Capacity of Cooperative Sensor Networks with Sensor Errors

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Abstract— **The communication from a cooperative sensor network to a mobile access point is considered in this paper. Sensors are assumed to be informed with a global message, and some nodes are misinformed with random messages. Three system configurations are discussed based on whether a polling channel is implemented and whether an energy limit is imposed on individual sensors. The capacity of these system configurations are derived. It is shown that if a polling channel is implemented and there is no energy constraint on individual nodes, there is no rate loss with the presence of sensor errors. If there is no polling channel implemented and there is no energy constraint on individual nodes, the rate loss is proportional to the probability that a node is misinformed.**

I. INTRODUCTION

A. Cooperative Sensor Networks with Sensor Errors

Information retrieval is a crucial component for large scale sensor networks in which nodes are constrained by transmission power and subject to various types of channel impairment. Thus the idea of cooperation among nodes for the purpose of delivering information reliably and efficiently has attracted much attention [1]–[5]. Cooperation can be made at different levels: a collection of nodes collaborating at the signal level, transmitting as if they are part of an antenna array and beaming a common message to the receiving node [4]. Nodes can also collaborate using information theoretic strategies [2], jointly encoding information and delivering the message at a rate that ensures reliable recovery at the receiver.

If nodes in a sensor network are made of low cost and low power sensors, it is inevitable that sensors make decision errors. The presence of sensor errors has an impact on cooperation. Suppose that a group of sensors are to transmit message W cooperatively. Suppose that in the process of distributing message W, some members of the group do not receive W correctly; decoding errors make their message something other than W. Will cooperative transmission help to mitigate such errors? Is there a price paid for the presence of sensor errors?

We consider cooperative information retrieval from an information theoretic point of view for a special type of sensor network: SEnsor Networks with Mobile Access (SENMA) [6]. As illustrated in Fig. 1, SENMA contains two types of nodes: a large number of low power geographically distributed sensors, and a mobile access point (AP) in charge of collecting data from sensors. By cooperative SENMA (C-SENMA) we mean that, in communicating a message to the mobile access point, each sensor transmits part of the codeword from an optimally designed code book. In other words, messages are coded across sensors, which makes the information retrieval robust against failures of individual sensors.

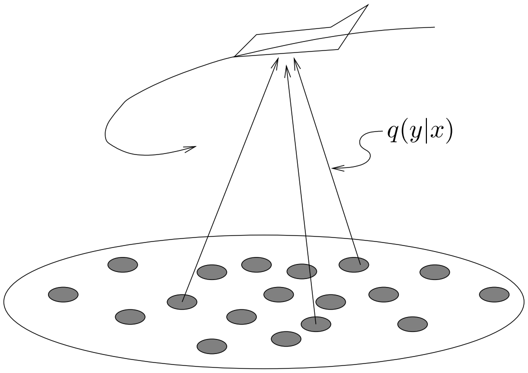

Fig. 1. SEnsor Networks with Mobile Access.

The presence of mobile AP makes the problem of cooperative information retrieval more tractable. Had there been no sensor error, the problem would have resembled one of communicating over a multi-input single-output channel. The information theoretical characterization of such a problem is well known. But the presence of sensor errors changes the basic setup of the standard information theory approach because one cannot simply incorporate sensor error into the channel model. Specifically, the sensor error depends on the code book used in the transmission[1].

B. Main Results

In this paper, we present capacity result for three configurations of C-SENMA, depending on whether the mobile AP can poll individual sensors and whether there is an energy constraint that limits the number of times a sensor can transmit.

If there is a polling channel, the mobile AP can decide adaptively whether to stay with a particular sensor or switch

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[1] When sensor makes a mistake, it chooses a wrong codeword. If the particular letter (bit) of the wrong codeword matches the letter of the correct codeword, the transmission is as if the sensor has the correct message.

to a different one during the information retrieval process. The design challenge for such systems is to choose an optimal strategy for adapting the polling sequence according to the previous receptions, and the achievable rate has to be optimal for all polling strategies. If there is no polling channel, then sensors transmit according to a fixed schedule. The design of the code book affects the interference seen at the receiver, making it more difficult to analyze than a regular discrete memoryless channel (DMC).

The first case is C-SENMA with Polling with No Energy constraint (PNE). If β is the probability that a sensor is misinformed[2] and only one sensor can transmit at a time, then sensor errors can be corrected completely via coding provided that $\beta < 1$. Specifically, if the channel between each sensor and the mobile access point is identical DMC $q(y|x)$, the capacity of C-SENMA PNE is, for all $\beta < 1$,

$$C^{\mathrm{PNE}} = \max_{p(x)} I(X;Y).$$

For C-SENMA with No Polling with No Energy constraint (NPNE), the capacity is shown to be

$$C^{\mathrm{NPNE}} = (1-\beta) \max_{p(x)} I(X;Y).$$

We also prove the capacity of C-SENMA with No Polling with Energy constraint (NPE).

C. Related Work

Cooperation among nodes for the purpose of communication can be found in many forms according to the traffic pattern, *e.g.*, *broadcast problem* where information is delivered from a source to all other nodes in the network with nodes relaying the information [7], [8], *relay problem* where information is delivered from a source to a destination with the help of relay nodes [9], [10], *multiple access problem* where information is delivered to a common destination from multiple nodes which assist each other in communication [2]. In this paper, we assume that nodes in the network have established a global message to deliver to the mobile access point and cooperation for delivering is done among nodes. This traffic pattern can also be found in [11]. If all the nodes are assumed to know the exact global message, there is no difference between multiple cooperating nodes and multiple transmitting antennas (in multiple-input single output or multiple-input multiple-output channels). In this paper, we take into account sensor errors in the cooperation process under a specific sensor network architecture.

C-SENMA with sensor errors was first considered in [12] and later in [13]. For C-SENMA without polling, achievable rate expressions were obtained in [12][3] for C-SENMA with multiple simultaneous transmissions and the effect of fading on the achievable rate was investigated in [13]. In this paper, the results are the *capacity* of C-SENMAs with one sensor transmitting at a time for the three system configurations.

II. MODEL

We first describe the channel model of C-SENMA PNE which implements a polling channel. The communication of the global message from the network to the mobile access point in C-SENMA PNE is divided into four steps as shown in Fig. 2: (a) orientation, (b) polling, (c) transmission and reception, and (d) decoding. In the first step, nodes are informed with the global message $W \in \{1, \ldots, M\}$ that is uniformly distributed. Due to the size of the network, a node may be informed incorrectly and end up with a different message. We assume that each node receives the global message correctly with a certain probability and the reception is independent of other nodes. More specifically, the reception of node i is controlled by a binary random variable U_i, independent of W and identically independently distributed (i.i.d.) across node index i with distribution

$$p(u_i) = \begin{cases} \beta & \text{if } u_i = 0 \\ 1-\beta & \text{if } u_i = 1 \end{cases}$$

where $\beta \in [0,1]$ is a constant. When $U_i = 1$, the received message at node i, $\widetilde{W}_i$, is equal to the global message W. When $U_i = 0$, $\widetilde{W}_i$ is uniformly distributed from 1 to M. Thus

$$p(\tilde{w}_i | \, w, u_i) = \begin{cases} \delta(\tilde{w}_i, w) & \text{if } u_i = 1 \\ \frac{1}{M} & \text{if } u_i = 0 \end{cases}$$

where $\delta(a,b)$ is equal to 1 if $a = b$, 0 otherwise. The constant β controls the reception of the global message by individual nodes and is referred to as the *orientation error probability* of the network.

The mobile access point comes to retrieve information from the field after the information orientation has been accomplished. The information retrieval process consists of Step 2 Polling and Step 3 Transmission and Reception. To avoid collision, a polling-based multiple access is employed: the mobile access point polls one node to transmit one symbol at each time slot. At time t, the receiver polls node K_t to transmit the t-th symbol of the codeword corresponding to the $\widetilde{W}_{K_t}$-th message. Since the mobile access point is usually not power-limited, we assume that the polling channel is error free. The uplink channels from each node to the receiver are assumed to be identical and modeled by a DMC $\{\mathcal{X}, \mathcal{Y}, q(y| \, x)\}$, where $\mathcal{X}$ and $\mathcal{Y}$ are the input and output alphabets respectively, and $q(y| \, x)$ is the transition probability of the channel. Node K_t, after receiving the polling signal, transmits the selected symbol to the uplink channel. Denote X_t and Y_t the input and output of the DMC at time t. After receiving Y_t, the mobile access point moves to the next time slot $t+1$ and starts the polling step again. It may poll a node that has or has not been polled before. Step 2 and Step 3 alternate until t reaches n, where n is the number of slots the receiver spends to retrieve information from the field.

In the last step, the receiver decodes the global message based on the channel outputs Y^n and the polling history K^n. The decoded message is denoted by $\hat{W} \in \{1, \ldots, M\}$. An decoding error occurs if $\hat{W} \neq W$. We assume that the sensor

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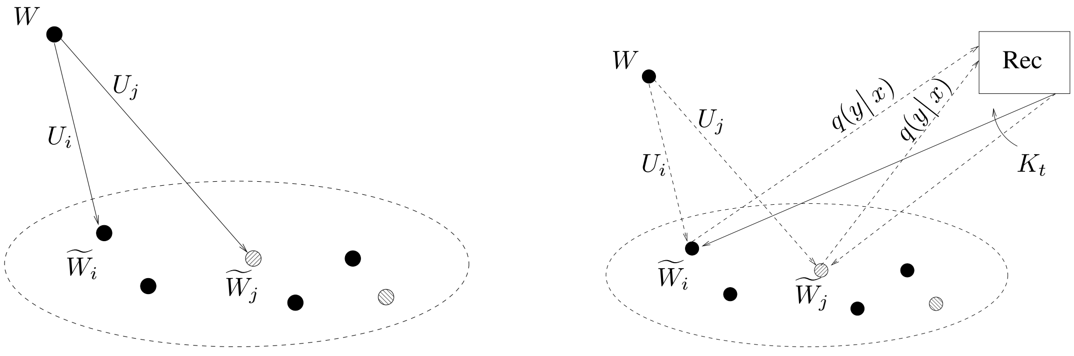
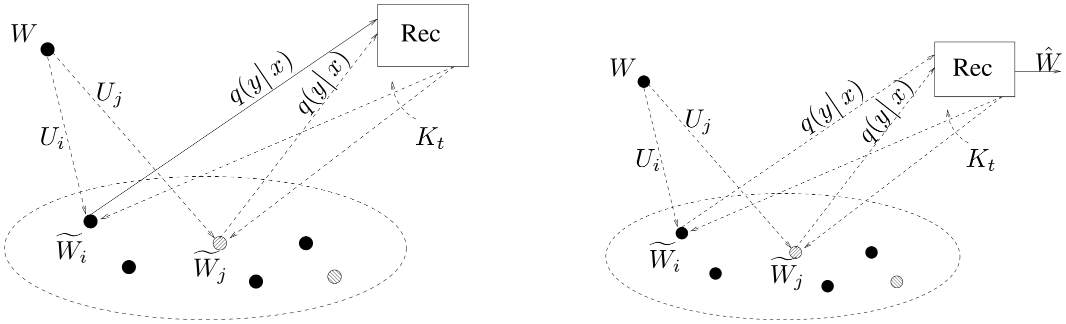
[2] When the sensor is misinformed, it chooses the message randomly with equal probability.

[3] Instead of the capacity of C-SENMA PNE, results presented in [12] are in fact achievable rates for C-SENMA NPNE with no fading.



(a) Step I: Orientation

(b) Step II: Polling



(c) Step III: Transmission and Reception

(d) Step IV: Decoding

Fig. 2. Communication steps.

network is large in the sense that there are infinite number of nodes[4].

In the above scheme, the mobile access point has a polling channel to address individual sensors. The polling channel could be costly to implement since the bandwidth needed to address individual sensors in a large network is huge. Therefore, it is also desired to consider schemes with predetermined scheduling, *i.e.*, the polling sequence is preset before the deployment of the sensors. Therefore, in C-SENMA NPNE and NPE, $\{K_t\}$ do not depend on the channel outputs.

For C-SENMA PNE and NPNE, we assume that there is no limit on how many times a sensor can transmit. In reality, battery-powered sensors are energy limited, which may post constraints on the lifetime of sensors. Thus, for C-SENMA NPE, we assume that each sensor has up to Q transmissions. Fig. 3 shows a transmission example of C-SENMA, where $\mathbb{C}$ represents the shared code book, and the actual transmitted symbols are labeled by solid dots in the code book.

The *rate* of a code book is defined as $R \triangleq \log(M)/n$, where M is the number of messages in the code book and n is the length of a codeword. The *probability of error* is defined as $P_e \triangleq \mathcal{P}(\hat{W} \neq W)$, where $W \in \{1, \cdots, M\}$ is uniformly distributed and $\hat{W}$ is the decoded message. A rate R is called *achievable* if for any given error $\epsilon > 0$, there exists a code book with rate larger than $R - \epsilon$ and probability of error less that ϵ. The capacity of a system configuration is defined as the maximum of all achievable rates for the system configuration.

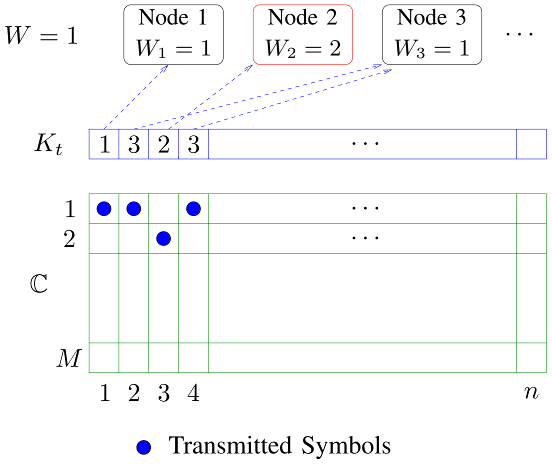


Fig. 3. Transmission example.

In the next three sections, we study the capacity of the three system configurations.

III. CAPACITY OF C-SENMA PNE

When the polling channel is implemented, it is possible for the mobile access point to first locate a sensor that, with a high probability, has the correct global message, and then retrieve the global message from that sensor. If the time slots needed to

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[4] The large network assumption is necessary for a non-zero capacity since, if the network has only finite nodes, there is a positive probability that all the nodes are misinformed.

locate such a sensor is only a function of the given probability of error, but not a function of the codeword length, then the capacity is just the capacity of the DMC $(\mathcal{X}, \mathcal{Y}, q(y|\, x))$

$$C^{(0)} = \max_{p(x)} I(X;Y),$$

since the overhead associated with the first phase can be made arbitrarily small by increasing the codeword length. This strategy is elaborated in the proof of the following theorem, which, along with a sketch of the proof, were suggested by an anonymous reviewer for 2004 IEEE ISIT.

Theorem 1: The capacity of C-SENMA PNE is

$$C^{\mathrm{PNE}} = \begin{cases} C^{(0)} & \text{if } \beta < 1, \\ 0 & \text{if } \beta = 1. \end{cases}$$

Proof: For all $\epsilon > 0$, for all $R_1 < C^{(0)}$, by the definition of the channel capacity of a DMC, there exists a n_0 such that for all $n_1 > n_0$, there exists a code $(2^{n_1 R_1}, n_1, \epsilon)$ over the DMC, where (M, n, ϵ) means a code with M messages, codeword length n, and probability of error less than ϵ. Let $M = 2^{n_1 R_1}$ be the total number of messages. Let $j > 1/\epsilon$. Select l large enough such that there exists a code (j, l, ϵ) over the DMC. Let $k > \log(\epsilon)/\log(1-(1-\epsilon)^2(1-\beta)^2)$. Divide the M messages evenly into j groups[5]. The mobile access point consecutively polls $2k$ sensors, each of which transmits its group index of its message using the (j, l, ϵ) code. The total number of channel uses for this phase is $2kl$, which is only a function of ϵ. The mobile access point decodes the $2k$ message group indices from the $2k$ sensors and checks if there exists $1 \le i \le k$ such that the $(2i-1)$-th decoded group index matches the $2i$-th decoded group index. If there is no such an i, the mobile access point declares an error; otherwise, the mobile access selects the smallest i and polls the $(2i-1)$-th sensor to transmit its message using the $(2^{n_1 R_1}, n_1, \epsilon)$ code. The decoded message is declared as the global message. The total channel uses are $n = 2kl + n_1$ and the communication rate is given by

$$R = \frac{n_1}{2kl + n_1} R_1,$$

which converges to R_1 as n_1 increases.

It can be shown[6] that the probability of error is upper bounded by

$$\mathcal{P}_e \le 2\epsilon + \frac{5\epsilon}{(1-\beta)^2}.$$

For $\beta < 1$, the probability of error $\mathcal{P}_e$ converges to zero as ϵ goes to zero and the communication rate R converges to R_1 as n_1 goes to infinity, where R_1 can be arbitrarily close to $C^{(0)}$. Therefore, $C^{(0)}$ is achievable for $\beta < 1$. Since the achievable rate of C-SENMA PNE cannot exceed $C^{(0)}$, we have proved Theorem 1. □

IV. CAPACITY OF C-SENMA NPE

With the polling channel, C-SENMA PNE achieves the channel capacity of the DMC. For systems without polling channels, the achievable rate is expected to be lower. Next, we study the capacity of C-SENMA NPE.

Theorem 2: The capacity of C-SENMA NPE is

$$C^{\mathrm{NPE}} = \max_{1 \le k \le Q} R_k$$

where Q is the maximum number of transmissions allowed to one sensor,

$$R_k = \frac{1}{k} \max_{p(s^k)} I(S^k; Y^k),$$

$S^k \in \mathcal{X}^k$, $Y^k \in \mathcal{Y}^k$, and

$$p(s^k, y^k) = p(s^k) \cdot \Big((1-\beta) \prod_{i=1}^{k} q(y_i|\, s_i) + \beta \sum_{s'^k \in \mathcal{X}^k} p(s'^k) \prod_{i=1}^{k} q(y_i|\, s'_i) \Big). \quad (1)$$

A. Achievability of Theorem 2

We first prove the achievability of R_1, from which we will derive the achievability of R_k. To achieve R_1, a new sensor is scheduled to transmit in every time slot.

Code book Generation: Generate a code book with $M = 2^{nR}$ messages and codeword length n at random according to the distribution $p(s)$. Let all sensors have the same code book.

Decoder: Typical set decoding is employed. Define the typical set $A_\epsilon^{(n)}$ with respect to the distribution $p(s, y)$ defined in (1) for $k = 1$,

$$\begin{aligned} A_\epsilon^{(n)} \triangleq \{ (s^n, y^n) \in \mathcal{X}^n \times \mathcal{Y}^n :& \\ & \Big| -\frac{1}{n} \log p(s^n) - H(S) \Big| \le \epsilon, \\ & \Big| -\frac{1}{n} \log p(y^n) - H(Y) \Big| \le \epsilon, \\ & \Big| -\frac{1}{n} \log p(s^n, y^n) - H(S,Y) \Big| \le \epsilon \}, \end{aligned}$$

where $p(s^n, y^n) = \prod_{i=1}^n p(s_i, y_i)$ [7]. Upon receiving channel outputs y^n, the receiver declares the message $\hat{w}$ as the received message if there is one and only one $\hat{w} \in \mathcal{W}$ such that $(s^n(\hat{w}), y^n) \in A_\epsilon^{(n)}$; otherwise, the receiver declares an error.

Due to the space limit, the error analysis is omitted. It can be shown that the average probability of error, averaged over all codewords and all code books, is bounded as

$$\mathcal{P}_e^{(n)} \le \epsilon + 2^{-n(I(S;Y) - R - 4\epsilon)},$$

for n large. If $R < I(S;Y)$, $\mathcal{P}_e^{(n)}$ can be made arbitrarily small by letting n go to infinity and ϵ go to zero, thus proving the achievability of $R_1 = \max_{p(s)} I(S;Y)$.

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[5] Some groups may have one more message than others. For j fixed, as n_1 increases, the ratio of the number of messages in each group over the total number of messages converges to $1/j$.

[6] Due to the space limit, some steps in the proofs of this paper are omitted. More detailed proofs are presented in [14].

[7] The distribution $p(s^n, y^n)$ is derived from $p(s, y)$, which is given by (1) for $k = 1$. It should not be confused with $p(s^k, y^k)$ in (1)

To prove the achievability of R_k, consider the k-th extended C-SENMA where the DMC is k-th extension of the original DMC,

$$q(y^k\,|\,x^k) = \prod_{i=1}^{k} q(y_i\,|\,x_i).$$

Schedule a new sensor to transmit in every time slot and let $p(s^k)$ be the input distribution in the k-th extended system. From the proof of the achievability of R_1, $I(S^k;Y^k)$ is achievable by the k-th extended system. Now consider scheduling a new sensor to transmit in every k time slots in the original system. The operation of the original C-SENMA with a new sensor in every k slots is equivalent to that of the k-th extended C-SENMA with a new sensor in every slot, except that, in the original system, it takes k times longer to transmit one codeword. Therefore, $R_k = \frac{1}{k}\max_{p(s^k)} I(S^k;Y^k)$ is achievable by the original system.

B. Converse

For a given n, suppose there is a total number of a nodes involved in the transmission of the codeword. Let $\mathcal{I}_i$ be the set of time slots allocated to node i, $1 \le i \le a$. By the energy constraint, $|\mathcal{I}_i| \le Q$ for all $1 \le i \le a$. Let S^n denote a codeword which is uniformly distributed among all codewords in the code book, and let Y^n denote the channel outputs. By Fano's inequality, for all achievable rate R,

$$\begin{aligned} nR &= H(S^n) \\ &= H(S^n\,|\,Y^n) + I(S^n;Y^n) \\ &\le 1 + \mathcal{P}_e^{(n)} nR + I(S^n;Y^n). \end{aligned}$$

Since $\mathcal{P}_e^{(n)}$ goes to zero as n goes to infinity, for n large, the achievable rate R is upper bounded by the C^{NPE} as follows,

$$\begin{aligned} R &\le \frac{1}{n} I(S^n;Y^n) \\ &= \frac{1}{n}\Big(H(Y^n) - H(Y^n\,|\,S^n)\Big) \\ &= \frac{1}{n}\sum_{i=1}^{a}\Big(H(Y_{\mathcal{I}_i}\,|\,Y_{\cup_{j<i}\mathcal{I}_j}) - H(Y_{\mathcal{I}_i}\,|\,S^n, Y_{\cup_{j<i}\mathcal{I}_j})\Big) \\ &= \frac{1}{n}\sum_{i=1}^{a}\Big(H(Y_{\mathcal{I}_i}\,|\,Y_{\cup_{j<i}\mathcal{I}_j}) - H(Y_{\mathcal{I}_i}\,|\,S_{\mathcal{I}_i})\Big) \qquad (2) \\ &\le \frac{1}{n}\sum_{i=1}^{a}\Big(H(Y_{\mathcal{I}_i}) - H(Y_{\mathcal{I}_i}\,|\,S_{\mathcal{I}_i})\Big) \\ &= \frac{1}{n}\sum_{i=1}^{a} I(S_{\mathcal{I}_i};Y_{\mathcal{I}_i}) \\ &\le \frac{1}{n}\sum_{i=1}^{a} |\mathcal{I}_i| C^{\mathrm{NPE}} \qquad (3) \\ &= C^{\mathrm{NPE}}, \end{aligned}$$

where (2) is because, conditioning on $S_{\mathcal{I}_i}$, $Y_{\mathcal{I}_i}$ is independent of $S^n \setminus S_{\mathcal{I}_i}$ and $Y_{\cup_{j<i}\mathcal{I}_j}$, and (3) is because

$$C^{\mathrm{NPE}} = \max_{1\le k\le Q} R_k \ge \frac{1}{|\mathcal{I}_i|} I(S_{\mathcal{I}_i};Y_{\mathcal{I}_i}).$$

V. CAPACITY OF C-SENMA NPNE

The capacity of C-SENMA NPNE is summarized in the following theorem:

Theorem 3: The capacity of C-SENMA NPNE is

$$C^{\mathrm{NPNE}} = (1-\beta)C^{(0)}.$$

To prove the direct part of the theorem, let $p^*(s)$ be the capacity achieving input distribution for the DMC $q(y\,|\,x)$. For a given k, schedule a new sensor to transmit in every k slots. Therefore, $R_k = \frac{1}{k}\max_{p(s^k)} I(S^k;Y^k)$ is achievable by the direct part of Theorem 2. Let the input distribution be $p^*(s^k) = \prod_{i=1}^{k} p^*(s_i)$. The resulting $R_k^* = \frac{1}{k} I(S^k;Y^k)$ is less than or equal to R_k and therefore is achievable. It can be shown that $\lim_{k\to\infty} R_k^* \ge C^{\mathrm{NPNE}}$. Therefore, C^{NPNE} is achievable.

Due to the space limit, we only give an intuitive explanation about the converse of the theorem. Since the transmission scheduling is pre-determined, β portion of the transmission is from mis-informed nodes. Therefore, this part of the channel use is wasted. Thus, $(1-\beta)C^{(0)}$ is the best possible rate.

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