On Market Dynamics of Electric Vehicle Diffusion

Zhe Yu[†], Shanjun Li[‡], and Lang Tong[†]

***Abstract*—A sequential game model is proposed to analyze a two-sided market and indirect network effects involving electrical vehicles (EV) and electric vehicle charging stations (EVCS). The investor maximizes his profit by choosing a set of locations to build charging stations or deferring his investment and earning interest at a fixed rate. The investor also decides the optimal pricing of charging. The consumer, on the other hand, decides whether to purchase an EV or a gasoline vehicle (GV) based on the price of EV, the cost of charging, and the availability of charging stations. The solution of the game provides a closed-form expression of the EV market share as a function of EV price, the price of charging, and the density of charging stations. An asymptotically optimal algorithm is proposed for solving the optimization of the investor's decision.**

***Index Terms*—Two-sided market; indirect network effects, product diffusion, electric vehicles, EV charging services.**

I. Introduction

The diffusion of electric vehicles has mixed results. The market share of electric vehicles (EV) in recent years has grown steadily, increasing almost 800% since 2011 [1]. Despite the growth, the overall EV market share remains less than 1% as of July 2014. The reason behind the growth of EV, or the lack of it, is multifaceted. The growth is driven partially by the increasing awareness of environmental impacts of fossil fuel vehicles, the superior design and performance of some EVs, and, by no small measure, the tax credits provided by the federal and state governments. On the other hand, the EV industry still faces strong skepticism due to the high cost of EV, the limited driving range, and the lack of adequate public charging facilities.

A similar trend exists in the deployment of public charging facilities. Since the first quarter of 2011, the number of public charging stations in US has grown 700% by the end of 2013, due in part to the direct and indirect investments of federal and local governments. The Department of Energy (DoE) of the United States, for example, has provided \$230 in 2013 to establish 13,000 charging stations [2]. It is hoped that such investments will stimulate the EV market, driving its market share toward long term growth and stability.

The growth trends of EV and EV charging station (EVCS) have strong temporal and geographical couplings. This is a result of the so-called indirect network effects; the growth of EV attracts investments on EVCS, and the increasing presence of EVCSs makes EV more attractive to consumers. Similarly, the lack of EVCSs limits the growth of EV market share, which in turns inhibits new investments essential to the healthy growth and the stability of the EV market.

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This paper focuses on the interactions between the two sides of EV-EVCS markets: the EV consumer on the one side and the investor/operator of EVCS on the other. In particular, we formulate a sequential game model for the two-sided EV-EVCS market, which allows us to address analytically and numerically some of the following issues: how does a consumer's decision of EV purchase interacting with that of the investor of EVCS facilities? How is the EV market share affected by the price of EV, the cost of EV charging, and the size of EVCS market? How does the EVCS investor maximize his profit by choosing sites of EVCS from a list of candidate locations?

A. Summary of results

The main contribution of this work is an analytical study about the indirect network effects between the EV consumer and the EVCS investor. To this end, we introduce a complete Stackelberg game model for the two-sided EV-EVCS market with the investor as the leader and the consumer the follower. Through profit maximization, the investor decides whether to build CSs chosen (optimally) from a list of candidate CS sites or defer his investment. The candidate sites are heterogeneous; each site may have different favorable rating and different operation and building cost from others. Observing investor's decision that defines the location of CSs and the cost of charging, the consumer decides whether to purchase an EV or a gasoline alternative.

We provide a solution of the Stackleberg game that includes the optimal decisions for the consumer and the investor. Under a random utility maximization (RUM) model of the consumer [3], we show that the optimal policy is a threshold policy on the consumer vehicle preference. A closed-form expression for the decision threshold t^* is obtained, as a function of the price of EV, the investor's decision on the number/location of charging stations, and the charging prices at those locations. The optimal decision threshold of purchasing an EV gives directly the EV market share $\eta = 1 - t^*$, which allows us to examine how the investor's decisions and EV price affect the overall EV market share.

To obtain optimal investor's decision, we first study the optimal operation decision by the investor by fixing the set of CS sites to build. We show that the optimal pricing for EV charging at these sites is such that profits generated from these sites are equal. We show further that the optimal pricing converges to a constant mark-up of the operating cost as the density of EV charging sites increases, which is a result of the monopolistic competition of EVCS market.

The optimal decision in choosing which CS sites to build (or defer investment) is more complicated and is combinatorial in nature. We provide a greedy heuristic and show that the heuristic is asymptotically optimal as the density of CS sites increases.

B. Related work

There is an extensive literature on the two-sided market and indirect network effects for various products; see *e.g.*, [4] on the CD player and CD title market, the video console and video game market [5], [6], [7], the hardware and software market [8], the credit card market [9], and the yellow page and advertisement market [10]. Rochet and Tirole in [11] proposed a restrictive definition of two-sided market. Caillaud and Jullien pointed out in [12] that, one side of the market always waits for the action from the other side. It is thus critical for players to take the right move in the initial stages of the product diffusion.

There is a growing literature on the EVCS investment from the operation research and engineering perspectives. For example, the charging station deployment has been formulated as an optimization problem from the social planner's point of view in [13], [14], [15]. A location competition problem of charging stations is considered in [16], where in consumer choice, a discrete decision model similar to this paper is used. Efficient design of large scale charging is presented in [17] and the competition of charging operations is considered in [18].

The work of Li *et al.* [19] and the current paper represent the first analyzing the two-sided EV and EVCS market and related indirect network effects. The work in [19] focuses on the empirical study of indirect network effects whereas the current paper focuses on the theoretical analysis.

C. Organization

This paper is organized as follows: the structure of the two-sided market and a Stackleberg game model are described in Sec. II. The solution of the game is obtained through a backward induction. In Sec. III, the consumers' model and the optimal decision are stated. The investor's model and optimal strategy are presented in Sec. IV. Sec. V concludes the paper.

II. A SEQUENTIAL GAME MODEL FOR THE EV MARKET

In this section, we formulate the two-sided market as a two-player Stackelberg (sequential) game with complete information. We first introduce the basic structure of the EV-EVCS market, define the players of the game, and specify the decision process.

A. Two-sided market structure

A two-sided market typically has a structure illustrated in Fig. 1 where we use a generic hardware-software market as an example to describe its basic components.

A two-sided market includes a set of platforms, say, Macbook™by Apple Inc. and the OS X operating system as a hardware-software platform vs. Dell's Inspiron™and the Windows 8 as its operating system as a different hardware-software platform.

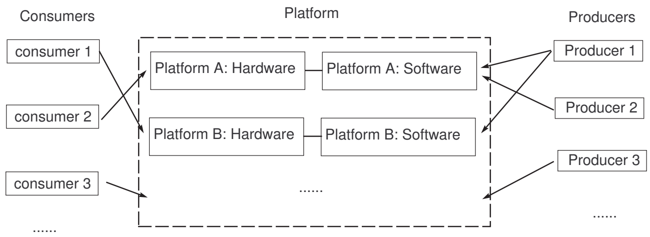

Fig. 1: The structure of two-sided market

On one side of the platforms is the consumer who makes her purchase decisions based on her preference and experience of a particular platform, the cost of the platform, and the available software for that platform. On the other side of the platforms are the software developers who invest time and money in developing softwares for one particular platform or multiple platforms. The software developer makes his decision based, among other factors, the cost of developing software and the popularity of the platform.

For the two sided EV-EVCS market studied in this paper, we consider two platforms: one is the EV as the hardware and the EVCS as the software. The other is the traditional gasoline vehicle (GV) as the hardware and gas station as the software. On one side of the platforms is the consumer on the market who decides which type of vehicle to purchase based on the cost of EV, the available charging stations, and the cost of charging. On the other side of the platforms is an investor who decides to build and operate charging stations or defer his investment and earn interest on a fixed rate*.

B. The investor's decision model

We assume that the investor is also the builder and the operator of the CSs. The investor decision has two components: the first is an *investment decision* on whether to build CSs from a list of candidate CS sites or defer his investment. The second is an *operating decision* on pricing the charging services at those selected locations.

Let $\mathscr{C} = \{s_i = (f_i, c_i), i = 1, \cdots, N_L\}$ be the set of candidate sites for charging stations known to the investor. The site $s_i = (f_i, c_i)$ has two attributes: the favorability rating f_i of the site and the marginal operating cost† c_i. For example, a site at a shopping center may be more attractive than a location that is less frequently visited by consumers. The cost of building and operating such sites are also different.

Given $\mathscr{C}$ and the utility function of the consumer, the investor's decision is given by $(\mathcal{C}, \vec{\rho}) \in 2^{\mathscr{C}} \times \mathcal{R}^{N_L}$ where $\mathcal{C} \subseteq \mathscr{C}$ is the set of locations selected to build charging stations, and $\vec{\rho} = (\rho_1, \cdots, \rho_{|\mathcal{C}|}) \in \mathcal{R}^{|\mathcal{C}|}$ is the vector of charging prices at the built locations.

*Because we focus on the early stage of EV diffusion in an environment that gas stations are already well established, the alternative of EVCS investor is not building additional gas stations.

†The marginal cost (\$/mile) here is the locational marginal price of wholesale electricity (\$/kWh) normalized by EV efficiency (miles/kWh).

Assuming the consumer maximizing her surplus, the investor chooses the investment sites and charging prices to maximize the investment profit within his budget B. The investment optimization is stated as

$$\begin{array}{ll} \max_{\mathcal{C},\vec{\rho}} & \Pi(\mathcal{C},\vec{\rho}) - \sum_{i=1}^{|\mathcal{C}|} F(s_i) \\ \text{subject to} & \sum_{i=1}^{|\mathcal{C}|} F(s_i) \le B \end{array} \tag{1}$$

where Π is the operational profit and $F(s_i)$ is the building cost of station i.

C. *The consumer's decision model*

The consumer observes the investor's decision on the location of charging stations $\mathcal{C} = \{s_1, \cdots, s_{N_E}\}$ and charging price vector $\vec{\rho} = (\rho_1, \cdots, \rho_{N_E})$. Here N_E is the number of charging stations normalized by the population of consumers. The consumer chooses the type of vehicle to purchase. If EV is the choice, the consumer also decides on the location of charging. The action of the consumer is given by $\{V, j\}$ where $V \in \{E, G\}$ is the vehicle choice (either EV or GV), and $j \in \{0, 1, \cdots, N_E\}$ is the preferred station for charging. We include $j = 0$ to indicate the home charging option. The consumer chooses $\{V, j\}$ by maximizing the overall vehicle surplus that includes the charging surplus for the EV purchase.

The consumer surplus model of purchasing a vehicle is assumed as follows:

$$\begin{array}{l} V_E = \beta \mathbb{E} U_E - p_E + \Phi + \epsilon_E \\ V_G = \beta \mathbb{E} U_G - p_G + \Phi + \epsilon_G \end{array} \tag{2}$$

where U_E is the (random) utility of consumer's best choice defined below, $\mathbb{E}U_E$ is the expected maximum charging utility, p_E is the price of an EV, Φ is the utility of owing a vehicle, and ϵ_E is a random vehicle preference. Variables $\mathbb{E}U_G$, p_G, and ϵ_G are similarly defined for the gasoline vehicle. The consumer decision is then defined by

$$\max\{V_E, V_G\}. \tag{3}$$

The optimization of consumer's vehicle decision also includes the optimization of charging locations. To this end, we assume a widely adopted discrete choice model with random utility functions. See [16]. Specifically, the consumer charging surplus at station i is assumed to be random in the following form

$$U_i = \alpha f_i - \rho_i + \epsilon_i, i = 0, \ldots, N_E \tag{4}$$

where f_i is the favorability rating, ρ_i the charging price determined by the investor, ϵ_i the random preference of the charging station i.

Given the realization of the charging preference $\vec{\epsilon} = (\epsilon_0, \ldots, \epsilon_{N_E})^T$, the EV owner chooses charging station $j \in \{0, 1, \ldots, N_E\}$ to maximize her charging utility, *i.e.,*

$$U_E = \max_{i \in \{0, \ldots, N_E\}} U_i. \tag{5}$$

For the option of choosing a gasoline vehicle, the utility of fueling will not change with the decisions of investor. Thus U_G is a constant.

D. *The sequential game model*

The sequential game structure for the two-sided EV-EVCS market is summarized as follows:

- The investor's decision is defined by the optimization in (1). Specifically, given locations $\mathscr{C}$, the investor determines to invest (build and operate) charging stations at $\mathcal{C} \subseteq \mathscr{C}$ and the price of charging $\vec{\rho}$. When $\mathcal{C} = \emptyset$, the investor defers investment by earning interest at a fixed rate.
- The consumer's decision is defined by (3-5). Specifically, having observed the investor's decision, $\{\mathcal{C}, \vec{\rho}\}$, the consumer chooses $V \in \{E, G\}$. If $V = E$, the consumer also chooses charging station to charge by maximizing her charging surplus.

The dynamic game is solved via backward induction. In particular, we first consider the consumer's decision by fixing the investor's choice of charging location and charging prices. The optimal consumer decision is given in Sec. III. In Sec. IV, the optimal investor's decision is presented.

III. Consumer Decisions

A. *Consumer Decision Model and Assumption*

We first summarize the assumptions on consumer model given in Sec II.C.

A1. Consumers are identical and their decisions are statistically independent. Without loss of generality, we focus on the decision of a single consumer.

A2. For a consumer who purchases an EV, the average charging demand is normalized to 1.

A3. The random preference of charging station i, ϵ_i, is independent and identically distributed (IID) and follows the extreme value type one distribution with the probability density function (PDF)

$$f(\epsilon) = e^{-\epsilon} e^{-e^{-\epsilon}}.$$

A4. The random vehicle preference of EV is uniformly distributed with $\epsilon_E = \phi t_E, t_E \sim \mathcal{U}(0,1)$ where ϕ is a coefficient that makes ϵ_E comparable with Φ in (2). The random preference of GV is given by $\epsilon_G = \phi t_G$, $t_G = 1 - t_E$.

The extreme value type one distribution is widely used in the discrete choice model. McFadden first introduce the extreme value distribution in the discrete choice model and showed it leads to the multinomial logit distribution across choices [20].

B. *Consumer Decision and EV Market Share*

The main result in this section is the structure of the optimal vehicle decision and the characterization of the EV market share as shown in the following theorem.

Theorem 1 (Consumer choice): The optimal consumer decision is a threshold policy on the realization of consumer preference t_E

$$\begin{cases} t_E \ge t^* & \text{purchase electric vehicle} \\ t_E < t^* & \text{purchase gasoline vehicle} \end{cases}$$

where

$$t^* = \left[\frac{\beta U_G - p_G}{2\phi} + \frac{1}{2} - \frac{\beta \ln(\sum_{i=0}^{N_E} \exp(\alpha f_i - \rho_i)) - p_E}{2\phi} \right]_0^1 \quad (6)$$

The EV market share for the optimal consumer choice is given by $\eta = (1-t^*)$. The market share of charging station i is given by

$$P_i = \frac{\exp(\alpha f_i - \rho_i)}{\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)} \triangleq \frac{q_i}{q}. \quad (7)$$

Proof: In deriving the optimal consumer vehicle decision from (2-3), we first compute the expected charging utility from (5) using the extreme value type one distribution of ϵ_i. Specifically,

$$\begin{aligned} \mathbb{E}(U_E) &= \ln(\textstyle\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)) \\ &\triangleq \ln(\textstyle\sum_{k=0}^{N_E} q_k) = \ln(q), \end{aligned} \quad (8)$$

where $q_k = \exp(\alpha f_k - \rho_k)$ is the exponential systematic surplus of the kth charging station.

Next, given the realization of the random EV preference t_E, by substituting $\mathbb{E}(U_E)$ and $t_G = 1 - t_E$ into (2), the consumer's optimal vehicle choice based on (3) is given by a threshold policy on t_E. In particular, the consumer purchases an EV if

$$\begin{aligned} t_E \geq & -\frac{\beta \ln(\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k)) - p_E}{2\phi} \\ & + \frac{\beta U_G - p_G}{2\phi} + \frac{1}{2}, \end{aligned} \quad (9)$$

provided that the right hand side is within $[0, 1]$. With t^* defined in (6), $1 - t^*$ gives the market share of EV. From [20] (chapter 4), the optimal charging station choice is given by (7). ∎

With Theorem 1, we can examine the trend of market share as a function of the density of charging stations (N_E), the charging price, and the price of EV.

First, the expression of η indicates that the market share is an increasing and concave function of charging station density N_E, which is also validated in Fig. 2. The concavity of market share implies that the marginal effect of building charging facilities decreases. In addition, the market share accelerates faster to 1 with lowered EV price.

Second, the market share has a dead zone effect in the density of charging stations. Specifically, there is a critical N_E below which the market share is zero. Note that the increase of EV price increases the critical charging station density. In fact, as shown in Fig. 3, the critical density of charging stations grows as a convex function of EV price. This suggests that reducing EV price (*e.g.,* via subsidy in the form of tax credit) is more effective than increasing the density of charging stations. Similar market share trends exist for the EV charging price.

IV. Investor Decisions

After the discussion about the consumer model and her decision, we now focus on the investor's decision model that includes the selection of the locations of charging stations to build and the optimal pricing of charging.

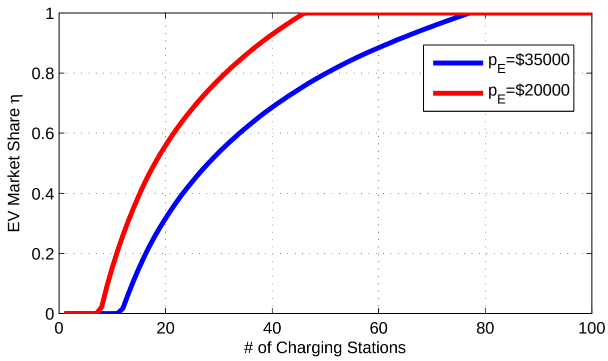

Fig. 2: EV market share vs. # of charging station. $p_G = \$17450$, $U_G = 4.5052$, $\rho_i = 0.2\$/\text{kWh}$.

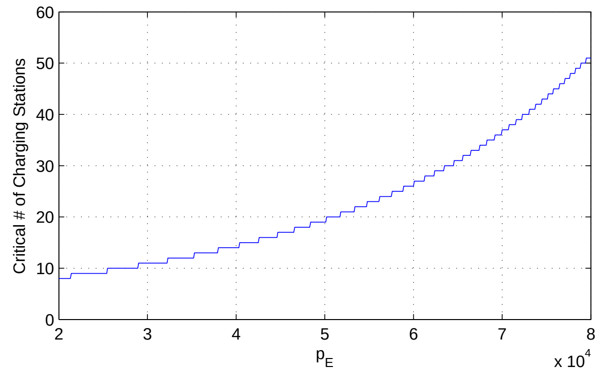

Fig. 3: Critical # of stations vs. EV price p_E. $p_G = \$17450$, $U_G = 4.5052$, $\rho_i = 0.2\$/\text{kWh}$.

A. Investor Decision Model and Assumptions

We make the following assumptions about the investor model:

B1. We consider a single investor who also operates all charging stations. This implies the monopolistic competition in the charging station market.

B2. We assume that the deferred investment earns an interest at the rate of γ.

B3. The investor knows the utility function of the consumer.

In solving the optimization in (1), we proceed with backward induction: in Sec IV-B, we find the optimal pricing with fixed EVCS locations and, in Sec IV-C, we optimize the location of charging stations.

B. Optimal Charging Price

Assuming the set of charging station locations $\mathcal{C}$ is given, the investor determines the optimal charging price $\vec{\rho}$ to maximize the total profit. Specifically, the investor has the following optimization

$$\max_{\vec{\rho}} \Pi = \max_{\vec{\rho}} \eta(\vec{\rho}) \sum_{i=1}^{N_E} P_i(\vec{\rho})(\rho_i - c_i), \quad (10)$$

where $\eta(\vec{\rho})$ is the expected EV market share given in Theorem 1 (here we make the dependency on charging price explicit), $P_i(\vec{\rho})$ the market share of station i, *i.e.*, the fraction of EV owners who charge at station i, and c_i the marginal operation cost of station i. The optimal charging price ρ_i^* is given by the following theorem.

Theorem 2 (optimal charging price): For fixed set of charging stations $\mathcal{C} = \{(f_i, c_i), i = 1, \cdots, N_E\}$, the optimal charging prices $\rho_i^*, i = 1, \cdots, N_E$, generates uniform profit across charging stations. In particular,

$$\rho_i^* - c_i = \frac{1}{\frac{\beta(1-P_0(\vec{\rho}^*))}{2\phi\eta(\vec{\rho}^*)} + P_0(\vec{\rho}^*)}, \tag{11}$$

where $P_0(\vec{\rho}^*) = \frac{\exp(\alpha f_0 - \rho_0^*)}{\sum_{k=0}^{N_E} \exp(\alpha f_k - \rho_k^*)}$ is the probability that the consumer charges at home and ρ_0^* the cost of charging at home.

Proof: This is a direct consequence of the first order optimality condition. ■

Note that the right hand side of (11) is the same for any i. The profit of each station from single consumer is the same. Equation (11) does not have a closed-form solution, but the optimal price can be solved numerically. Since $P_0 \geq 0$, the revenue is strict positive. As $N_E \to \infty$, the EV market share $\eta \to 1$ and $P_0 \to 0$, we have the limit of the marginal charging profit

$$\rho_i^* - c_i \to \frac{2\phi}{\beta} \tag{12}$$

as $N_E \to \infty$. This means that, when the consumer is sensitive to the utility of charging ($\beta \gg 1$), the optimal charging price is closer to the marginal operating cost.

C. Investor Charging Locations

After obtained the optimal charging price, the investor needs to decide the set of charging locations to invest. Given the location candidates $\mathscr{C} = \{s_i = (f_i, c_i),\ i = 1, \cdots, N_L\}$, the investor has the following optimization

$$\begin{array}{ll} \max_{\mathcal{C}\subseteq\mathscr{C}} & \left\{\Pi(\mathcal{C}) - \sum_{i=1}^{|\mathcal{C}|} F(s_i)\right\}, \\ \text{subject to} & \sum_{i=1}^{|\mathcal{C}|} F(s_i) \leq B \end{array} \tag{13}$$

where $F(s_i)$ is the building cost of charging station i and $\Pi(\mathcal{C})$ the operational profit. Note that, for convenience, we ignore the dependencies of the optimal charging price $\vec{\rho}^*$ (which is a function of $\mathcal{C}$) in Π.

In general, the optimal investment decision from (13) requires combinatorial search of $\mathcal{C}$, which is not tractable. However, the convergence of optimal prices to a constant across charging stations in (12) makes it possible to separate the price decision and the location choice, which leads to a linear complexity heuristic algorithm.

The Greedy Investment Algorithm (GIA) given in Algorithm 1 first ranks the charging station according to the systematic part of the charging surplus $v_i = \exp(\alpha f_i - c_i)$. It then adds charging station to the investment list one at a time in the order of decreasing systematic surplus v_i. It computes the cumulated profit until either the budget is exhausted or the cumulated profit starts to decrease. Note that the revenue of charging is a concave and the cost of building charging stations grows linearly.

Algorithm 1 Greedy Investment Algorithm

1. Compute the exponential of systematic surplus $v_i = \exp(\alpha f_i - c_i)$ and sorted list $\{v_{(i)}\}$.
2. Set $N = 1$.

while $N \leq N_L$ and $\sum_{i=1}^{N} F(s_i) \leq B$ **do**
  Compute $\tilde{P}_N \triangleq \Pi(s_1, \cdots, s_N) - \sum_{i=1}^{N} F(s_i)$.

  if $\tilde{P}_N < \tilde{P}_{N-1}$ or $\sum_{i=1}^{N} F(s_i) > B$ **then**
    STOP;
  else
    $N \leftarrow (N+1)$.
  end if
end while

By ignoring the dependencies of charging locations in the term $(\rho_i - c_i)$ of $\Pi(\mathcal{C})$ in (13), GIA is not optimal in general. As N_E increases, such dependencies vanish, which makes the algorithm asymptotically optimal.

Theorem 3 (Asymptotic optimality): If the cost of charging stations is constant, *i.e.*, $F(s_i) = F_0$, then the greedy algorithm is asymptotically optimal (as $N \to \infty$).

Proof: See Sec. VI-A. ■

After obtained the optimal set of charging stations $\mathcal{C}^*$ and the optimal charging price vector $\vec{\rho}^*$, the investor will make the investment if the investmet profit $(\Pi(\mathcal{C}^*, \vec{\rho}^*) - \sum_{i=1}^{|\mathcal{C}^*|} F(s_i))$ is positive. Otherwise, the investor will defer his investment and earn interest on a fixed rate.

To make $(\Pi(\mathcal{C}^*, \vec{\rho}^*) - \sum_{i=1}^{|\mathcal{C}^*|} F(s_i))$ positive, the EV price and the building cost of charging stations need to be low enough, which implies the subsidies to EV purchase and charging stations is necessary to the successful launch of EV.

V. Conclusion

In this paper, the two-sided market problem of EV-EVCS is considered in this paper. A sequential Stackelberg game is formulated to analyze the indirect network effect between charging station investor and consumers. The optimal operation decision of charging stations is shown as locational equal profit pricing. An asymptotic optimal algorithm of investment decision is proposed which reduces the computation complexity significantly.

As an analytical approach to understanding the market dynamics of EV diffusion, this paper assumes a stylized model for both the consumer and the investor. Here we aim to capture major factors in the interactions between the consumer and the investor, including the EV price, the coverage of the charging stations, and the price of charging. Ignored in the model includes several nontrivial and practically significant factors. For instance, the price of EV is assumed exogenous, and the EV consumers and charging stations are mostly homogeneous (except that the operating cost and pricing of charging are different across locations). Competitions among investors and a multi-stage counter part of this work are to be reported in the future.

VI. Appendix

A. Proof of Theorem 3

As $N_E \to \infty$, $(\rho_i^* - c_i) \to \frac{2\phi}{\beta}$. So for any $\varepsilon > 0$, $\exists M$, such that when $N_E > M$, $|\rho_i^* - c_i - \frac{2\phi}{\beta}| < \varepsilon$. If the optimal charging price is approximated by $\rho_i^* \approx c_i + \frac{2\phi}{\beta}$, the difference between the real profit Π and the approximation $\tilde{\Pi}$ is

$$|\Pi - \tilde{\Pi}| = |\eta \sum_{i=1}^{N_E} P_i(\rho_i^* - c_i) - \frac{2\phi}{\beta_1}\eta \sum_{i=1}^{N_E} P_i| < \varepsilon$$

when $N_E > M$. So in the following analysis, the optimal charging price ρ_i^* will be approximated by $c_i + \frac{2\phi}{\beta}$.

First, fixing N_E charging stations to build, we examine where to build these stations. Denote the exponential of systematic surplus of station i by $q_i = \exp(\alpha f_i - \rho_i^*) \approx \exp(\alpha f_i - (c_i + \frac{2\phi}{\beta_1}))$ and the sum of q_i as $q = \sum_{i=0}^{N_E} q_i$. The approximated charging profit of the investor can be stated as

$$\begin{aligned} \tilde{\Pi}(q) &= \eta(q) \sum_{i=1}^{N_E} P_i(q_i, q)(\rho_i^* - c_i) \\ &= \frac{2\phi}{\beta_1}\eta(q) \sum_{i=1}^{N_E} \frac{q_i}{q} \end{aligned}. \quad (14)$$

Take partial derivative of the revenue with respect to q_i, we have the following lemma.

Lemma 1: The revenue of the investor is strictly increasing in q_i:

$$\frac{\partial \tilde{\Pi}(q)}{\partial q_i} > 0.$$

Lemma 1 implies that, if given two station candidates j and j', fixing the other $(N_E - 1)$ stations, the one with larger $q_i = \exp(\alpha f_i - (c_i + \frac{2\phi}{\beta}))i \in \{j, j'\}$, should be built. So we have the following optimal strategy about where to build stations.

Lemma 2: Fixing the number of stations to build as N_E, the optimal strategy of building is to pick N_E candidates with largest $v_i = \exp(\alpha f_i - c_i)$ to build.

Next, after we sort the N_L candidate locations by q_i, we can present the cost $\tilde{F}(q) \triangleq \sum_{i=1}^{N_E} F(s_i) = (1+\gamma)F_0 N_E$ as a function of q. Since $q_i \geq q_{i+1}$, the cost $\tilde{F}(q)$ is a piece wise linear concave function of q. The partial derivative is piece wise constant and increasing in q. The operating profit $\tilde{\Pi}(q)$ is increasing in q. The second order derivative of $\tilde{\Pi}(q)$ with respect to q is stated as

$$\frac{\partial^2 \Pi}{\partial q^2} = \frac{2\phi}{q^3 \beta}(\frac{3\beta q_0}{2\phi} - \frac{\beta q}{2\phi} - 2q_0(\frac{\beta \ln(q)}{2} + C)), \quad (15)$$

where $(C = -\frac{p_E}{2\phi} - \frac{\beta \mathbb{E} U_G - p_G}{2\phi} + \frac{1}{2})$ is a constant. The second order derivative is a decreasing function of q and crosses the zero point. The convexity of $\tilde{\Pi}$ is stated in the following lemma.

Lemma 3: As q increases, $\tilde{\Pi}(q)$ is first a convex function, then a concave function.

The trends of $\tilde{\Pi}(q)$, $\tilde{F}(q)$ and the derivative are plotted in Fig 4 and 5. In Fig. 4, q^* is the optimal point to maximize the profit $(\tilde{\Pi}(q) - \tilde{F}(q))$. Fig. 5 shows the derivative of $\tilde{F}(q)$ is increasing and the marginal profit $\frac{\partial \tilde{\Pi}(q)}{\partial q}$ is first increasing then decreasing. There are at most two cross points in the derivative and the latter one is the optimal point. Combining Lemma 1, 2, 3, we have the asymptotic optimality.

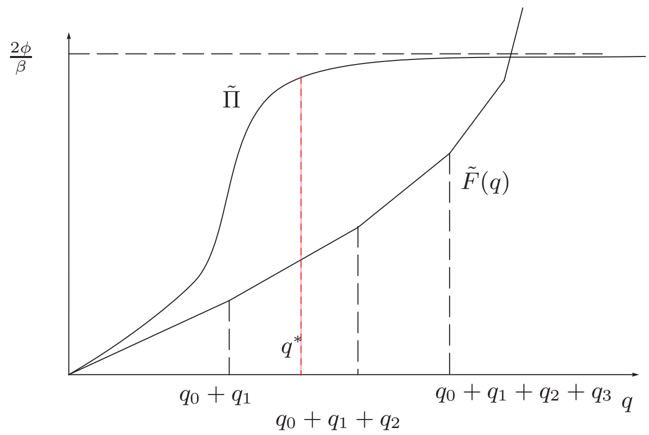

Fig. 4: Profit and cost of charging stations in (13).

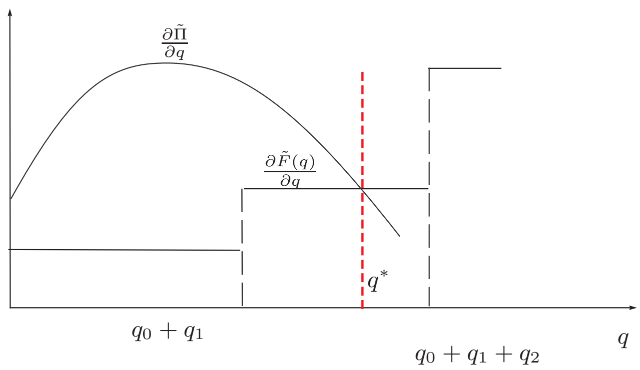

Fig. 5: Profit and cost derivatives of charging stations.

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