

The Effect of Fading on the Achievable Rate of Cooperative Sensor Networks with Misinformed Sensors

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Abstract—Communication from a cooperative sensor network to a mobile access point is considered in this paper. Sensors are assumed to be informed with a global message and some nodes are misinformed with random messages. The multiple access channel is i.i.d. fading and the realization of the channel state is known to the access point only. An achievable rate is derived for the information retrieval process when multiple sensors are activated at a time. For a Gaussian multiple access channel under a total network power constraint, the optimal number of simultaneous transmissions is investigated under three fading scenarios: non-fading, unit-gain, and Rayleigh-fading. With Gaussian code books, the optimal number of simultaneous transmissions varies in the three fading environments.

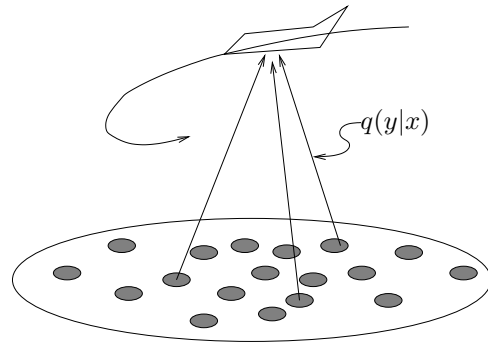


Fig. 1. Sensor Network with Mobile Access.

I. INTRODUCTION

We consider information retrieval in a cooperative Sensor Network with Mobile Access (SENMA) [1]. As illustrated in Fig. 1, SENMA contains two types of nodes: a large number of low power geographically distributed sensors, and a mobile access point in charge of collecting data from sensors. By cooperative SENMA (C-SENMA) we mean that, in communicating to the mobile access point, sensors may reach an agreement on a message and transmit using an appropriate coding scheme. Such cooperative scheme, as alternative to collaborative transmission at the signal level, makes information retrieval robust against failures of individual sensors. A coding-across-sensors scheme to cope with packet losses is presented in [2].

The process of reaching agreement, referred to as orientation, is nontrivial. Orientation can be carried out in many ways. For example, nodes may exchange information via conference links among themselves and establish a global message. Alternatively, the global message may be propagated by a software agent that travels across the sensor network. When sensor networks are viewed as a form of storage devices in which one mobile access point deposits information meant to be retrieved by other mobile access points at a different time, the process of orientation is simply the broadcast of messages from a mobile access point.

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For large scale sensor networks, perfect orientation may not be possible. In practice, there is always a possibility that some sensors do not have the correct message for transmission. We refer to such sensors as misinformed. How to retrieve information reliably from the field with the presence of misinformed sensors is not obvious. The maximum achievable rates of different system configurations are addressed in [3] and [4]. This paper focuses on C-SENMA with No Polling with No Energy constraint and studies the effect of fading on the achievable rate of a Gaussian multiple access channel under a network-wise power constraint. The optimal number of simultaneous transmissions is investigated under different fading environments: non-fading, unit-gain, and Rayleigh-fading. For the non-fading case, it is shown that the maximum achievable rate increases to infinity as the number of simultaneous transmissions increases to infinity. For the unit-gain case, the optimal number of simultaneous transmissions is one, while the optimal number of simultaneous transmissions for the Rayleigh-fading case varies from infinity to one, depending on the probability that a node is misinformed.

II. MODEL

The communication of the global message from the network to the mobile access point is divided into four steps as shown in Fig. 2: (a) orientation, (b) activation, (c) transmission and reception, and (d) decoding. In the first step, nodes are informed with the globe message $W \in \{1, \dots, M\}$ that is uniformly distributed. Due to the size of the network, a node may be

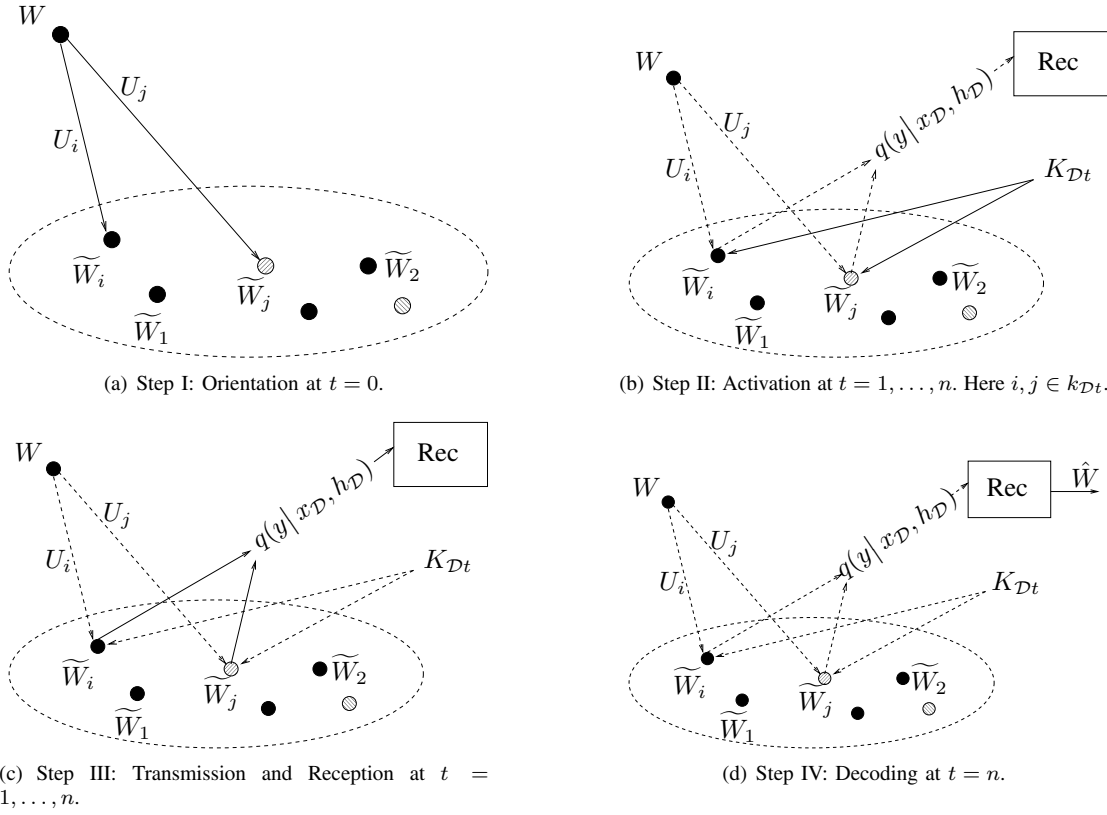


Fig. 2. Communication steps.

informed incorrectly and end up with a different message. We assume that each node receives the globe message correctly with some probability, and the reception is independent of other nodes. More specifically, the reception of node i is controlled by a binary random variable U_i , independent of W and identically independently distributed (i.i.d.) across node index i with distribution

$$p(u_i) = \begin{cases} \beta & \text{if } u_i = 0 \\ 1 - \beta & \text{if } u_i = 1 \end{cases}$$

where $\beta \in [0, 1]$ is a constant. When $U_i = 1$, the received message at node i , \tilde{W}_i , is equal to the global message W . When $U_i = 0$, \tilde{W}_i is uniformly distributed from 1 to M . Thus

$$p(\tilde{w}_i | w, u_i) = \begin{cases} \delta(\tilde{w}_i, w) & \text{if } u_i = 1 \\ \frac{1}{M} 1_{1 \leq \tilde{w}_i \leq M} & \text{if } u_i = 0 \end{cases}$$

where $\delta(a, b)$ is equal to 1 if $a = b$, 0 otherwise, and the indicating function 1_A equal to 1 if event A is true, 0 otherwise. The constant β controls the reception of the globe message by individual nodes and is referred to as the orientation error probability of the network.

The mobile access point comes to retrieve information from the field after the information orientation has been accomplished. The information retrieval process consists of Step 2 Activation and Step 3 Transmission and Reception. We assume that d nodes are scheduled to transmit at each time slot, each transmitting one symbol to the channel. Denote

K_{jt} the j -th node among the d nodes activated at time t , $1 \leq j \leq d$. The transmission from node K_{jt} at time t , denoted by X_{jt} , depends on j , the node index among the activated group, as well as the local message at this node and the time it is activated. Let $\mathcal{D} \triangleq (1, \dots, d)$ and $Z_{\mathcal{D}}$ denote (Z_1, \dots, Z_d) , where Z_1, \dots, Z_d are generic symbols. Denote $K_{\mathcal{D}t}$ the activation vector (K_{1t}, \dots, K_{dt}) and $X_{\mathcal{D}t}$ the transmission vector. The activation signals $K_{\mathcal{D}t}$ may be preset before the deployment of the sensors, thus it does not require a polling channel from the mobile access point to the sensors. We refer to this configuration as with No Polling.

Another possible system configuration is to implement a polling channel from the mobile access point to the sensors. If a polling channel is implemented, the mobile access point has the ability to poll individual sensors and the activation vector $K_{\mathcal{D}t}$ is generated by mobile access point on the fly. A polling channel enables the mobile access point to adaptively adjust the activation signal to previous receptions. Since the mobile access point is usually equipped with high-gain antennas and high-power transmitter, we assume the polling channel, if implemented, is error-free. We refer to this configuration as with Polling.

Another system configuration option is energy constraint. We refer to a system as with Energy Constraint if each node has only up to Q times of transmissions. If each node does not have such a limit, we refer to the system as with No Energy Constraint.

The uplink multiple access channel (MAC) with fading is modeled as follows. Denote $\tilde{H}_{it} \in \mathcal{H}$ the channel state associated with node i at time t . Assume that the channel states associated with nodes $K_{\mathcal{D}t}$ at time t , $\tilde{H}_{K_{\mathcal{D}t}t} \in \mathcal{H}^d$, have distribution $p(h_{\mathcal{D}})$, i.i.d. across t . The fading process $\tilde{H}_{K_{\mathcal{D}t}t}$ is independent of the transmissions from the sensors and the access point. For convenience, denote $H_{\mathcal{D}t} \triangleq \tilde{H}_{K_{\mathcal{D}t}t}$ the channel states associated with nodes $K_{\mathcal{D}t}$ activated at time t . We assume that the realization of $H_{\mathcal{D}t}$, unknown to the sensors, is known to the mobile access point at the end of time slot t . The memoryless channel output, conditioning on the simultaneous transmissions from nodes $K_{\mathcal{D}t}$ and the associated channel states, is governed by the transition probability $q(y|x_{\mathcal{D}}, h_{\mathcal{D}})$, where $y \in \mathcal{Y}$ is the channel output to the mobile access point, $h_j \in \mathcal{H}$ the channel state associated with the j -th node among the d transmitting nodes, $1 \leq j \leq d$, and $x_j \in \mathcal{X}$ the transmission from the j -th among the d nodes.

Let $\pi_{\mathcal{D}}$ be a permutation in the domain \mathcal{D} . From the above assumption, the channel states associated with vector $(K_{\pi_1 t}, \dots, K_{\pi_d t})$ has the identical distribution $p(h_{\mathcal{D}})$. Therefore, a necessary condition for $p(h_{\mathcal{D}})$ is that $p(h_{\mathcal{D}})$ is symmetrical with respect to input permutations, i.e., $p(h_{\pi_1}, \dots, h_{\pi_d})$ is identical for all permutations $\pi_{\mathcal{D}}$. Similarly, $q(y|x_{\mathcal{D}}, h_{\mathcal{D}})$ needs to be symmetrical with respect to node permutation, i.e., $q(y|x_{\pi_1}, \dots, x_{\pi_d}, h_{\pi_1}, \dots, h_{\pi_d})$ is identical for all permutations $\pi_{\mathcal{D}}$.

After receiving the channel output Y_t and channel states $H_{\mathcal{D}t}$, the mobile access point moves to the next time slot $t+1$ and the activation step starts again. Step 2 and Step 3 alternate until t reaches n , the number of time slots the mobile access point spends to retrieve information from the field.

In the last step, the access point decodes the globe message based on the observation of Y^n , $H_{\mathcal{D}}^n$ and the activation history $K_{\mathcal{D}}^n$. The decoded message is denoted by $\hat{W} \in \{1, \dots, M\}$. We assume that the sensor network is large in the sense that there are infinite number of nodes. The large network assumption is to make sure that the probability of all nodes being misinformed is zero. Thus it is possible to have a positive achievable rate. A transmission example when $d=1$ is shown in Fig. 3, where \mathbb{C} represents the shared code book, and the actual symbols transmitted are labeled by solid dots in the code book.

The rate of a code book is defined as $R \triangleq \log_2(M)/n$, where M is the number of messages in the code book and n is the length of a codeword. The probability of error is defined as $\mathcal{P}_e \triangleq \mathcal{P}(\hat{W} \neq W)$, where $W \in \{1, \dots, M\}$ is uniformly distributed and \hat{W} is the decoded message. A rate R is called achievable if for any given error probability $\epsilon > 0$, there exists a code book with rate larger than $R - \epsilon$ and probability of error less than ϵ .

III. ACHIEVABLE RATE RESULTS

For $d=1$, i.e., activate one node at a time, the MAC reduces to discrete memoryless channel (DMC) with fading $q(y|x, h)$. We have the following results for $d=1$:

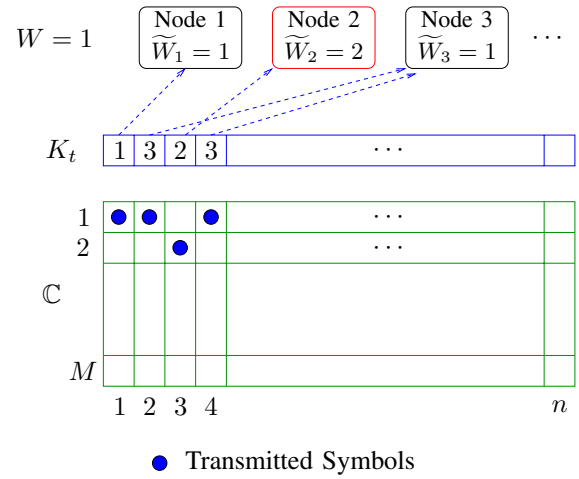


Fig. 3. Transmission example ($d=1$).

Theorem 1: For $d=1$, the maximum achievable rate of C-SENMA with No Polling with No Energy constraint (NPNE) is

$$C_1^{\text{NPNE}} = (1 - \beta)C_1^{(0)},$$

where $C_1^{(0)} = \max_{p(x)} I(X; Y|H)$ is the capacity of the DMC with fading.

Theorem 2: For $d=1$, the maximum achievable rate of C-SENMA with No Polling with Energy constraint (NPE) is

$$C_1^{\text{NPE}} = \max_{1 \leq k \leq Q} R_k$$

where Q is the maximum number of transmissions allowed to one sensor,

$$R_k = \frac{1}{k} \max_{p(s^k)} I(S^k; Y^k | H^k),$$

$S^k \in \mathcal{X}^k$, $Y^k \in \mathcal{Y}^k$, $H^k \in \mathcal{H}^k$, and

$$p(s^k, y^k, h^k) = p(s^k) \cdot \prod_{i=1}^k p(h_i) \cdot \left((1 - \beta) \prod_{i=1}^k q(y_i | s_i, h_i) + \beta \sum_{s'^k \in \mathcal{X}^k} p(s'^k) \prod_{i=1}^k q(y_i | s'_i, h_i) \right).$$

For $d \geq 1$, the following results are obtained:

Theorem 3: Let

$$C_d^{(0)} = \max_{p(x_{\mathcal{D}})} I(X_{\mathcal{D}}; Y | H_{\mathcal{D}})$$

where $p(x_{\mathcal{D}}, h_{\mathcal{D}}, y) = p(x_{\mathcal{D}})p(h_{\mathcal{D}})q(y|x_{\mathcal{D}}, h_{\mathcal{D}})$. The maximum achievable rate for C-SENMA with Polling with No Energy constraint (PNE) with d simultaneous transmissions is

$$C_d^{\text{PNE}} = \begin{cases} C_d^{(0)} & \text{if } \beta < 1, \\ 0 & \text{if } \beta = 1. \end{cases}$$

Theorem 4: For a given d , consider random variables $X_{\mathcal{D}}$ with distribution $p(x_{\mathcal{D}})$. Denote $p_{\mathcal{I}}(x_{\mathcal{I}})$ the marginal distribution of $X_{\mathcal{I}}$ for $\mathcal{I} \subset \mathcal{D}$. The following rate is achievable for C-SENMA NPNE with d simultaneous transmissions,

$$C_d^{\text{NPNE}} = \max_{p(x_{\mathcal{D}})} \sum_{\mathcal{I} \subset \mathcal{D}} (1 - \beta)^{|\mathcal{I}|} \beta^{d-|\mathcal{I}|} I(X_{\mathcal{I}}^{(\mathcal{I})}; Y | H_{\mathcal{D}}), \quad (1)$$

where $X_{\mathcal{D}}^{(\mathcal{I})} = (X_1^{(\mathcal{I})}, \dots, X_d^{(\mathcal{I})})$ are derived from $X_{\mathcal{D}}$ with distribution

$$p^{(\mathcal{I})}(x_{\mathcal{D}}^{(\mathcal{I})}) = p_{\mathcal{I}}(x_{\mathcal{I}}^{(\mathcal{I})}) \prod_{j \notin \mathcal{I}} p_j(x_j^{(\mathcal{I})}),$$

$H_{\mathcal{D}}$, independent of $X_{\mathcal{D}}^{(\mathcal{I})}$, has distribution $p(h_{\mathcal{D}})$, and $p(y | x_{\mathcal{D}}^{(\mathcal{I})}, h_{\mathcal{D}}) = q(y | x_{\mathcal{D}}^{(\mathcal{I})}, h_{\mathcal{D}})$.

Due to the space limit, the proofs, which are available in [3], are not presented here.

IV. GAUSSIAN MULTIPLE ACCESS CHANNELS

Consider the following Gaussian multiple access channel

$$Y = V + \sum_i \tilde{H}_i \tilde{X}_i$$

where $V \in \mathcal{C}$ is the additive white Gaussian noise with zero mean and unit variance, $\tilde{X}_i \in \mathcal{C}$ the input from the i -th sensor, $\tilde{H}_i \in \mathcal{C}$ the fading channel gain associated with the i -th sensor, and $Y \in \mathcal{C}$ the channel output. We impose a total power constraint P on the network¹, i.e., the total transmit power from all sensors is less than or equal to P :

$$\frac{1}{n} \sum_{t=1}^n \sum_{j=1}^d |x_{jt}|^2 \leq P$$

where x_{jt} is the transmission from the j -th node among the d transmitting nodes at time t . We assume that the channel gains \tilde{H}_i 's have distribution $p(h)$, i.i.d. across sensors and time. For convenience, denote X_j and H_j the transmission from and the channel gain associated with the j -th node among the activated group respectively, $1 \leq j \leq d$. Let $H_{\mathcal{D}} = (H_1, \dots, H_d)$ be the channel gains associated with the d transmitting sensors. The realization of the channel states $H_{\mathcal{D}}$ is assumed to be known to the mobile access point. We focus on C-SENMA NPNE, and an achievable rate is given by (1) with the power constraint on the input distribution, $\sum_{j=1}^d E[|X_j|^2] \leq P$. We consider three types of fading channels: non-fading, unit-gain, and Rayleigh-fading. The optimal number of simultaneous transmissions is studied for Gaussian code books.

A. Non-fading

In the non-fading case, $p(h) = \delta(h - 1)$, i.e., the channel gain for each sensor is 1. From the multiple-input-single-output (MISO) channel capacity, we know that $C_d^{(0)} = \log_2(1 + dP) = O(\log_2 d)$, which goes to infinity as d

increases. Next we show that C_d^{NPNE} is still $O(\log_2 d)$ as long as $\beta < 1$.

Consider activating d sensors at a time. Let the input random vector $X_{\mathcal{D}} = (X_1, \dots, X_d) \sim \mathcal{N}_C(\mathbf{0}, \frac{P}{d} \mathbf{1}\mathbf{1}^T)$, where $\mathbf{1} = [1, 1, \dots, 1]^T$. For $\mathcal{I} \subset \mathcal{D}$, since the derived random variables $X_{\mathcal{I}}^{(\mathcal{I})}$, independent of each other, are independent of $X_{\mathcal{I}}^{(\mathcal{I})}$, $X_{\mathcal{I}}^{(\mathcal{I})}$ contribute $(d - |\mathcal{I}|)P/d$ power to the additive noise. Therefore,

$$I(X_{\mathcal{I}}^{(\mathcal{I})}; Y) = \log_2 \left(1 + \frac{|\mathcal{I}|^2 P/d}{1 + (d - |\mathcal{I}|)P/d} \right).$$

From Theorem 4, the following rate is achievable,

$$R_d^{\text{NF}} = \sum_{i=0}^d (1 - \beta)^i \beta^{d-i} \binom{d}{i} \log_2 \left(1 + \frac{i^2 P/d}{1 + (d - i)P/d} \right).$$

The next proposition shows that $R_d^{\text{NF}} = O(\log_2 d)$. Since $R_d^{\text{NF}} \leq C_d^{\text{NPNE}} \leq C_d^{(0)} = O(\log_2 d)$, we have $C_d^{\text{NPNE}} = O(\log_2 d)$.

Proposition 1: For $\beta \in [0, 1)$ and $P > 0$,

$$\lim_{d \rightarrow \infty} (R_d^{\text{NF}} - \log_2 d) = \log_2 \left(\frac{(1 - \beta)^2 P}{1 + \beta P} \right).$$

Due to the space limit, refer to [3] or [4]² for the proof. Proposition 1 implies that for the non-fading case, R_d^{NF} grows at the rate of $\log_2 d$, increasing to infinity as d goes to infinity. Let $f(a, b) = \log_2(a + \frac{b^2 P}{1 + (1 - b)P})$. Fig. 4(a) shows the achievable rate R_d^{NF} and the approximation function $\log_2 d + f(0, 1 - \beta)$ versus d when $P = 10$ and $\beta = 0.1, 0.3, 0.5$. As shown in Fig. 4(a), R_d^{NF} and the approximation function converge as d increases. As expected, the achievable rate is higher for a network with a smaller orientation error probability.

B. Unit-gain

In the unit-gain case, $p(h)$ is a uniform distribution on the unit circle.

Proposition 2: Suppose d sensors are activated at a time. With Gaussian code books $X_{\mathcal{D}} \sim \mathcal{N}_C(\mathbf{0}, \Sigma)$ where $\text{Tr}(\Sigma) \leq P$, the maximum achievable rate, optimized over Σ , is

$$R_{\max}^{\text{UG}} = (1 - \beta) \log_2(1 + P).$$

The maximum is achieved when Σ has only one non-zero diagonal entry, which is equal to P .

Due to the space limit, refer to [3] for the proof. Proposition 2 suggests that, with Gaussian code books, the optimal d for the unit-gain case is one, i.e., it is better to activate one sensor at a time. To see how the achievable rate depends on

¹If, instead, a power constraint is posted on individual nodes, the total transmit power can reach infinity if more and more nodes transmit simultaneously.

²Instead of the maximum achievable rate for C-SENMA PNE, results presented in [4] are in fact achievable rates for C-SENMA NPNE with no fading.

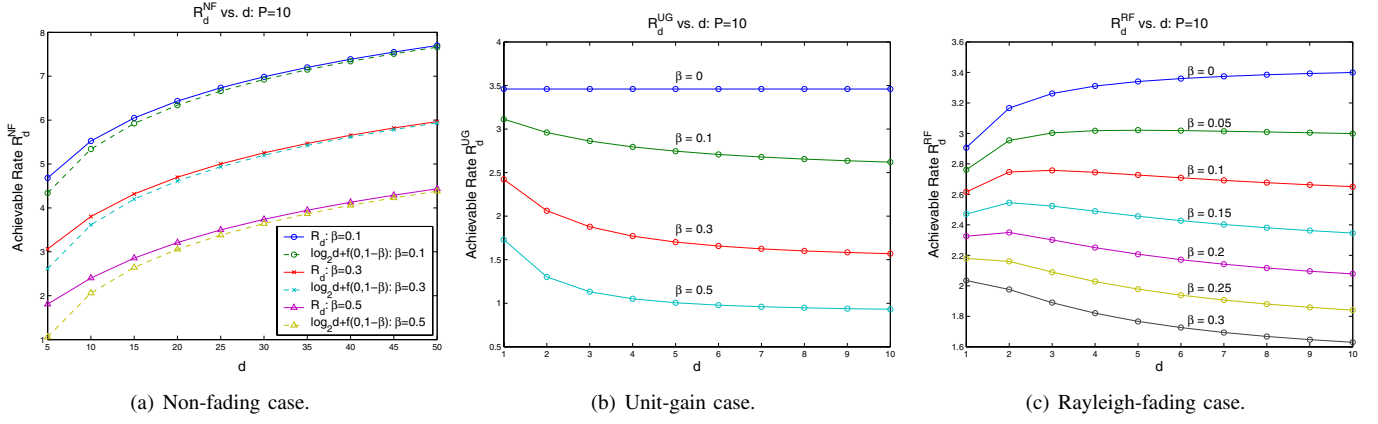


Fig. 4. Achievable rates versus d : $P = 10$ and various β 's.

d , consider input distribution $X_D \sim \mathcal{N}_C(\mathbf{0}, \frac{P}{d} \mathbf{I})$ for a given d . From Theorem 4, the achievable rate is given by

$$R_d^{\text{UG}} = \sum_{i=1}^d (1-\beta)^i \beta^{d-i} \binom{d}{i} \log_2 \left(1 + \frac{iP/d}{1 + (d-i)P/d} \right).$$

Fig. 4(b) shows the achievable rate R_d^{UG} versus d when $P = 10$ and $\beta = 0, 0.1, 0.3, 0.5$. As expected from Proposition 2, R_d^{UG} achieves maximum when $d = 1$.

C. Rayleigh-fading

In the Rayleigh-fading case, $p(h)$ is the distribution of a complex Gaussian with zero mean and unit variance. Consider activating d sensors at a time and using a Gaussian code book $X_D \sim \mathcal{N}_C(\mathbf{0}, \frac{P}{d} \mathbf{I})$. For $\mathcal{I} \subset \mathcal{D}$, let $i = |\mathcal{I}|$. We have

$$\begin{aligned} I(X_{\mathcal{I}}^{(T)}; Y | H_{\mathcal{D}}) &= E \log_2 \left(1 + \frac{\sum_{j \in \mathcal{I}} |H_j|^2 P/d}{1 + \sum_{j \in \mathcal{I}^c} |H_j|^2 P/d} \right) \\ &= \begin{cases} \int_0^\infty \int_0^\infty \log_2 \left(1 + \frac{z_1 P/d}{1 + z_2 P/d} \right) \frac{z_1^{i-1} e^{-z_1}}{(i-1)!} \frac{z_2^{d-i-1} e^{-z_2}}{(d-i-1)!} dz_1 dz_2 & \text{if } 0 < i < d \\ \int_0^\infty \log_2 \left(1 + \frac{z_1 P}{d} \right) \frac{z_1^{d-1} e^{-z_1}}{(d-1)!} dz_1 & \text{if } i = d \\ 0 & \text{if } i = 0 \end{cases} \quad (2) \\ &\triangleq F(d, i), \end{aligned}$$

where (2) uses the fact that $Z = \sum_{j \in \mathcal{I}} |H_j|^2$ is chi-square distributed with $2i$ degrees of freedom and distribution $\frac{z^{i-1} e^{-z}}{(i-1)!}$, [5]. From Theorem 4, the following rate is achievable,

$$R_d^{\text{RF}} = \sum_{i=1}^d (1-\beta)^i \beta^{d-i} \binom{d}{i} F(d, i).$$

Fig. 4(c) shows the achievable rate R_d^{RF} versus d when $P = 10$ and $\beta = 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$. As shown in Fig. 4(c), when $\beta = 0$, R_d^{RF} increases monotonically and converges to the limit $\log_2(1 + P)$ as d increases, which is expected because of the diversity gain. With $\beta > 0$, R_d^{RF} is no longer monotonically increasing. R_d^{RF} first increases and

then decreases as d increases. As β increases, the optimal d decreases, reaching one when $\beta \geq 0.25$ in Fig. 4(c).

The optimal number of sensors to activate at a time is infinity for the non-fading case, one for the unit-gain case. For the Rayleigh-fading case, the optimal number decreases as the orientation error probability increases. The phenomenon is related to the channel diversity gain associated with activating more than one sensor at a time. For the unit-gain case, there is no channel diversity gain. Activating more than one node at a time reduces the achievable rate due to the interference transmission from misinformed nodes. For the non-fading case, the diversity gain is so strong that it counteracts the rate loss due to interference from misinformed sensors. For the Rayleigh-fading case, the diversity gain is in between. The optimal d has a tradeoff between the diversity gain and the interference loss.

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