligned} \tag{23}$$

where D_1 (respectively, D_2) denotes the event that channel c_1 (respectively, c_2) is available[6].

As in the case with a single interface[7], we obtain that for SU i,

$$E[L_i] = \frac{\lambda}{-2\lambda + 2\alpha_c}, \quad E[L_i^2] = \frac{\lambda(\lambda + 2\alpha_c)}{6(\lambda - \alpha_c)^2}, \tag{24}$$

where

$$\alpha_c = \sum_{m=1}^{M} 2P_I p\Big(1 - \frac{2p}{N}\Big)^{m-1} P_m. \tag{25}$$

Following similar steps as in the previous cases, we obtain the optimal contention probability to be

$$p = \min\left\{\frac{N}{2M(1-p_0)}, 1\right\}, \tag{26}$$

from where we note that $\frac{M(1-p_0)}{N/2}$ is intuitively the average number of backlogged SUs per PU channel. Adaptive algorithms similar to that described by (13) and (14) can be devised to find the optimal p.

Different from the multi-channel gain, the power of two choices is typically analyzed in the regime where $N >> M$. With this insight, we next focus on the case where $N \geq 2M$ and characterize the gain of using two interfaces per SU. It is clear that when $N \geq 2M$, the optimal contention probability is given by $p = 1$. It follows that $\alpha_{\mathcal{M}}$ and α_c can be rewritten

[6]Specifically, we have $D_1 = (1 - \mathcal{I}_{H_{c_1}(t)}) \prod_{k\in\{1,\ldots,M\}\setminus\{i\}} [1 - \mathcal{I}_{L_k(t)}\mathcal{I}(\xi_{kc_1}(t)=1)\mathcal{I}(\zeta_k(t)=1)]$, and D_2 is obtained similarly.

[7]See [2] for detailed derivations.

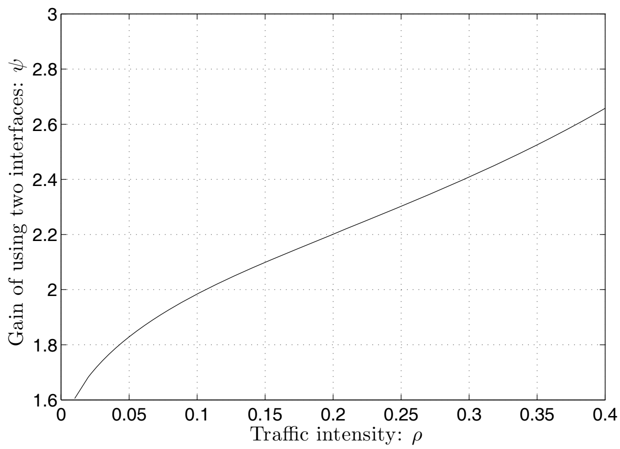


Fig. 6. Gain of using two interfaces.

as[8]

$$\alpha_{\mathcal{M}}=\sum_{m=1}^{M} P_I\Big(1-\frac{1}{N}\Big)^{m-1}P_m,\quad \alpha_{\mathcal{C}}=\sum_{m=1}^{M} 2P_I\Big(1-\frac{2}{N}\Big)^{m-1}P_m. \tag{27}$$

Also, the mean service rate and the empty probability of SUs are updated accordingly (detailed expressions can be found in [2]). The mean queue lengths of SUs for the two cases can be readily derived subsequently.

We next characterize the gain of using two interfaces under this regime. Let $\psi \triangleq \frac{E[L_i]^{(\mathcal{M})}}{E[L_i]^{(\mathcal{C})}}$, where $E[L_i]^{(\mathcal{M})}$ and $E[L_i]^{(\mathcal{C})}$ denote the mean queue lengths of SUs for the cases with a single interface and two interfaces respectively. When M is fixed and $N \to \infty$, we have that

$$\lim_{N\to\infty} \psi = \lim_{N\to\infty} \frac{\alpha_{\mathcal{C}}-\lambda}{\alpha_{\mathcal{M}}-\lambda} = \frac{2\big(1-(1-\sqrt{\rho/2})^M\big)-\rho}{\big(1-(1-\sqrt{\rho})^M\big)-\rho}, \tag{28}$$

where $\rho = \lambda/P_I$ is the traffic intensity. Fig. 6 depicts the gain as a function of the traffic intensity. As expected, the application of two interfaces provides significant gain by decreasing the mean queue lengths, and as the traffic intensity grows larger, the gain increases as well.

V. Adaptive Packet Generation Control Under Delay Constraints

In previous sections, we analyzed SUs' delay performance for different scenarios in the light traffic regime. As expected, larger delay can occur with increased arrival rate or decreased spectrum opportunities. Accordingly, when a stringent delay requirement is imposed on the SUs, effective control mechanisms (e.g., rate-limiting) are called for to regulate SUs' traffic in order to meet the requirement. In this section, we turn our attention to study such scenarios and design traffic control strategies that regulate SUs' packet generation to satisfy the delay constraint. In particular, we are interested in packet generation control, where the SUs either use a randomized strategy, or a queue-length-based control mechanism. In the following, we focus on the case with a single PU channel. The analysis readily extends to the case with multiple PU channels.

[8]See [2] for more details.

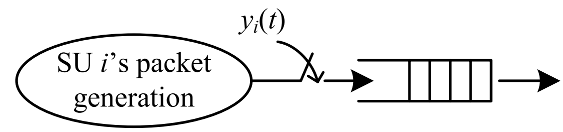


Fig. 7. An illustration of packet generation control.

For notational convenience, let $y_i(t)$ be the control process that regulates SU i's packet generation, i.e., $y_i(t)$ is a Bernoulli random variable taking two values: 0 or 1. When $y_i(t)=1$, SU i generates new packets, according to the Poisson distribution with rate λ, at time t; otherwise, no new packets are produced, as illustrated in Fig. 7.

Applying fluid approximation, the PDSDE of SU i is written as

$$dL_i(t) = y_i(t)dN_i(t) - F(t)dt. \tag{29}$$

Based on the properties of PDSDE, we obtain for $n \geq 2$,

$$nE[L_i^{n-1}F] - \sum_{k=1}^{n}\binom{n}{k}E[L_i^{n-k}y^k]\lambda = 0. \tag{30}$$

Suppose that the delay requirement on the SUs is given as

$$\Pr(D \geq D_0) \leq \delta, \tag{31}$$

where D denotes the queueing delay of one SU; $D_0 \in \mathbb{N}$ and $\delta \in (0,1)$ are positive constants and known to all users *a priori*. Appealing to Markov's Inequality and Little's Law, a sufficient condition in meeting the delay constraint (31) can be expressed in terms of the SUs' mean queue length as follows:

$$E[L_i] = \lambda_0 E[D] \leq \lambda_0\delta D_0, \tag{32}$$

where λ_0 is the average packet arrival rate of each SU, under the delay constraint.

A. Randomized Packet Generation Control by SUs

In the randomized packet generation control, SUs generate new packets with probability q, independently across users and time, i.e.,

$$y_i(t) = \begin{cases} 1, & \text{w.p. } q, \\ 0, & \text{w.p. } 1-q, \end{cases} \tag{33}$$

Based on (30), we obtain

$$E[L_i] = \frac{\lambda E[y_i^2]}{-2\lambda E[y_i] + 2\alpha_{\mathcal{S}}} = \frac{\lambda q}{-2\lambda q + 2\alpha_{\mathcal{S}}}, \tag{34}$$

where $\alpha_{\mathcal{S}}$ is given by (11) and P_m by (8), with $p_{_0} = 1-\frac{\lambda_0}{\mu} = 1-\frac{\lambda_0}{pP_I(1-p_0)(1-p+pp_0)^{M-1}}$ and $\lambda_0 = \lambda q$.

Using a similar approach, it can be shown that the optimal contention probability p is the same as given in (12), and the corresponding stochastic algorithm given by (13) and (14) can be applied to update $p_{_0}$ and p by the SUs.

Intuitively, the larger the control parameter q, the higher the buffer occupancy and SUs' queueing delay. In particular, we are interested in finding out the maximum q satisfying (32),

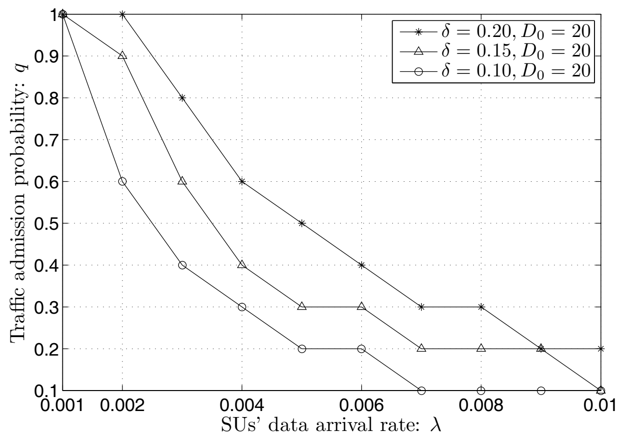


Fig. 8. q_{max} under different SU arrival rates and delay requirements.

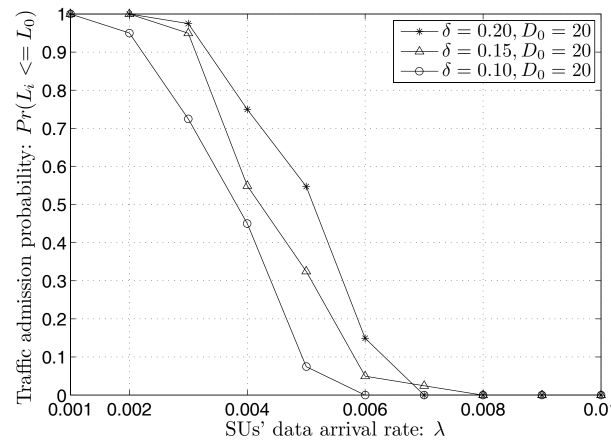


Fig. 9. $\Pr(L_i \leq L_0)$ under different SU arrival rates and delay requirements.

i.e.,

$$q_{max} = \max\{q \in [0,1] : E[L_i] \leq q\lambda\delta D_0\}. \quad (35)$$

Since a closed-form expression for q_{max} is not attainable, we next conduct numerical study to find q_{max} under different delay requirements and SUs' data arrival rates. As shown in Fig. 8 (with $M = 20$), when the arrival rate increases, or the delay requirement becomes more strict (i.e., with a smaller value of the product δD_0), the maximum traffic admission probability decreases.

B. Threshold-based Packet Generation Control by SUs

Different from the randomized control strategy outlined above, in the threshold-based control scheme, each SU decides whether to generate new packets by comparing its current queue length with a threshold L_0: if the queue length is smaller than or equal to L_0, the SU generates data at rate λ; otherwise, no traffic is generated, i.e.,

$$y_i(t) = \begin{cases} 1, & L_i(t) \leq L_0, \\ 0, & L_i(t) > L_0. \end{cases} \quad (36)$$

Correspondingly, we have

$$p_0 = 1 - \frac{\lambda \Pr(L_i \leq L_0)}{\mu}, \quad (37)$$

and the mean queue length of the SUs can be derived as (we omit details here for brevity)

$$\begin{aligned} E[L_i] &= \frac{\lambda}{2\alpha_s}\left(E[Ly] + E[y^2]\right) \\ &= \frac{\lambda}{2\alpha_s}\left(2\sum_{k=1}^{L_0}\Pr(k \leq L_i \leq L_0) + \Pr(L_i \leq L_0)\right). \end{aligned} \quad (38)$$

As in (32), a sufficient condition for meeting the delay constraint is

$$E[L_i] \leq \lambda \Pr(L_i \leq L_0)\delta D_0. \quad (39)$$

Clearly, based on (38), a closed-form expression on the average queue length is not attainable. However, distributed adaptive learning, similar to (13) and (14), can be performed by the SUs to dynamically adjusting the threshold L_0 and control the traffic accordingly. We next carry out simulations (over 4×10^4 trials) to study the performance of the threshold-based control mechanism. Again, we are interested in obtaining the best L_0 that leads to the maximum traffic admission probability $\Pr(L_i \leq L_0)$ with which the sufficient condition can still be satisfied. Figs. 9 and 10 depict a few simulation results on the maximum probability $\Pr(L_i \leq L_0)$ and the corresponding threshold L_0, respectively. It can be seen that when λ increases, or the delay requirement becomes more stringent, the traffic admission probability $\Pr(L_i \leq L_0)$ decreases, and so does the threshold L_0.

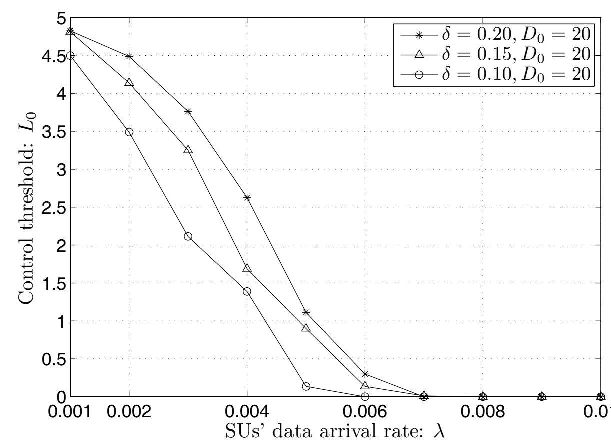


Fig. 10. Control threshold L_0 under different SU arrival rates and delay requirements.

VI. Conclusions

In this paper, we have carried out delay analysis for a cognitive radio network. We took a stochastic fluid queue approach and modeled the system using Poisson driven stochastic differential equations. We characterized the moments of the queue lengths of SUs, for cases with a single PU channel and multiple PU channels. The impact of the PU traffic on SUs' queue lengths and the gain of using multiple PU channels were examined. Also, we explored the gain of using two interfaces per SU. Adaptive algorithms, using local information only, have been developed to find the optimal contention probabilities that achieve the minimum mean queue lengths and thus the minimum queueing delays of SUs.

Our analysis and numerical examples revealed that the mean queueing delay of SUs increases as the duty cycle of the PUs' traffic increases, pointing to the impact of PU activity on the delay performance of SUs. Also, when multiple PU channels were employed, we observed a decrease in the mean queueing delay, indicating a multi-channel gain. Moreover, if each SU is equipped with two interfaces, there is a decrease in the mean queueing delay because of the gain of using two choices.

Finally, we also studied packet generation control on the SUs, when delay constraints were imposed. We developed two control mechanisms, one randomized and the other utilizing SUs' queue lengths, and evaluated their performance.

APPENDIX

Lemma 6.1: The fixed point equation (9) has a unique solution p_0.

Proof: Let $\Gamma(\gamma) = (1-\gamma)(1-p+p\gamma)^{M-1}$, $\gamma \in [0,1)$. The first-order derivative of Γ w.r.t. γ is given by

$$\frac{d\Gamma(\gamma)}{d\gamma} = (1-p(1-\gamma))^{M-2}(Mp(1-\gamma)-1). \qquad (40)$$

Recall from (12), we have $p \leq \frac{1}{M(1-p_0)}$, indicating that $\frac{d\Gamma(p_0)}{dp_0} \leq 0$ and $\Gamma(p_0)$ is nonincreasing in p_0. It follows that $1 - \frac{\lambda}{\Gamma(p_0)}$ is nonincreasing in p_0 as well. Based on this monotonicity property (cf.[27]), we conclude that there is one unique solution to the the fixed point equation given by (9). ■

TABLE I
MAIN NOTATION USED IN THE PAPER.

N	number of PU channels
M	number of SUs
$x_j(t)$	the ON-OFF process controlling PU j's data generation [9]
r_j	the data generation rate of PU j
$1/\mu_{H_j}$	the average length of $x_j(t)$'s ON periods
$1/\lambda_{H_j}$	the average length of $x_j(t)$'s OFF periods
λ	SUs' data arrival rate
μ	SUs' average service rate
p	SUs' contention probability
$\mathcal{I}_{f(t)}$	indicator function $\mathbf{1}(f(t)>0)$
$\zeta_i(t)$	indicator random variable denoting whether SU i contends at time t
$\xi_{ij}(t)$	indicator random variable denoting whether SU i chooses channel j at time t
$H_j(t)$	queue length of PU j at time t
$L_i(t)$	queue length of SU i at time t
P_I	PUs' idle probability
p_0	SUs' idle probability
$N_i(t)$	Poisson counter of SU i's arrival process
P_m	probability that the number of backlogged SUs equals m
$\Phi_i(t)$	indicator random variable denoting whether SU i's queue is empty at time t
$y_i(t)$	packet generation control process for SU i
D_0	delay constraint on SUs
q	control probability at which the SU generates new packets
L_0	control threshold beyond which the SU cannot generate new packets

REFERENCES

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access: signal processing, networking, and regulatory policy," *IEEE Signal Process. Mag.*, pp. 79–89, May 2008.

[2] S. Wang, J. Zhang, and L. Tong, "Delay analysis for cognitive radio networks with random access: a fluid queue view," in *Proc. 2010 IEEE INFOCOM*.

[3] A. Laourine, S. Chen, and L. Tong, "Queueing analysis in multichannel cognitive spectrum access: a large deviation approach," in *Proc. 2010 IEEE INFOCOM*.

[4] S. Chen and L. Tong, "Multiuser cognitive access of continuous time markov channels: maximum throughput and effective bandwidth regions," in *Proc. 2010 UCSD Workshop Inf. Theory Appl.*.

[5] A. Ephremides and R.-Z. Zhu, "Delay analysis of interacting queues with an approximate model," *IEEE Trans. Commun.*, vol. 35, no. 2, pp. 194–201, 1987.

[6] E. Modiano and A. Ephremides, "A method for delay analysis of interacting queues in multiple access systems," in *Proc. 1993 IEEE INFOCOM*, vol. 2, pp. 447–454.

[7] Y. Liu and W. Gong, "On fluid queueing systems with strict priority," *IEEE Trans. Automatic Control*, vol. 48, no. 12, pp. 2079–2088, Dec. 2003.

[8] N. Bisnik and A. Abouzeid, "Queueing network models for delay analysis of multihop wireless ad hoc networks," in *Proc. 2006 International Conf. Wireless Commun. Mobile Comput.*, pp. 773–778.

[9] D. Miorandi, A. A. Kherani, and E. Altman, "A queueing model for HTTP traffic over IEEE 802.11 WLANs," in *Proc. 2004 ITC Specialist Seminar*, vol. 50, no. 1, pp. 63–79.

[10] O. Tickoo and B. Sikdar, "Queueing analysis and delay mitigation in IEEE 802.11 random access MAC based wireless networks," in *Proc. 2004 IEEE INFOCOM*, vol. 2, pp. 1404–1413.

[11] T. K. Apostolopoulos and E. N. Protonotarios, "Queueing analysis of buffered slotted multiple access protocols," *Comput. Commun.*, vol. 8, no. 1, pp. 9–21, 1985.

[12] E. D. Sykas, D. E. Karvelas, and E. N. Protonotarios, "Queueing analysis of some buffered random multiple access schemes," *IEEE Trans. Commun.*, vol. 34, no. 8, pp. 790–798, 1986.

[13] T. Wan and A. U. Sheikh, "Performance and stability analysis of buffered slotted ALOHA protocols using tagged user approach," *IEEE Trans. Veh. Technol.*, vol. 49, no. 2, pp. 582–593, 2000.

[14] S. Meyn, *Control Techniques for Complex Networks*. Cambridge University Press, 2008.

[15] D. Anick, D. Mitra, and M. M. Sondhi, "Stochastic theory of a data-handling system with multiple sources," *Bell System Tech. J.*, vol. 61, no. 8, pp. 1871–1894, 1982.

[16] A. Eryilmaz, P. Marbach, and A. Ozdaglar, "A fluid-flow model for backlog-based CSMA policies," in *Proc. 2008 IEEE WICON*.

[17] A. Eryilmaz and R. Srikant, "Fair resource allocation in wireless networks using queue-length-based scheduling and congestion control," in *Proc. 2005 IEEE INFOCOM*, vol. 3, pp. 1794–1803.

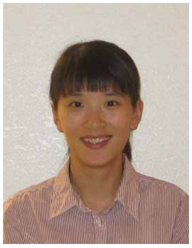
[18] X. Li, Q. Zhao, X. Guan, and L. Tong, "Optimal cognitive access of markovian channels under tight collision constraints," to appear in *IEEE J. Sel. Areas Commun.*, 2011.

[19] W. Luo and A. Ephremides, "Stability of N interacting queues in random access systems," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1579–1587, July 1999.

[20] R. R. Rao and A. Ephremides, "On the stability of interacting queues in a multiple access system," *IEEE Trans. Inf. Theory*, vol. 34, no. 5, pp. 918–930, 1988.

[9] We may drop the subscript j for the case with a single PU channel.

[21] R. W. Brockett, "Lecture notes: stochastic control," Harvard University, Tech. Rep., 1983.
[22] R. W. Brockett, W. Gong, and Y. Guo, "Stochastic analysis for fluid queueing systems," in *Proc. 1999 IEEE Decision Control*, pp. 3077–3082.
[23] H. J. Kushner and G. Yin, *Stochastic Approximation and Recursive Algorithms and Applications*, 2nd edition. Springer, 2003.
[24] D. Bertsekas and R. Gallager, *Data Networks*, 2nd edition. Prentice Hall, 1992.
[25] Y. Azar, A. Broder, A. Karlin, and E. Upfal, "Balanced allocations," in *Proc. 1994 ACM Symp. Theory Comput.*, vol. 2, pp. 593–602.
[26] M. D. Mitzenmacher, "The power of two choices in randomized load balancing," Ph.D. dissertation, University of California at Berkeley, 1996.
[27] A. Kumar, E. Altman, D. Miorandi, and M. Goyal, "New insights from a fixed point analysis of single cell IEEE 802.11 WLANs," *IEEE/ACM Trans. Netw.*, pp. 588–601, June 2007.



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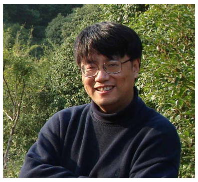

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