On the Operation and Value of Storage in Consumer Demand Response

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Abstract—We study the optimal operation and economic value of energy storage operated by a consumer who faces (possibly random) fluctuating electricity prices and seeks to reduce its energy costs. The value of storage is defined as the consumer’s net benefit obtained by optimally operating the storage. We formulate the operation problem as a dynamic program. Under the assumption that consumer utility (received from electricity consumption) is additively separable over time, we establish a threshold structure of the optimal operation policy for a consumer who faces random electricity prices and stochastic demand. For a more general setting that incorporates the inter-temporal substitution effect in consumer demand, if the consumer always faces the same (realized) purchasing and selling price, then it is optional for her to use the storage only for arbitrage, and therefore the VoS does not depend on the consumer’s demand.

Index Terms—Value of storage, Energy storage operation, Demand response, Dynamic programming.

I. INTRODUCTION

Renewable generation capacity is expanding rapidly to potentially reduce carbon dioxide emissions and dependence on fossil fuels. As non-dispatchable generation, renewable energy introduces variability into the energy portfolio, and further amplifies the difficulty of matching demand with supply in the real time. Fast-response energy storage devices (e.g., batteries, flywheels, and plug-in hybrid electric vehicles (PHEVs)) are environmentally friendly candidates that can provide flexibility to the system and mitigate the impact of volatile renewable generations.

The focus of this paper is on the operation of electric storages owned by a load-serving entity who seeks to reduce its energy costs. Our motivation stems from the potential of electricity consumers to own and use storage devices (e.g., major consumers like data centers [1], [2] and individual consumers who own PHEVs [3], [16]) and a recent study that shows consumer ownership of storage can be socially beneficial [5].

In this paper, we study the operation of a finite-capacity electric storage owned by an electricity consumer who is faced with a challenging sequential decision making problem: up to what level to charge or discharge the storage, whether to satisfy demand directly from the grid or from the storage, and sell how much energy back to the grid.

A. Summary of Results

The main contribution of this work is twofold. First, we formulate in Section II the storage operation problem as a finite-horizon Markov decision process. For a fairly general setting we show in Section III that an optimal policy can be characterized by two thresholds: if the storage level is between the two thresholds, keep the current storage level and do not charge or discharge. When the storage level is below the lower threshold, it is optimal to charge the storage up to this threshold; and when the storage level is above the the higher threshold, discharge the storage to meet the demand or/and to sell back to the market.

Second, we consider a more general setting that incorporates the inter-temporal substitution effect in consumer demand. We formally define the value of storage (VoS) as the consumer’s net benefit obtained by optimally operating the storage. For an important special case where the (realized) purchasing price always equals the price of selling stored electricity (back to the grid), we show that the VoS does not depend on the consumer’s demand, because it is then optimal for the consumer to use the storage only for arbitrage (not for consumption); further, if the storage can be fully charged or discharged within one time period, then the value of storage is shown to increase linearly in the storage capacity.

Finally, we illustrate the analysis developed in this paper using several numerical examples where we consider a real-time pricing scheme used in practice in the US.

B. Related Work

There is a growing literature on the use of storage in a smart grid. We restrict our discussion on those most relevant to our work. For energy storage owned by system operators, there exists a substantial literature on its operation [6], [7], [13] and its value to system efficiency and reliability [9]. Different from the setting studied in this paper, the models used in these works do not allow the option of selling power back to the grid. Another well studied application of energy storage is the use of storages to arbitrage [10], [3]. A few recent works conduct a dynamic programming (DP) approach to derive the arbitrage value of electric storage, in the presence of dynamic pricing [11], [12]. Different from the present paper, the aforementioned works assume that owners (of electric storages) put no value on electricity consumption and use storages only for arbitrage.

Closer to the present paper, there have been recent works on the operation of consumer-owned electric storages. Leveraging on Lyapunov optimization techniques, a few recent papers propose a variety of on-line algorithms that are shown to be asymptotically optimal, as storage capacity increases to
infinity [13], [14]. These algorithms are expected to perform well when storage capacity is significantly larger than the maximum charging/discharging rates (i.e., if it takes many hours to fully charge and discharge the storage). The authors of [15] derive a threshold structure for the optimal control policy, under the assumptions that the demand is inelastic and the price of selling stored electricity (back to the grid) is constantly zero. These assumptions are relaxed in this work.

The rest of the paper is organized as follows. In Section II, we introduce the system model, and formulate the storage operation problem as a finite-horizon Markov decision process. In Section III, we derive the main results of this paper, including a threshold characterization of the optimal operation policy. In Section IV, we consider a more general setting that incorporates the inter-temporal substitution effect in consumer demand. We show that the optimal control policy can be obtained by solving a sequence of (deterministic) convex optimization problems. In Section V, we define the value of storage and characterize some of its properties. In Section VI, we present numerical results on the value of storage using real pricing data from the New England ISO. Finally, in Section VII, we make some brief concluding remarks.

II. Model Formulation

We study the operation of a finite-capacity storage owned by an electricity consumer. The consumer has the options of discharging the storage for its own consumption, charging its battery from purchased power, and selling its stored electricity back to the grid. The detailed model of this decision process has the following elements:

1) **Discrete time:** Time periods are indexed by \( t = 0, \ldots, T \).

2) **Storage capacity:** At each stage \( t = 0, \ldots, T \), let \( x_t \in [0, B] \) denote the storage level at the beginning of stage \( t \), where \( B > 0 \) is the storage capacity.

3) **Randomness:** For each stage \( t = 0, \ldots, T \), let \( s_t \in S \) denote the global state. We assume that the set \( S \) is finite. The global state evolves as an exogenous Markov chain, of which the transition probability is independent of the consumer’s action. The global state is used to model exogenous (deterministic and random) factors, such as the current time, the demand of other electricity customers, and weather conditions, which have impacts on both the electricity prices and the consumer’s utility.

4) **Prices:** For \( t = 0, \ldots, T - 1 \), and every global state \( s_t \), let \( p_t(s_t) \in (0, \infty) \) denote the purchasing price and \( q_t(s_t) \in (0, p_t(s_t)) \) denote the selling price.

5) **Electricity consumption:** For \( t = 0, \ldots, T - 1 \), let \( c_t \in [0, \infty) \) denote the amount of electricity purchased (from the market) for consumption, and let \( d_t \in [0, \infty) \) be the amount of electricity withdrawn from the storage for consumption.

6) **Consumer utility:** For \( t = 0, \ldots, T - 1 \), the consumer receives a utility \( u_t(c_t + d_t, s_t) \), which depends on the global state and her electricity consumption. We assume that, for every \( t \in \{0, \ldots, T - 1\} \) and every \( s \in S \), the utility function \( u_t(\cdot, s) \) is non-decreasing, concave, and continuously differentiable, with

\[
\lim_{y \to \infty} u_t'(y, s) = 0, \quad t = 0, \ldots, T - 1, \quad \forall s \in S.
\]  

7) **Charging and discharging:** For \( t = 0, \ldots, T - 1 \), the consumer purchases \( a_t \in \mathbb{R} \) amount of electricity for storage. For the rest of the paper, we will use \( a_t^+ \) and \( a_t^- \) denote the positive and negative part of \( a_t \), respectively, i.e., \( a_t^+ = \max\{0, a_t\} \) and \( a_t^- = -\min\{0, a_t\} \).

8) **System evolution:** Given the storage level and the action taken at stage \( t \), the storage level at the next stage is

\[
x_{t+1} = \lambda \left( x_t + \gamma a_t^+ - (d_t + a_t^-) / \eta \right),
\]  

where \( \gamma \in (0, 1] \) is the charging efficiency, \( \eta \in (0, 1] \) is the discharging efficiency, and \( \lambda \in (0, 1] \) is the storage efficiency. We assume that the storage level at the initial stage 0, \( x_0 \), is exogenous and independent of the consumer’s decision.

9) **Rate constraints:** There are maximum charging and discharging rates of the energy storage, \( R^C \) and \( R^D \), i.e.,

\[
\gamma a_t^+ \leq R^C, \quad (d_t + a_t^-) / \eta \leq R^D.
\]  

We are ready to formulate the operation problem as a \((T + 1)\)-stage Markov decision process (MDP) by providing its state space, action sets, transition probabilities, and stage cost. At each stage \( t \), the system state consists of the current storage level, \( x_t \), and the global state \( s_t \). For \( t = 0, \ldots, T - 1 \), a feasible action is a vector \((a_t, c_t, d_t)\) that satisfies the rate constraints in (3), and the following one:

\[
(d_t + a_t^-) / \eta \leq x_t, \quad x_t + \gamma a_t^+ - (d_t + a_t^-) / \eta \leq B,
\]  

where the first constraint is to avoid storage underflow, and the second constraint is to prevent storage overflow. Without loss of generality, we have assumed (in the first constraint in (4)) that the consumer never charges and withdraws the storage simultaneously, i.e., \( d_t > 0 \) implies \( a_t \leq 0 \). We let \( A(x_t) \) denote the set of feasible actions at storage level \( x_t \).

The evolution of storage levels is deterministic, and is governed by (2). The evolution of the global state is random and independent of the current storage level and the actions taken by the consumer. Finally, for \( t = 0, \ldots, T - 1 \), the stage payoff is given by

\[
w_t(x_t, s_t, a_t, c_t, d_t) = u_t(c_t + d_t, s_t) - p_t(s_t)(a_t^+ + c_t) + q_t(s_t)a_t^-.
\]  

At the terminal stage \( T \), no action is available, and the stage payoff (salvage value) is given by

\[
w_T(x_T, s_T) = V_T(x_T, s_T),
\]  

where \( V_T : [0, B] \times S \to [0, \infty) \) is assumed to be non-decreasing, concave and continuously differentiable in \( x_T \), for every \( s_T \in S \).

A policy \( \pi = (\mu_0, \ldots, \mu_{T-1}) \) is a sequence of decision rules such that \( \mu_t(x_t, s_t) \in A(x_t) \), for all \( x_t, s_t \) and \( t \). We
will use \( V^π_t(x_t, s_t) \) to denote the payoff-to-go function under a policy \( π \), and the current system state \((x_t, s_t)\):

\[
V^π_t(x_t, s_t) = u_t(x_t, s_t, μ_t(x_t, s_t))
+ \mathbb{E} \left\{ \sum_{\tau = t+1}^{T-1} w_\tau(x_\tau, s_\tau, μ_\tau(x_\tau, s_\tau)) + w_T(x_T, s_T) \right\},
\]

where the expectation is over the sequence of global states \{\( s_\tau \)\}_{\tau = t+1}^T. We note that since the marginal utility converges to zero (cf. (1)), and the purchasing price is always positive, the payoff-to-go function is always bounded, under any policy \( π \). By a slight abuse of notation, we will use \( V_t(x_t, s_t) \) to denote the optimal payoff-to-go function, i.e.,

\[
V_t(x_t, s_t) = \sup_π \{ V^π_t(x_t, s_t) \}.
\]

Note that the optimal payoff-to-go function for the terminal stage \( T \) is given in (6). We say a policy \( π^* \) is optimal, if it attains the optimal payoff-to-go defined above, i.e.,

\[
V^*_t(x_0, s_0) = V_0(x_0, s_0),
\]

for all initial states \((x_0, s_0)\).

### III. Optimal Storage Operation

In this section, we derive the main theoretic results of this paper. We first show that optimal payoff-to-go is concave in current storage level, which enables us to establish the threshold structure of an optimal policy in Theorem 3.1.

**Lemma 3.1:** For every \( s \in S \) and \( t = 0, \ldots, T-1 \), the optimal payoff-to-go function \( V_t(x, s) \) is concave in \( x \).

For \( t = 0, \ldots, T-1 \), Bellman’s equation yields

\[
V_t(x_t, s_t) = \max_{a_t, c_t, d_t} \left\{ u_t(x_t, s_t, a_t, c_t, d_t) + V_{t+1}(s_{t+1}) \right\},
\]

where \( x_{t+1} \) is determined by Eq. (2), and \( V_{t+1}(s_{t+1}) \) denote the (conditional) expected payoff-to-go function at stage \( t + 1 \), i.e., for \( t = 0, \ldots, T-1 \)

\[
V_{t+1}(s_{t+1}) = \mathbb{E}\left\{ V_{t+1}(x_{t+1}, s_{t+1}) \mid s_t \right\},
\]

where the expectation is over the global state \( s_{t+1} \), conditioning on the global state at stage \( t, s_t \). Proposition 3.1 implies that given the optimal payoff-to-go at stage \( t + 1 \), the optimization problem on the right hand side of (9) is convex. As a result, the optimal payoff-to-go \( V_0(0, s_0) \) can be obtained by solving a sequence of convex optimization problems, through the backward recursions defined in (9).

We will characterize in Theorem 3.1 an optimal policy that solves the optimization problem on the right hand side of (9).

**Lemma 3.1** shows that the optimal payoff-to-go function \( V_t(\cdot, s_t) \) is concave, for \( t = 0, \ldots, T \) and every \( s \in S \). It follows from the definition in (10) that \( \bar{V}_{t+1}(\cdot) \) must be concave, too. We therefore have the existence of its right and left directional derivatives\(^1\). For the rest of this paper, we will use the notation \( \partial^+ \bar{V}_{t+1}(x) \) and \( \partial^- \bar{V}_{t+1}(x) \) to denote the right and left directional derivative of \( \bar{V}_{t+1}(x) \), respectively.

The following lemma shows that it is optimal not to discharge the storage, if the marginal value of storage is greater than both the selling price and the minimum of the marginal utility at zero energy consumption and the purchasing price.

**Lemma 3.2:** At a system state \((x_t, s_t)\), if

\[
\eta \max\{q_t(s_t), \min\{p_t(s_t), u'_t(0, s_t)\}\} \leq \lambda \partial^- \bar{V}_{t+1}(\lambda x_t),
\]

then it is optimal not to discharge the storage, i.e., \( a_t^* \geq 0 \).

Motivated by the above lemma, we define some notations that will be useful in Theorem 3.1. We let \( k_t(s_t) \) denote the maximum \( \lambda x_t \) in the interval \([0, \lambda B]\) such that the condition in (11) holds; if the condition in (11) does not hold even for \( x_t = 0 \), we let \( k_t(s_t) = 0 \). We next introduce more notations that will be useful in Theorem 3.1, the main result of this section. For \( t = 0, \ldots, T-1 \) and every \( s \in S \), we define a real number \( y_t(s) \) such that

\[
y_t(s) = \begin{cases} (u_t')^{-1}(p_t(s)), & \text{if } u_t'(0, s) > p_t(s), \\ 0, & \text{otherwise,} \end{cases}
\]

where \( (u_t')^{-1}(p_t(s)) \Delta \inf\{y : u_t'(y, s) = p_t(s)\} \).

The scalar \( y_t(s) \) defined in (12) is actually the minimum optimizer that maximizes the stage payoff of a consumer without storage, \( u_t(y, s) - y p_t(s) \), over \( y \in [0, \infty) \). Similarly, we define \( z_t(s) \) as the minimum optimizer that maximizes \( u_t(z, s) - z q_t(s) \), over \( z \in [0, \infty) \). Since \( p_t(s) \geq q_t(s) \), we have \( z_t(s) \geq y_t(s) \).

For \( t = 0, \ldots, T-1 \) and every \( s \in S \), we let \( h_t(s) \in [0, B] \) denote the maximum storage level at which the expected marginal storage value (at stage \( t + 1 \)) is no less than the adjusted purchasing price \( p_t(s)/(\gamma \lambda) \). Formally,

\[
h_t(s) = \begin{cases} 0, & \text{if } \partial^+ \bar{V}_{t+1}|_{s}(0) \leq p_t(s)/(\gamma \lambda), \\ \lambda B, & \text{if } p_t(s)/(\gamma \lambda) \leq \partial^- \bar{V}_{t+1}|_{s}(\lambda B), \\ \sup\{h \in [0, \lambda B): \lambda \gamma \partial^+ \bar{V}_{t+1}|_{s}(h) \geq p_t(s)\}, & \text{otherwise.} \end{cases}
\]

Similarly, for \( t = 0, \ldots, T-1 \) and every \( s \in S \), we let \( g_t(s) \in [0, B] \) denote the maximum storage level at which the expected marginal storage value (at stage \( t + 1 \)) is no less than the adjusted selling price \( q_t(s)/\lambda \), i.e.,

\[
g_t(s) = \begin{cases} 0, & \text{if } \partial^+ \bar{V}_{t+1}|_{s}(0) \leq q_t(s)/\lambda, \\ \lambda B, & \text{if } q_t(s)/\lambda \leq \partial^- \bar{V}_{t+1}|_{s}(\lambda B), \\ \sup\{g \in [0, \lambda B): \lambda \partial^+ \bar{V}_{t+1}|_{s}(g) \geq q_t(s)\}, & \text{otherwise.} \end{cases}
\]

\(^1\)Indeed, since \( \bar{V}_{t+1}(\cdot) \) is continuous and concave, it must be continuously differentiable at all points in the interval \((0, B)\) but at most countably many points.
The concavity of \( V_{t+1}(\cdot) \) and the fact that \( q_t(s) \leq p_t(s) \)
implies that \( g_t(s) \geq h_t(s) \). We indeed have \( h_t(s_t) \leq k_t(s_t) \leq \gamma g_t(s_t) \).

**Theorem 3.1:** There exists an optimal policy \( \pi^* = (\mu^*_0, \ldots, \mu^*_{T-1}) \) characterized by the two thresholds \( h_t(s_t)/\lambda \) and \( k_t(s_t)/\lambda \).

(a) If the storage level \( x_t \) is below the threshold \( h_t(s_t)/\lambda \), then greedily charge the storage up to this level, and purchase electricity for consumption, i.e., \( c^*_t = y_t(s_t), d^*_t = 0 \), and
\[
a^*_t = \min \left\{ \frac{R^C}{\gamma}, \frac{h_t(s_t)}{\lambda} - x_t / \gamma \right\},
\]
where \( R^C \) is the maximum charging rate.

(b) If the storage level \( x_t \) is between the two thresholds, i.e., if \( x_t \in [h_t(s_t)/\lambda, k_t(s_t)/\lambda] \), then keep the storage level, and purchase electricity for consumption, i.e., \( a^*_t = 0, c^*_t = y_t(s_t), \) and \( d^*_t = 0 \).

(c) If the storage level \( x_t \) is above the threshold \( k_t(s_t)/\lambda \), then greedily discharge the storage to meet demand and/or sell back to the market. Let \( d^* \) be given in Eq. (16) (locating at the top of next page), \( a^*_t = \min \{0, d^*_t - \eta(x_t - g_t(s_t))\} \) and \( c^*_t = (y_t(s_t) - d^*_t)^+ \).

**Remark 3.1:** Through an approach similar to the proof of Lemma 3.2, it is easy to show that if the storage level \( x_t \) is below the threshold \( h_t(s_t)/\lambda \), then it is optimal to charge the storage to this threshold at which the marginal value of storage equals the purchasing price \( p_t(s_t) \). Part (b) of Theorem 3.1 follows from Part (a) and Lemma 3.2.

If the storage level \( x_t \) is above the threshold \( k_t(s_t)/\lambda \), then the condition in (11) is violated, and the consumer should discharge the storage. If \( u^*_t(0, s_t) \leq q_t(s_t) \), then the consumer should sell (instead of consuming) the stored electricity, and therefore we have \( d^*_t = 0 \) in (16); otherwise, the consumer consumes the stored electricity until its marginal value is no less than either the marginal utility or the purchasing price (cf. Eq. (16)). After consuming \( d^*_t \) amount of electricity, if the marginal value of storage remains lower than the selling price, i.e., if
\[
x_t - d^*_t / \eta > g_t(s_t)/\lambda,
\]
then sell \(-a^*_t\) amount of stored electricity to the market, until the marginal value of storage equals the selling price (at which we have \( x_{t+1} = g_t(s_t) \)).

Finally, if \( d^*_t < g_t(s_t) \), then the marginal utility \( u^*_t(d^*_t, s_t) \) is higher than the purchasing price \( p_t(s_t) \), and the consumer should purchase electricity to meet the demand up to the consumption level \( y_t(s_t) \) (cf. its definition in (12)).

**IV. INTER-TEMPORAL SUBSTITUTION IN CONSUMER DEMAND**

In this section, we consider a more general setting that incorporates the inter-temporal substitution effect in consumer demand. Formally, we let \( m_t \) denote the state of consumer, which has influence on her utility received from electricity consumption: the consumer receives a utility of \( u_t(c_t + d_t, s_t, m_t) \), at stage \( t = 0, \ldots, T - 1 \). All possible consumer states lie in a compact subset of \( \mathbb{R} \), \([0, M]\).

At the terminal stage \( T \), the value function \( V_T(x_T, s_T, m_T) \) is allowed to depend on the consumer’s final state \( m_T \). For the general model considered in this section, system state at stage \( t \) becomes \((x_t, s_t, m_t)\), which includes the consumer’s state \( m_t \). The stage payoff is given by
\[
 w_t(x_t, s_t, m_t, a_t, c_t, d_t) = u_t(c_t + d_t, s_t, m_t) - p_t(s_t) a^*_t + q_t(s_t) a^*_t. 
\]

We assume that the transition of the consumer’s state is affine, i.e.,
\[
 m_{t+1} = \alpha_t m_t + \beta_t (c_t + d_t) + \vartheta_t, \quad t = 0, \ldots, T-1, 
\]
where \( \alpha_t, \beta_t, \) and \( \vartheta_t \) are given real numbers. We note that a broad array of electric demands can be modeled by the linear state transition defined above; two concrete examples are listed below.

1) For deferrable loads such as Plug-in Hybrid Electric Vehicles (PHEVs), dish washers, or clothes washers, consumer usually only cares whether a task is completed before a certain time [18], [19]. The consumer state \( m_t \) can be set as the amount of electricity needed to complete the task.

2) For thermal loads such as air conditioners and heaters, consumer’s satisfaction depends on the current indoor temperature, whose transition could be approximated by a linear dynamic [20], [21]. For these appliances, the consumer state \( m_t \) is the indoor temperature at stage \( t \).

Through an approach similar to the proof of Lemma 3.1, one can prove that the optimal payoff-to-go function is concave in the vector \((x_t, m_t)\), as stated in the following proposition.

**Proposition 4.1:** Suppose that \( V_T(x_T, s_T, m_T) \) is concave in the vector \((x_T, m_T)\), and that the utility function \( u_t(\cdot, s_t, m_t) \) is concave, for \( t = 0, \ldots, T - 1 \) and every pair of \((s_t, m_t)\). Then, for \( t = 0, \ldots, T \) and every \( s \in \mathcal{S} \), the optimal payoff-to-go function, \( V_t(x, s, m) \), is concave in the vector \((x, m)\).

The optimal policy can be calculated through backward induction, i.e.,
\[
 \mu^*_t(x_t, s_t, m_t) \in \max_{a_t, c_t, d_t} \left\{ w_t(x_t, s_t, m_t, a_t, c_t, d_t) + \mathbb{E}[V_{t+1}(x_{t+1}, s_{t+1}, m_{t+1})] \right\},
\]
where the expectation is over the next global state \( s_{t+1}, x_{t+1} \) is determined by (2), and \( m_{t+1} \) is given by (17). Proposition 4.1 states that the optimization problem on the right hand side of (18) is convex. Therefore, the optimal operation policy and the optimal payoff-to-go can be calculated through backward recursion, by solving a sequence of convex optimization problems (on the right hand side of (18)).
\[
D_t^* = \begin{cases} 
\sup_{d \leq \min\{\eta r_t, z_t(s_t), \eta R^+\}} \left\{d : \lambda \partial^T V_{t+1}(\lambda(x_t - d/\eta)) \leq \eta \min\{u^*_t(d, s_t), p_t(s_t)\}\right\}, & \text{if } u^*_t(0, s_t) > q_t(s_t), \\
0, & \text{otherwise},
\end{cases}
\] (16)

V. VALUE OF STORAGE

In this section, we formally define the value of storage (VoS) and provide some characterization of VoS for an important special case where the purchasing price always equals the selling price. Formulation and results established in this section are based on the general setting introduced in Section IV.

The optimal payoff-to-go $V_0(0, s_0, m_0)$ is the maximum expected payoff the consumer could obtain (with a storage of capacity $B$), under an initial consumer state $m_0$ and an initial global state $s_0$. We note that $V_0(0, s_0, m_0)$ is closely related to the value of storage (VoS), which is defined as the difference between the maximum expected payoffs achieved by two consumers, the former of whom owns a storage of capacity $B$, and the latter does not own a storage. Formally, for a consumer with initial state $m_0$, the value of a capacity-$B$ storage is defined by

\[
\text{VoS}(B, m_0) = E\{V_0(0, s_0, m_0) - V_0(0, s_0, m_0)\}, \quad (19)
\]

where the expectation is over the initial global state $s_0$, and $V_0(0, s_0, m_0)$ is the optimal payoff-to-go (of the dynamic program defined in this section) with $B = 0$.

Next, we derive a strong result for an important case where the purchasing price always equals the selling price. In this case, we argue that the value of storage is independent of the consumer’s demand. It is worth noting that this result holds for a more general setting where the consumer’s state transition is arbitrary, i.e., the next state $m_{t+1}$ could be an arbitrary function of the current state $m_t$ as well as the actions $(c_t, d_t)$ (not necessarily according to (17)).

**Theorem 5.1:** Suppose that the selling price always equals the purchasing price, i.e., $p_t(s_t) = q_t(s_t)$ for every $t$ and $s_t$. The value of storage (defined in (19)) is independent of the consumer’s initial state $m_0$ and utility functions $\{u_t\}_{t=0}^{T-1}$.

**Theorem 5.1** follows from the existence of an optimal policy that never withdraws energy from storage for consumption. We omit the detailed proof due to page limit.

**Theorem 5.1** leads to the following corollary: the value of storage increases linearly with storage capacity, if the charging rate constraints will never be binding. Like Theorem 5.1, Corollary 5.1 holds under arbitrary evolution of the consumer’s state. The setting with no rate constraints is motivated by the fact that fast-response storage devices are rapidly becoming available; for example, the lithium-ion titanate batteries are capable of recharging in approximately 10 minutes to 95% of full capacity [22].

**Corollary 5.1:** Suppose that the selling price always equals the purchasing price, i.e., $p_t(s_t) = q_t(s_t)$ for every $t$ and $s_t$, charging and discharging rates are sufficiently high such that $R^C = R^D = B$, and that the salvage value $V_T$ is linear in $x_T$. The value of storage increases linearly with the capacity $B$.

**Proof:** Theorem 5.1 shows that for the purpose of calculating value of storage, it is sufficient to consider a model without demand. In such a model where storage is purely used for arbitrage, since we have assumed that the salvage value of storage is linear in the storage level, it follows from the backward induction process defined in (18) that the value of storage at any stage $t$ increases linearly with the battery level $x_t$. It is therefore always optimal to charge or discharge the storage until reaching the capacity limit (either $0$ or $B$). It follows that value of storage increases linearly with the capacity $B$.

VI. NUMERICAL EXAMPLES

In this section, we present several numerical examples that compute the value of a finite-capacity storage under different parameter settings (e.g. the storage capacity and prices during peak hours). We will consider a simple setting where the consumer is faced with inelastic demand and deterministic prices. The setting with deterministic time-variant prices is motivated by the day-ahead hourly pricing program that has been widely implemented in the US, mainly for large commercial and industrial consumers [23], [24], [25].

We let each stage last for one hour, and set the storage efficiency $\lambda = 0.97$, the charging efficiency $\gamma = 0.85$, and the discharging efficiency $\eta = 0.85$. These are typical parameters of the sodium sulfur (NaS) battery [26]. We will consider fast-charging storage devices that can be fully charged within one hour, i.e., $R^C = R^D = B$.

We calculate the VoS for a consumer who is faced with the two trajectories of day-ahead hourly purchasing prices presented in Fig. 1, for the time period between 11AM and 10PM. If we refer to the hour starting at 11AM as stage 0, then the last hour (starting at 9PM) is stage 10, i.e., $T = 11$ in this example. We assume that the selling price equals the purchasing price, at every stage $t$ except the peaking hour (i.e., stage 3 on Aug. 1 and stage 7 on Feb. 16). For the peak hour of each of these two price trajectories, the ratio of the purchasing price to the selling price is set to be 0.6, 0.8, and 1. The consumer has an inelastic demand of 10MWh (the marginal utility is higher than the purchasing price until consuming 10MWh of energy), and no demand during other hours.

Fig. 2 depicts the VoS under the two different price trajectories and different (purchasing/selling) peak price ratios. For the case where the purchasing price equals the selling price, Theorem 5.1 and Corollary 5.1 show that the VoS does
not depend on the consumer’s demand, and increases linearly with the capacity $B$, as shown in Fig. 2. If the selling price is lower than the purchasing price, then the VoS becomes a piecewise linear function of the capacity $B$. The price trajectory on Aug. 1, 2011 yields a higher VoS because of the extremely high price during 2PM-3PM (stage 3).

VII. CONCLUSION

We study the economic value and optimal operation of energy storage at consumer locations, through a dynamic programming formulation. For a model with stochastic consumer demand and electricity prices, we characterize an important threshold structure of the optimal operation policy.

We define the value of storage for an even more general setting that incorporates the inter-temporal effects in consumer demand. The value of storage reflects the expected net benefit obtained by the consumer if she optimally operates the storage. We show that if the realized purchasing price always equals the selling price, then the value of storage is independent of the consumer’s demand, since it is optimal for the consumer to use the storage only for arbitrage. Under an additional assumption that the storage can be fully charged and discharged within one time period, we further show that the value of storage increases linearly with the storage capacity.

REFERENCES


