Semi-Blind Collision Resolution in Random Access Wireless Ad Hoc Networks

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***Abstract*—A new signal processing based collision resolution technique for random access wireless ad hoc networks is proposed in this paper. Without assuming the knowledge of propagation channels and signal waveforms, the proposed algorithm is capable of separating colliding packets by exploiting channel diversities and known symbols embedded in data packets. Compared with training-based methods, the proposed algorithm requires considerably fewer known symbols. This algorithm can be applied to various spread spectrum and narrowband systems along with existing medium access control protocols.**

***Index Terms*—Collision resolution, random access network, semi-blind approach.**

I. Introduction

A. Packet Collision and Multiple Packet Reception in Random Access Networks

IN A SLOTTED random access ad hoc network, all users share a common radio channel for immediate packet transmission. A packet collision occurs when more than one user transmits in the same slot. For conventional narrowband networks, this concurrent channel access by more than one user results in the destruction of all colliding packets. To recover the information in the colliding packets, they have to be retransmitted in later time slots, which has adverse effects on the network throughput and delay.

One effective way of improving the performance of random access networks is to introduce multiple packet reception (MPR) to the receivers. MPR enables correct receiving of some or all colliding packets without retransmission. In addition to the direct throughput and delay improvement brought by the recovery of colliding packets, the traffic load caused by retransmissions is reduced, which further decreases the frequency of collision occurrence. Indeed, it has been shown that MPR capability significantly improves the network performance [2], [3], [7], [8], [14].

B. Collision Resolution at the Modulation Level

The use of code division multiple access (CDMA) in random access networks can provide MPR by properly designed transmission protocols [13]. In general, spread spectrum transmission protocols can be classified into three major modes [15].

1) *Transmitter-oriented spread spectrum transmission protocol*
   With this protocol, each user is assigned a unique transmitting code. If the transmitter-based codes used by different users are orthogonal, all colliding packets can be recovered under the assumption of perfect synchronization and ideal channel conditions. However, if multipath fading destroys the orthogonality or when nonorthogonal codes are employed by the network, packet collisions cannot be resolved at the modulation level.
2) *Receiver-oriented spread spectrum transmission protocol*
   In this case, each user is assigned a unique receiving code; all transmissions to a particular user must use that user's spreading code. With this protocol, packet collisions cannot be completely resolved by CDMA modulation, even if the receiver-based codes are orthogonal. When more than one user transmits to a particular user in the same slot, the packets intended for this user are lost.
3) *Network-wide spread spectrum transmission protocol*
   This is perhaps the simplest transmission protocol, where a common code, such as the time of day, is employed by all users in the network. This protocol facilitates the transmission of broadcast messages, but it does not provide multiaccess capability at the modulation level. The simultaneous transmission from different users results in the destruction of all transmitted information.

The above discussion shows that even under the assumption of perfect synchronization and ideal channel conditions, packet collisions cannot be completely resolved at the modulation level when a common code, receiver-based codes, or nonorthogonal transmitter-based codes are employed.

C. Collision Resolution at the Signal Processing Level

Collision resolution at the signal processing level aims to provide the MPR capability to random access networks with various transmission protocols. Recent work by Tsatsanis *et al.* [20], [21], [24] is perhaps the first that applies signal separation techniques for collision resolution in cellular systems. This approach relies on multiple copies of the colliding packets from the same set of users, which can be achieved by the central control of base stations. Due to the distributed nature of ad hoc networks, we cannot assume that the same set of users are active in consecutive time slots. Consequently, collision resolution in ad hoc networks needs to be achieved on a slot-by-slot basis.

Collision resolution in ad hoc networks may be achieved by embedding known symbols in data packets. These known

Manuscript received October 5, 1999; revised June 20, 2000. This work was supported in part by the National Science Foundation under Contract CCR-9804019 and the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564. The associate editor coordinating the review of this paper and approving it for publication was Prof. Michail K. Tsatsanis.

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Publisher Item Identifier S 1053-587X(00)07678-9.

symbols can be used to design training-based packet separation algorithms. In Section III, we consider one such approach, where we show that the number of known symbols required by training-based methods is related to the number of colliding packets and their channel lengths. In heavily loaded networks with severe multipath fading, the number of known symbols required by training-based methods can be considerably large. Incorporating a large amount of known symbols in data packets is not efficient in bandwidth utilization, especially in time-varying scenarios or in cases with relatively small data packets.

The elimination of training makes blind collision resolution an appealing alternative. Quite a few blind multiuser detection methods proposed for CDMA systems—such as [6], [10], [19], and [23], and references therein—provide possible solutions to collision resolution. Unfortunately, relying on the distinction among all users' codes, many existing blind techniques are not able to resolve packet collisions in networks with a common code or receiver-based codes. The algorithms proposed in [12] and [22] exploit the finite alphabet property of the input signals; they do not rely on code discrimination for signal separation. The main difficulties for these algorithms, however, are their complexity and the existence of local optima.

In most communication signals, there are known symbols embedded in data packets for purposes of synchronization and user identification. Based on this observation, many researchers [4], [5], [9], [11], [16] considered semi-blind techniques for channel estimation and equalization. Most existing semi-blind channel estimation and equalization methods are proposed for single input systems and do not directly apply to collision resolution problem in ad hoc networks.

D. Contributions

Our goal is to provide a signal processing based collision resolution technique that can be applied to random access networks with receiver-oriented or network-wide transmission protocols. To this end, we utilize the embedded known symbols that may not be sufficient for training-based methods.

In random access networks employing a common code or receiver-based codes, all or some of the colliding packets may be spread by a common code; spreading codes do not provide sufficient information for packet separation. However, in multipath fading scenarios, the overall channel impulse responses which include spreading codes, propagation channels, and front-end filters are generally distinct among different colliding packets. Based on this observation, we exploit the diversity of propagation channels for packet separation. In particular, we group colliding packets according to their channel orders and extract them sequentially. Packets coming through channels with the smallest channel order are obtained first and then subtracted from the observation. This successive demodulation makes the proposed algorithm particularly attractive in near–far scenarios where users with smaller channel order are usually closer to the receiver, hence have higher SNR. The subtraction of stronger users from the observation facilitates the detection of weaker ones.

In addition to the diversity of channel conditions, the proposed algorithm also exploits embedded known symbols for packet separation. In order to reduce the number of required known symbols, we obtain colliding packets from an innovation sequence generated from the observation by a smoothing operation. Since the innovation sequence contains less interference than the observation, our approach requires considerably fewer known symbols than training-based methods.

The rest of the paper is organized as follows. Section II presents the system model and the assumptions used in this paper. In Section III, we present the training-based least squares (LS) receiver and analyze the minimum number of required known symbols. The semi-blind least squares smoothing (LSS) approach for collision resolution is proposed in Section IV. The minimum number of known symbols required by the LSS receiver is also derived. Theoretical resolvability comparison between the training-based LS receiver and the semi-blind LSS receiver is discussed in Section V. In Section VI, we present the simulation results on the resolvability comparison of the LS receiver and the proposed semi-blind LSS receiver. Given the same number of known symbols, it is shown that the semi-blind LSS receiver provides significantly improved resolvability over the LS receiver. The effect of error-control codes on the resolvability and the near–far resistance of the proposed algorithm are also studied in simulations.

II. Problem Statements

A. Notations and Definitions

Notations used in this paper are mostly standard. Upper- and lower-case bold letters denote matrices and vectors with $(\cdot)^t$ and $(\cdot)'$ denoting the transpose and Hermitian operations, respectively. Given a matrix $\boldsymbol{A}$, $\mathcal{R}\{\boldsymbol{A}\}$ is the row space of $\boldsymbol{A}$. For a matrix $\boldsymbol{X}$ having the same number of columns as $\boldsymbol{A}$, $\mathcal{P}_{\boldsymbol{A}}^{\perp}\{\boldsymbol{X}\}$ is the projection error of $\boldsymbol{X}$ into the row space of $\boldsymbol{A}$.

B. Model

Consider a collision event that involves K users in a random access ad hoc network. Resolution of this collision can be modeled as a packet detection problem in K-input P-output finite impulse response systems, as shown in Fig. 1. The system inputs correspond to K colliding packets, and the outputs come from P diversity channels that may include spreading gain, oversampling factor, and possible antenna array. Suppose that each packet contains N_0 symbols $s^{(k)}(0), \cdots, s^{(k)}(N_0-1)$, and $s^{(k)}(t) = 0$ for $t < 0$. The noiseless channel output $\boldsymbol{x}(t) \in \mathcal{C}^{P\times 1}$ and the received signal $\boldsymbol{y}(t) \in \mathcal{C}^{P\times 1} (t = 0, \cdots, N_0 - 1)$ from the slot when collision occurs can be written as

$$\boldsymbol{x}(t) \stackrel{\Delta}{=} [x_1(t), \cdots, x_P(t)]^t = \sum_{k=1}^{K}\sum_{i=0}^{L_k} \boldsymbol{h}_k(i) s^{(k)}(t-i) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{x}(t) + \boldsymbol{n}(t) \tag{2}$$

where $\{\boldsymbol{h}_k(i) \in \mathcal{C}^{P\times 1}, i = 0, \cdots, L_k\}$ is the kth user's vector channel impulse response, which includes the spreading code, the propagation channel, and the front-end filters at the transmitter and the receiver.

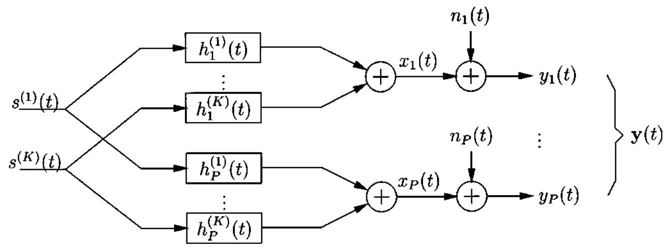

Fig. 1. Multiple-input multiple-output system.

Consider the output $\boldsymbol{x}(t) \in \mathcal{C}^{P\times 1}$ collected from N symbol intervals, where $N < N_0$ will be specified soon. We define the output block row vector $\boldsymbol{x}_t$ and the input row vector $\boldsymbol{s}_t^{(k)}$ as

$$\begin{aligned} \boldsymbol{x}_t &\triangleq [\boldsymbol{x}(t), \cdots, \boldsymbol{x}(t+N-1)] \in \mathcal{C}^{P\times N} \\ \boldsymbol{s}_t^{(k)} &\triangleq \left[s_t^{(k)}, \cdots, s_{t+N-1}^{(k)}\right] \in \mathcal{C}^{1\times N}. \end{aligned} \tag{3}$$

From (1) and (2), we have

$$\boldsymbol{x}_t = \sum_{k=1}^{K}\sum_{i=0}^{L_k} \boldsymbol{h}_k(i)\boldsymbol{s}_{t-i}^{(k)}, \qquad \boldsymbol{y}_t = \boldsymbol{x}_t + \boldsymbol{n}_t \tag{4}$$

where $\boldsymbol{y}_t$ and $\boldsymbol{n}_t$ are defined similarly as $\boldsymbol{x}_t$. Considering w consecutive output block row vectors, we define

$$\boldsymbol{X}_w(t) \triangleq \begin{pmatrix} \boldsymbol{x}_t \\ \vdots \\ \boldsymbol{x}_{t+w-1} \end{pmatrix} \in \mathcal{C}^{wP\times N}. \tag{5}$$

To contain in $\boldsymbol{X}_w(t)$ all the output from the slot when collision occurs, we have $t = 0$ and $N = N_0 - w + 1$. As discussed in Section II-C, the selection of w should be such that certain assumptions hold. For the convenience of notation, we work on $\boldsymbol{X}_w(t)$ instead of $\boldsymbol{X}_w(0)$ so that we do not need to worry about boundaries. However, it should be noted that the collision resolution techniques considered in this paper only rely on data obtained from one slot.

The kth $(1 \le k \le K)$ user's input symbols involved in $\boldsymbol{X}_w(t)$ and the corresponding channel matrix are defined as

$$\boldsymbol{S}_w^{(k)}(t) \triangleq \begin{pmatrix} \boldsymbol{s}_{t-L_k}^{(k)} \\ \vdots \\ \boldsymbol{s}_t^{(k)} \\ \vdots \\ \boldsymbol{s}_{t+w-1}^{(k)} \end{pmatrix} \in \mathcal{C}^{(w+L_k)\times N}$$

$$\boldsymbol{H}_w^{(k)} \triangleq \begin{pmatrix} \boldsymbol{h}_k(L_k) & \cdots & \boldsymbol{h}_k(0) & & \\ & \ddots & & \ddots & \\ & & \boldsymbol{h}_k(L_k) & \cdots & \boldsymbol{h}_k(0) \end{pmatrix} \in \mathcal{C}^{wP\times (w+L_k)}.$$

With $\boldsymbol{Y}_w(t)$ and $\boldsymbol{N}_w(t)$ similarly defined, we have, from (4)

$$\begin{aligned} \boldsymbol{X}_w(t) &= \sum_{k=1}^{K} \boldsymbol{H}_w^{(k)}\boldsymbol{S}_w^{(k)}(t) = \boldsymbol{H}_w\boldsymbol{S}_w(t) \\ \boldsymbol{Y}_w(t) &= \boldsymbol{X}_w(t) + \boldsymbol{N}_w(t) \end{aligned} \tag{6}$$

where

$$\boldsymbol{H}_w \triangleq \left[\boldsymbol{H}_w^{(1)}, \cdots, \boldsymbol{H}_w^{(K)}\right] \in \mathcal{C}^{wP\times(Kw+L)} \tag{7}$$

$$\boldsymbol{S}_w(t) \triangleq \begin{pmatrix} \boldsymbol{S}_w^{(1)}(t) \\ \vdots \\ \boldsymbol{S}_w^{(K)}(t) \end{pmatrix} \in \mathcal{C}^{(Kw+L)\times N} \tag{8}$$

$$L \triangleq \sum_{i=1}^{K} L_i.$$

Our goal here is to estimate $\boldsymbol{s}_t^{(k)}$ from $\boldsymbol{Y}_w(t)$ without knowing $\boldsymbol{h}_k \triangleq [\boldsymbol{h}_k'(0), \cdots, \boldsymbol{h}_k'(L_k)]'$ $(k = 1, \cdots, K)$.

C. Assumptions

Three assumptions are made in this paper.

A1) There exists a w such that $\boldsymbol{H}_w$ [as defined in (7)] has full column rank.

A2) For the w specified in A1, $\boldsymbol{S}_w(t)$ [as defined in (8)] has full row rank.

A3) There are known symbols embedded in the data packets.

A direct consequence of A1 is the isomorphic relation between the input and output subspaces

$$\mathcal{R}\{\boldsymbol{X}_w(t)\} = \mathcal{R}\{\boldsymbol{S}_w(t)\}. \tag{9}$$

This isomorphism indicates that without knowing the input sequences, the row span of the input matrix $\boldsymbol{S}_w(t)$ can be obtained from the output $\boldsymbol{X}_w(t)$. As will be detailed in Section IV, this row space information on the input matrix is utilized by the proposed algorithm to reduce the number of known symbols required by training-based collision resolution methods.

For A1 to hold, it is necessary that $\boldsymbol{H}_w$ has more rows than columns. This necessary condition leads to a lower bound on w, as given by

$$w \ge w_0 \triangleq \begin{cases} \left\lceil \dfrac{L}{P-K} \right\rceil, & \text{if } L \ne 0 \\ 1, & \text{if } L = 0. \end{cases} \tag{10}$$

A sufficient condition for A1 is that $H(z) \triangleq [\boldsymbol{h}_1(z), \cdots, \boldsymbol{h}_K(z)]$ is irreducible and column reduced [1].

A2 implies that the input sequences are persistently exciting. A necessary condition for A2 is that $\boldsymbol{S}_w(t)$ has more columns than rows, i.e., $N \ge Kw + L$.

A3 is a key assumption for all training-based and semi-blind methods. This assumption holds in most communication systems, where known symbols are inserted in the data stream for synchronization and user identification. We assume that each packet contains an equal number of known symbols that may not be consecutive. The minimum amount of known symbols required for collision resolution is discussed in the next two sections, where we consider, respectively, the training-based LS receiver and the proposed semi-blind LSS receiver. One problem with training-based and semi-blind collision resolution techniques is that the receiver needs to know which users are involved in the collision in order to determine which sets of known symbols should be utilized. However, in random access networks, the active user profile may not be available to the receiver. One simple but perhaps computationally expensive solution to this problem is to exhaust every set of known symbols in the packet recovery. To reduce the computational cost, we present an active user detection scheme in the following section.

III. Training-Based Least Squares Receiver

In this section, we consider the LS receiver that relies on training symbols. The minimum number of known symbols required by the LS receiver is analyzed. A simple active user detection scheme is also presented.

A. An Example

Consider an example where we have three colliding packets with channel order 1, 1, and 2, respectively. Suppose that $\boldsymbol{H}_w$ has full column rank and that $\boldsymbol{S}_w(t)$ has full row rank for $w = 2$. From (6), we have

$$\boldsymbol{X}_2(t) \triangleq \begin{pmatrix} \boldsymbol{x}_t \\ \boldsymbol{x}_{t+1} \end{pmatrix} = \boldsymbol{H}_2 \boldsymbol{S}_2(t). \tag{11}$$

From the isomorphism between $\mathcal{R}\{\boldsymbol{X}_2(t)\}$ and $\mathcal{R}\{\boldsymbol{S}_2(t)\}$, as given in (9), we have

$$\boldsymbol{s}_t^{(k)} \in \mathcal{R}\{\boldsymbol{X}_2(t)\}, \qquad k = 1, 2, 3 \tag{12}$$

which implies that all colliding packets can be obtained as linear combinations in the row space of $\boldsymbol{X}_2(t)$. With a sufficient number of known symbols to construct linear equations, we can solve for the combination coefficients and resolve the collision. Under the assumption that $\boldsymbol{S}_2(t)$ has full row rank, we can show that $\mathcal{R}\{\boldsymbol{X}_2(t)\}$ has dimension 10 by the isomorphic relation and (11). Let $\{\boldsymbol{u}_1, \cdots, \boldsymbol{u}_{10}\}$ denote a basis of $\mathcal{R}\{\boldsymbol{X}_2(t)\}$. We have, from (12)

$$\begin{aligned} \boldsymbol{s}_t^{(k)} &= \left[a_1^{(k)}, \cdots, a_{10}^{(k)}\right] \begin{pmatrix} \boldsymbol{u}_1 \\ \vdots \\ \boldsymbol{u}_{10} \end{pmatrix} \\ &\triangleq \boldsymbol{a}^{(k)} \boldsymbol{U}, \qquad k = 1, 2, 3 \end{aligned} \tag{13}$$

where $\boldsymbol{a}^{(k)}$ is the receiver coefficient vector for estimating $\boldsymbol{s}_t^{(k)}$. For the LS receiver, $\boldsymbol{a}^{(k)}$ can be obtained by imposing the least squares criterion on the known symbols embedded in $\boldsymbol{s}_t^{(k)}$. Specifically, let α_k denote the vector containing the positions of the known symbols in $\boldsymbol{s}_t^{(k)}$. From (13), we have

$$\boldsymbol{s}_t^{(k)}(\alpha_k) = \boldsymbol{a}^{(k)} \boldsymbol{U}(\alpha_k) \tag{14}$$

where $\boldsymbol{A}(\alpha_k)$ denotes the matrix that consists of the columns in $\boldsymbol{A}$ whose indices are in α_k. If $\boldsymbol{U}(\alpha_k)$ is of full row rank, the optimal LS receiver for $\boldsymbol{s}_t^{(k)}$ is given by

$$\boldsymbol{a}^{(k)} = \boldsymbol{s}_t^{(k)}(\alpha_k) \boldsymbol{U}'(\alpha_k) (\boldsymbol{U}(\alpha_k) \boldsymbol{U}'(\alpha_k))^{-1}. \tag{15}$$

Because $\{\boldsymbol{u}_1, \cdots, \boldsymbol{u}_{10}\}$ is also a basis for $\mathcal{R}\{\boldsymbol{S}_2(t)\}$, a necessary and sufficient condition for $\boldsymbol{U}(\alpha_k)$ being of full row rank is that $\boldsymbol{S}_2(t, \alpha_k)$ [defined as the matrix that consists of the columns in $\boldsymbol{S}_2(t)$ whose indices are in α_k] has full row rank. This implies that the minimum number of known symbols required for obtaining $\boldsymbol{s}_t^{(k)}$ is determined by the number of rows in $\boldsymbol{S}_2(t, \alpha_k)$, which is equal to the dimension of $\mathcal{R}\{\boldsymbol{X}_2(t)\}$. The full row rank condition on $\boldsymbol{S}_2(t, \alpha_k)$ also indicates that the known symbols of any user can not be all zero. Furthermore, if all users' known symbols are inserted at the same place, i.e., $\alpha_1 = \cdots = \alpha_K \triangleq \alpha$, then their known symbol vectors $\boldsymbol{s}_t^{(1)}(\alpha), \cdots, \boldsymbol{s}_t^{(K)}(\alpha)$ should be linearly independent.

We point out that since $N < N_0$ [see (3) and (5)], $\boldsymbol{s}_t^{(k)}$ does not contain all the symbols in the kth colliding packets. However, with the estimate of $\boldsymbol{s}_t^{(k)}$, the rest of the $N_0 - N$ symbols can be obtained.

B. Minimum Number of Known Symbols Required by the LS Receive

The above example shows that to obtain $\boldsymbol{s}_t^{(k)} (k = 1, \cdots, K)$ from $\boldsymbol{X}_w(t)$, the minimum number of known symbols required by the LS receiver is the dimension of $\mathcal{R}\{\boldsymbol{X}_w(t)\}$. Consequently, we have the following proposition.

Proposition 1: Let $w_{\min}$ denote the minimum w that makes $\boldsymbol{H}_w$ full column rank and $\boldsymbol{S}_w(t)$ full row rank. Assume that for $w < w_{\min}$, no column of $\boldsymbol{H}_w$ is linearly independent of other columns in $\boldsymbol{H}_w$. Then, the minimum number $\mathcal{N}_{\text{LS}}$ of known symbols required by the LS receiver to recover all colliding packets in the absence of noise is given by

$$\mathcal{N}_{\text{LS}} = K w_{\min} + L \geq \begin{cases} K \left\lceil \dfrac{L}{P - K} \right\rceil + L, & \text{if } L \neq 0 \\ K, & \text{if } L = 0. \end{cases} \tag{16}$$

The proof of Proposition 1 is based on the following lemma (the proof of Lemma 1 is given in the Appendix).

Lemma 1: When $\boldsymbol{S}_w(t)$ has full row rank, the necessary and sufficient condition for $\boldsymbol{s}_{t-d}^{(k)} \in \mathcal{R}\{\boldsymbol{X}_w(t)\}$ is that the $(L_k + 1 - d)$th column of $\boldsymbol{H}_w^{(k)}$ is linearly independent of other columns in $\boldsymbol{H}_w$.

Proof of Proposition 1: Lemma 1 states that when $\boldsymbol{S}_w(t)$ has full row rank, the necessary and sufficient condition for the ith row vector of $\boldsymbol{S}_w(t)$ being contained in $\mathcal{R}\{\boldsymbol{X}_w(t)\}$ is that the ith column vector of $\boldsymbol{H}_w$ is linearly independent of other columns in $\boldsymbol{H}_w$. Hence, under the assumption that no column of $\boldsymbol{H}_w$ is linearly independent of other columns in $\boldsymbol{H}_w$ for $w < w_{\min}$, no colliding packets can be obtained from $\boldsymbol{X}_w(t)$ by the LS receiver when $w < w_{\min}$. Proposition 1 then follows directly from the fact that the minimum number of known symbols required by the LS receiver to recover the colliding packets from $\boldsymbol{X}_{w_{\min}}(t)$ is equal to the dimension of $\mathcal{R}\{\boldsymbol{X}_{w_{\min}}(t)\}$. The lower bound on $\mathcal{N}_{\text{LS}}$ comes from the lower bound on w given in (10). □□□

It is interesting to note that under the assumptions of Proposition 1, the conventional LS receiver does not have partial resolvability. Here, we define partial resolvability of a receiver as the ability to recover some of the colliding packets when the number of available known symbols is smaller than the minimum amount required for resolution of all colliding packets. For the conventional LS receiver, when the number of known symbols is equal to or larger than $\mathcal{N}_{\text{LS}}$, all colliding packets can be obtained. Otherwise, they are all lost. The lack of partial resolvability in the LS receiver results from the fact that every colliding packet is obtained from $\boldsymbol{X}_w(t)$ with $w \geq w_{\min}$. Because no specific information about the propagation channel is available, the receiver may have to assume that no column of $\boldsymbol{H}_w$ is linearly independent of other columns in $\boldsymbol{H}_w$ for $w < w_{\min}$, i.e., no packet can be obtained from $\boldsymbol{X}_w(t)$ by the LS receiver when $w < w_{\min}$.

We point out that in Proposition 1 and other theoretical results that follow, we consider a packet recovered if it is obtained exactly by a *linear* FIR filter in the absence of noise. If the LS receiver is followed by a nonlinear filter, such as a quantizer that maps the symbol estimates to their nearest constellation points, the minimum number of required known symbols may be smaller than $\mathcal{N}_{\mathrm{LS}}$, as illustrated by the simulation results in Section VI.

C. Active User Detection Scheme

In order to know which sets of known symbols we need to use in the collision resolution, the receiver needs the active user profile, which may not be available in random access networks. Exhausting every set of known symbols is a computationally expensive way of solving this problem. Here, we present a simple scheme to detect those users that are involved in a collision.

Suppose that there are total M users in the network. Let $\alpha_k (k = 1, \cdots, M)$ be the vector containing the positions of the known symbols in $\boldsymbol{s}_t^{(k)}$. If the kth user is active in the time slot when a particular collision occurs, then from (14), we have

$$\boldsymbol{s}_t^{(k)}(\alpha_k) \in \mathcal{R}\{\boldsymbol{U}(\alpha_k)\}. \tag{17}$$

It follows that the projection error of $\boldsymbol{s}_t^{(k)}(\alpha_k)$ into $\mathcal{R}\{\boldsymbol{U}(\alpha_k)\}$ is zero. In contrast, if the kth user is not involved in the collision, the projection error of $\boldsymbol{s}_t^{(k)}(\alpha_k)$ into $\mathcal{R}\{\boldsymbol{U}(\alpha_k)\}$ cannot be zero under the assumption that $\boldsymbol{s}_t^{(k)}(\alpha_k)$ is linearly independent of any row vector in the input matrix $\boldsymbol{S}_w(t, \alpha_k)$. Hence, the active users can be detected by investigating the projection error of $\boldsymbol{s}_t^{(k)}(\alpha_k)$ into $\mathcal{R}\{\boldsymbol{U}(\alpha_k)\}$ for $k = 1, \cdots, M$. In the noisy case, we need to compare the projection error of $\boldsymbol{s}_t^{(k)}(\alpha_k)$ into $\mathcal{R}\{\boldsymbol{U}(\alpha_k)\}$ with a certain threshold to determine whether the kth user is active.

IV. Semi-Blind Least Squares Smoothing Approach

The least squares smoothing (LSS) approach was originally proposed for blind identification of single input multiple output channels [17], [18], [28]. It was then realized in [26] and [27] that the LSS approach can be applied to blind or semi-blind detection in multiple-input multiple-output systems. The filtering effect of the LSS approach makes it particularly attractive for successive detection, where users are extracted sequentially, based on their channel conditions. In this section, we propose a semi-blind collision resolution technique based on the LSS approach. We will show that the semi-blind LSS receiver provides significant improvement in resolvability over the training-based LS receiver.

A. Basic Idea

Proposition 1 shows that the minimum number of known symbols required by the LS receiver is a monotone increasing function of the number of colliding packets (K) and the summation of the channel orders (L). This observation suggests that in order to reduce the number of required known symbols, we should reduce MAI (decrease K) and ISI (decrease L) in the received signal. The basic idea of the semi-blind LSS receiver is to generate from the received signal an innovation sequence that contains less MAI and ISI. The colliding packets can then be obtained from this innovation sequence with fewer known symbols.

We illustrate the basic idea of the semi-blind LSS receiver with the same example given in Section III. Consider first the detection of the two packets $\boldsymbol{s}_t^{(1)}$ and $\boldsymbol{s}_t^{(2)}$ with the smallest channel order from $\boldsymbol{X}_2(t)$. The key observation here is that although the input vectors with different time indices are linearly independent, the channel memory brings time dependency, hence, redundancy, into the output. To reduce MAI and ISI, we introduce a smoothing operation on $\boldsymbol{X}_2(t)$ to obtain the innovation with respect to the future and past data. Specifically, consider $w = 2$ consecutive future and past data vectors $\boldsymbol{X}_2(t+2)$ and $\boldsymbol{X}_2(t-2)$ given by

$$\boldsymbol{X}_2(t+2) \triangleq \begin{pmatrix} \boldsymbol{x}_{t+2} \\ \boldsymbol{x}_{t+3} \end{pmatrix} = \boldsymbol{H}_2 \boldsymbol{S}_2(t+2)$$
$$\boldsymbol{X}_2(t-2) \triangleq \begin{pmatrix} \boldsymbol{x}_{t-2} \\ \boldsymbol{x}_{t-1} \end{pmatrix} = \boldsymbol{H}_2 \boldsymbol{S}_2(t-2).$$

Under A1, we have the following two isomorphic relations:

$$\begin{aligned} \mathcal{R}\{\boldsymbol{X}_2(t+2)\} &= \mathcal{R}\{\boldsymbol{S}_2(t+2)\} \\ \mathcal{R}\{\boldsymbol{X}_2(t-2)\} &= \mathcal{R}\{\boldsymbol{S}_2(t-2)\}. \end{aligned} \tag{18}$$

The overall data matrix $\boldsymbol{X}_6(t-2)$ that contains the future, the current, and the past data is given by

$$\boldsymbol{X}_6(t-2) \triangleq \begin{pmatrix} \boldsymbol{X}_2(t-2) \\ \boldsymbol{X}_2(t) \\ \boldsymbol{X}_2(t+2) \end{pmatrix} = \boldsymbol{H}_6 \boldsymbol{S}_6(t-2). \tag{19}$$

Under the assumption that $\boldsymbol{H}_6$ has full column rank and $\boldsymbol{S}_6(t-2)$ has full row rank, $\boldsymbol{X}_6(t-2)$ spans a 22-dimensional row space, which is isomorphic to $\mathcal{R}\{\boldsymbol{S}_6(t-2)\}$. This isomorphism is illustrated in Fig. 2, where the output block row vectors in $\boldsymbol{X}_6(t-2)$ and the input row vectors in $\boldsymbol{S}_6(t-2)$ are plotted. The other two pairs of isomorphic spaces specified in (18) are also illustrated with right and left slashes, respectively. The input row vectors involved in the current data $\boldsymbol{X}_2(t)$ are shaded with horizontal lines. Fig. 2 shows that among all input vectors contained in $\boldsymbol{X}_2(t)$, only $\boldsymbol{s}_t^{(1)}$ and $\boldsymbol{s}_t^{(2)}$ are outside the space spanned by the future and past data. All the other input vectors in $\boldsymbol{X}_2(t)$ are contained in either $\mathcal{R}\{\boldsymbol{X}_2(t+2)\}$ or $\mathcal{R}\{\boldsymbol{X}_2(t-2)\}$. For uncorrelated and zero-mean input signals, $\boldsymbol{s}_t^{(1)}$ and $\boldsymbol{s}_t^{(2)}$ are, asymptotically, the innovations of $\boldsymbol{X}_2(t)$ with respect to $\boldsymbol{X}_2(t+2)$ and $\boldsymbol{X}_2(t-2)$. It then follows that the asymptotic smoothing error

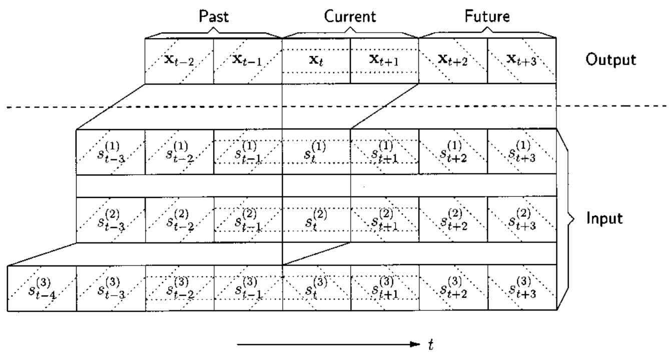

Fig. 2. Isomorphism between input and output subspaces.

$\boldsymbol{E}_2(t)$ of $\boldsymbol{X}_2(t)$ by the future data $\boldsymbol{X}_2(t+2)$ and the past data $\boldsymbol{X}_2(t-2)$ has the following form:

$$\begin{aligned}\boldsymbol{E}_2(t) &\triangleq \mathcal{P}^{\perp}_{[\boldsymbol{X}_2'(t+2),\, \boldsymbol{X}_2'(t-2)]'}\{\boldsymbol{X}_2(t)\}\\ &= \boldsymbol{h}_1 \boldsymbol{s}_t^{(1)} + \boldsymbol{h}_2 \boldsymbol{s}_t^{(2)} \end{aligned} \tag{20}$$

where $\boldsymbol{h}_i \triangleq [\boldsymbol{h}_i'(0), \cdots, \boldsymbol{h}_i'(L_i)]'$ is the vector of channel coefficients for the ith user.

From (20), we note that the smoothing error $\boldsymbol{E}_2(t)$ contains only two multiaccess interferers; the MAI from the third user and the ISI of the first two users are completely removed from $\boldsymbol{X}_2(t)$ by the smoothing operation. The original system with $K=3$, $L=4$ has been converted to a system with $K=2$ and $L=0$. With $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ linearly independent, $\boldsymbol{E}_2(t)$ spans a 2-D row space from which $\boldsymbol{s}_t^{(1)}$ and $\boldsymbol{s}_t^{(2)}$ can be obtained with two known symbols from each.

After recovering the first two users' signals, we can also obtain their channel coefficients from $\boldsymbol{E}_2(t)$ as

$$[\boldsymbol{h}_1, \boldsymbol{h}_2] = \boldsymbol{E}_2(t)\boldsymbol{S}'_{t,1:2}(\boldsymbol{S}_{t,1:2}\boldsymbol{S}'_{t,1:2})^{-1} \tag{21}$$

where

$$\boldsymbol{S}_{t,1:2} = \begin{pmatrix} \boldsymbol{s}_t^{(1)} \\ \boldsymbol{s}_t^{(2)} \end{pmatrix}.$$

Hence, signals from the first two users can be subtracted from the output, and we then have a system with $K=1$ and $L=2$. With the output after the subtraction denoted as $\overline{\boldsymbol{x}}_t$, we now apply the same process to the third user. Consider the smoothing error of $\overline{\boldsymbol{X}}_3(t)$ by $\overline{\boldsymbol{X}}_2(t+3)$ and $\overline{\boldsymbol{X}}_2(t-2)$. With similar analysis, we have the asymptotic smoothing error $\boldsymbol{E}_3(t)$ as

$$\begin{aligned}\boldsymbol{E}_3(t) &\triangleq \mathcal{P}^{\perp}_{[\overline{\boldsymbol{X}}_2'(t+3),\, \overline{\boldsymbol{X}}_2'(t-2)]'}\{\overline{\boldsymbol{X}}_3(t)\}\\ &= \boldsymbol{h}_3 \boldsymbol{s}_t^{(3)}. \end{aligned}$$

With one known symbol from the third user to remove the scalar ambiguity, both $\boldsymbol{s}_t^{(3)}$ and $\boldsymbol{h}_3$ can be obtained from $\boldsymbol{E}_3(t)$.

B. Semi-Blind LSS Receiver with Successive Demodulation

Here, we consider the general case where we have K packets involved in a collision. Suppose that $L_k \in \{l_1, \cdots, l_J\}$ with $l_1 < l_2 < \cdots < l_J$. The number of colliding packets that come from channels with order l_i is k_i ($\sum_{i=1}^{J} k_i = K$). Without loss of generality, we assume that the packets are arranged according to their channel orders and that the packets from channels with order l_1 are the first k_1 packets. Then, we have the following theorem that characterizes the smoothing error.

Theorem 1: Suppose that A1 holds for w and $2w + l_1 + 1$. Then, for uncorrelated and zero-mean input signals, the asymptotic smoothing error $\boldsymbol{E}_{l_1+1}(t)$ is given by

$$\begin{aligned}\boldsymbol{E}_{l_1+1}(t) &\triangleq \mathcal{P}^{\perp}_{[\boldsymbol{X}_w'(t+l_1+1),\, \boldsymbol{X}_w'(t-w)]'}\{\boldsymbol{X}_{l_1+1}(t)\}\\ &= \boldsymbol{h}_1 \boldsymbol{s}_t^{(1)} + \cdots + \boldsymbol{h}_{k_1} \boldsymbol{s}_t^{(k_1)}. \end{aligned} \tag{22}$$

Theorem 1, which can be obtained by applying the single-user result in [18] to multiuser cases as shown in [26], summarizes the key result on which the semi-blind LSS approach is based. From (22), we note that the smoothing error contains $k_1 \leq K$ multiaccess interferers and no ISI. With k_1 known symbols from the first k_1 users, their packets can be obtained as linear combinations in the k_1-dimensional row space spanned by the smoothing error. After recovering $\boldsymbol{s}_t^{(1)}, \cdots, \boldsymbol{s}_t^{(k_1)}$, we can also obtain $\boldsymbol{h}_1, \cdots, \boldsymbol{h}_{k_1}$ from $\boldsymbol{E}_{l_1+1}(t)$ as shown in (21). Consequently, interference from the first k_1 users can be subtracted from the received signal, and the same process can be applied to users with channel order l_2 that is now is the smallest channel order.

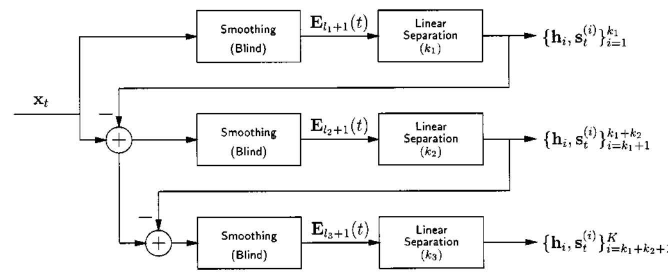

Fig. 3. Schematic diagram of the semi-blind LSS approach.

A schematic diagram of this approach is shown in Fig. 3, where we assume, without loss of generality, that $J = 3$. Fig. 3 shows that the semi-blind LSS receiver consists of two operations: a blind smoothing and a nonblind linear separation. The minimum number of known symbols required by each linear separator is also marked in Fig. 3.

One possible implementation of the semi-blind LSS approach with successive demodulation is summarized in Fig. 4.

We point out that when the channel orders are unknown to the receiver, the semi-blind LSS approach can be implemented by starting the successive demodulation from a lower bound to an upper bound of the channel orders. At each stage, an energy detector can be built on the smoothing error to test whether there are users with this order present. If there are, the same active user detection scheme presented in Section III-C can be implemented on the smoothing error, and users with this order can be obtained by exploiting their known symbols. It is possible that a weak user eludes the energy detector. In this case, this user appears in the smoothing errors of later stages. The dimension of the space spanned by the smoothing error in these stages increases; hence, more known symbols are required by the linear separators.

Semi-blind LSS with Successive Demodulation

Choose $w \geq w_0$ such that A1 holds for w and $2w + l_k + 1$ $(1 \leq k \leq K)$.

For $i = 1 : J$

1. Form $\mathbf{Y}_w(t + l_i + 1)$, $\mathbf{Y}_w(t - w)$, and $\mathbf{Y}_{l_i+1}(t)$.
2. Obtain the orthogonal basis $\{\mathbf{u}_1, \cdots, \mathbf{u}_r\}$ that spans the r-dimensional signal row space of $[\mathbf{Y}'_w(t + l_i + 1), \mathbf{Y}'_w(t - w)]'$, where $r = (K - \sum_{j=1}^{i-1} k_j)(2w + l_i + 1) + \sum_{j=i}^{J} k_j l_j - k_i$.
3. Obtain the projection error of $\mathbf{Y}_{l_i+1}(t)$ onto $sp\{\mathbf{u}_1, \cdots, \mathbf{u}_r\}$:

$$\mathbf{E}_{l_i+1}(t) \triangleq \mathbf{Y}_{l_i+1}(t) - \mathbf{Y}_{l_i+1}(t)\mathbf{U}'\mathbf{U}, \quad \mathbf{U} = \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_r \end{pmatrix}.$$

4. Obtain the orthogonal basis $\{\mathbf{v}_1, \cdots, \mathbf{v}_{k_i}\}$ that spans the k_i-dimensional signal row space of $\mathbf{E}_{l_i+1}(t)$.
5. Obtain the k_i packets that have order l_i from $\{\mathbf{v}_1, \cdots, \mathbf{v}_{k_i}\}$ with k_i known symbols from each of these k_i packets.
6. Obtain the corresponding channel vectors from $\mathbf{E}_{l_i+1}(t)$ as shown in (21).
7. Subtract these k_i packets from the observation.

end

Fig. 4. Semi-blind least squares smoothing algorithm.

C. Minimum Number of Known Symbols Required by the Semi-Blind LSS Receiver

Theorem 1 and the example given in Section IV-A show that unlike the LS receiver, the minimum number of known symbols required by the semi-blind LSS with successive demodulation does not depend on the specific values of $L_k (1 \leq k \leq K)$. Instead, it is determined by the diversity of the channel orders as specified in the following proposition.

Proposition 2: Suppose that A1 holds for w and $2w + l_k + 1 (1 \leq k \leq K)$. Then, for uncorrelated and zero-mean input signals, the minimum number $\mathcal{N}_{\rm LSS}$ of known symbols required by the LSS receiver with successive demodulation to recover all colliding packets in the absence of noise is given by

$$\mathcal{N}_{\rm LSS} = \max\{k_1, \cdots, k_J\} \leq K. \tag{23}$$

Proposition 2 is a direct consequence of Theorem 1 and successive demodulation. As illustrated in Fig. 3, the J linear separators in the LSS receiver require $k_1, \cdots, k_J$ known symbols, respectively. Hence, the minimum number of known symbols required for recovering all colliding packets is $\max\{k_1, \cdots, k_J\}$, as given in (23). The upper bound on $\mathcal{N}_{\rm LSS}$ follows directly from $k_i \leq K (i = 1, \cdots, J)$. This upper bound is achieved only when $L_1 = L_2 = \cdots = L_K$, i.e., $J = 1$. In the other extreme case when the channel orders of the colliding packets are distinct, the LSS receiver can resolve the collision blindly if the scalar ambiguity is not taken into consideration.

V. Comparison of the LS and the Semi-Blind LSS Receivers

From (23) and (16), we can see that the upper bound K on $\mathcal{N}_{\rm LSS}$ is a lower bound on $\mathcal{N}_{\rm LS}$, which can be achieved only when $L = 0$. Consequently, we have the following proposition.

Proposition 3: Under the assumptions of Proposition 1 and Proposition 2, we have

$$\mathcal{N}_{\rm LSS} \leq \mathcal{N}_{\rm LS} \tag{24}$$

with equality if and only if $L = 0$.

The above proposition states that the semi-blind LSS receiver requires fewer known symbols than the LS receiver, except when $L = 0$. When $L = 0$, the received signal is a static linear mixture of the K colliding packets. There is no channel order diversity, and the ideal channels do not introduce redundancy into the observation. The innovation sequence generated by the smoothing operation is, asymptotically, the observation itself. Hence, the LSS receiver requires the same number of known symbols as the LS receiver.

Another property of the semi-blind LSS receiver that distinguishes it from the LS receiver is its partial resolvability. We have shown in Section III that the LS receiver does not offer partial resolution of the colliding packets. However, taking advantage of the channel order diversity and successive subtraction, the semi-blind LSS receiver possesses partial resolvability. As illustrated in Fig. 3, the first k_1 packets can be obtained by the LSS receiver, as long as $k_1 \leq \mathcal{N}_{\rm LSS}$ known symbols are available. In the following proposition, we compare the number of packets that can be recovered by the semi-blind LSS receiver and the LS receiver given m known symbols.

Proposition 4: Suppose that there are m known symbols available. Under the assumptions of Proposition 1 and Proposition 2, the LS receiver and the semi-blind LSS receiver can recover, respectively, $\mathcal{K}_{\rm LS}(m)$ and $\mathcal{K}_{\rm LSS}(m)$ colliding packets, as given by

$$\mathcal{K}_{\rm LS}(m) = \begin{cases} 0, & \text{if } m < \mathcal{N}_{\rm LS} \\ K, & \text{if } m \geq \mathcal{N}_{\rm LS} \end{cases} \tag{25}$$

$$\mathcal{K}_{\rm LSS}(m) = \begin{cases} 0, & \text{if } m < k_1 \\ \displaystyle\sum_{i=1}^{j} k_i, & \text{if } m \geq k_1 \end{cases} \tag{26}$$

where

$$j = \max_l \{k_i \leq m \quad \text{for } i = 1, \cdots, l\}. \tag{27}$$

Proposition 4 gives a quantitative characterization of the partial resolvability of the semi-blind LSS receiver. It also demonstrates that given the same number of known symbols, the LSS receiver achieves noticeable improvement in resolvability over the LS receiver. Specifically, from $k_1 \leq \mathcal{N}_{\rm LSS} \leq \mathcal{N}_{\rm LS}$, we have

$$\mathcal{K}_{\rm LSS}(m) \geq \mathcal{K}_{\rm LS}(m) \tag{28}$$

with equality if and only if $m < k_1$ [when $\mathcal{K}_{\rm LSS}(m) = \mathcal{K}_{\rm LS}(m) = 0$] or $m \geq \mathcal{N}_{\rm LS}$ [when $\mathcal{K}_{\rm LSS}(m) = \mathcal{K}_{\rm LS}(m) = K$]. We illustrate the two formulas given in Proposition 4 in Fig. 5, where we consider a typical example with $J = 4$, $k_2 < k_1 < k_3 < k_4$, and $L \neq 0$.

We point out that Proposition 1 holds for a finite number of data samples, while Propositions 2–4 are asymptotic results. The loss of finite sample convergence property in the semi-blind LSS receiver is a price we paid for the reduction of required known symbols. However, the simulation results shown in Section VI demonstrate that even with a relatively small packet size (for example, 250 QPSK symbols per packet), the semi-blind

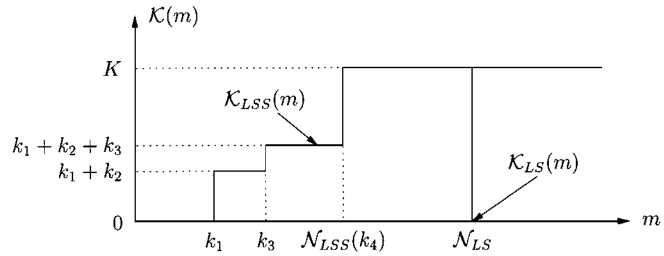

Fig. 5. Typical example of $\mathcal{K}_{\rm LSS}(m)$ and $\mathcal{K}_{\rm LS}(m)$.

TABLE I
SETUP OF MULTIPATH CHANNEL PARAMETERS

Chip pulse shaping filter:	raised-cosine function with roll-off factor $\beta = 0.25$.
Number of rays:	Uniformly distributed among $\{1, 2, 3, 4, 5\}$.
Delay of each ray:	uniformly distributed on $[0, T]$, where T was the symbol period. Delays of two different rays were independent with each other.
Amplitude of each ray:	Real and imaginary parts were independent Gaussian random variables with zero mean and unit variance. Amplitudes of two different rays were independent with each other.
Spreading gain:	$P_0 = 32$.
Over sampling factor:	$P_1 = 2$.
Number of outputs:	$P = P_0 P_1 = 64$.

LSS receiver offers significant improvement in collision resolvability over the LS receiver.

VI. Simulation Examples

A. Setup

Simulation studies on the collision resolvability of the proposed semi-blind LSS receiver in random access networks with a common code are presented in this section. We randomly generated 100 realizations of packet collision where three packets, each containing 1000 QPSK symbols, were involved in each collision, i.e., a total of 300 colliding packets were tested in the simulations. We assumed that the active user profile was known to the receiver. The propagation channels of these 300 colliding packets were randomly generated multipath channels whose parameters are listed in Table I. With this setup of channel parameters, the possible channel orders are 1 and 2 (in symbol period).

In the simulations, we chose $w = w_0 = 1$, where w_0 is defined in (10). The semi-blind LSS approach was implemented as a front-end filter to obtain sufficient known symbols. The LS receiver was then implemented in the second step, based on the known symbols obtained by the LSS approach. The performance of this implementation of the semi-blind LSS receiver was compared with that of the conventional LS receiver with optimal delay.

B. Resolvability Comparison

The concept of resolvability was introduced in [25] and [29] as a performance measure for receivers with collision resolution capability. The collision resolvability function $\mathcal{R}_K(k, m, q)$ of a receiver is defined as the probability that given m known symbols, at least $1 \leq k \leq K$ colliding packets can be detected with no more than q errors at a certain SNR.

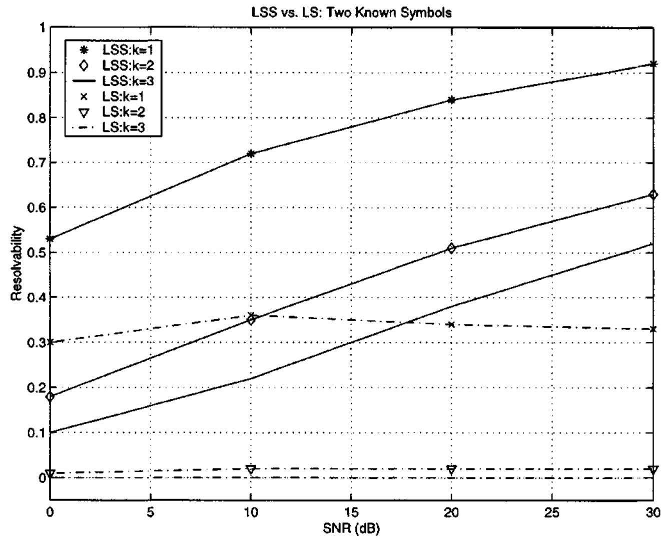

Fig. 6. Resolvability comparison ($m = 2, q = 10$, perfect power control).

In this simulation example, perfect power control was assumed, and two known symbols ($m = 2$) were considered to be available in each packet. We considered a colliding packet to be resolved if it was detected with no more than ten errors ($q = 10$). The resolvability functions ($k = 1, 2, 3$) of the LS receiver and the semi-blind LSS receiver obtained from this simulation are shown in Fig. 6, which demonstrates that the proposed semi-blind LSS receiver provides significantly improved resolvability over the conventional LS receiver.

The simulation result shown in Fig. 6 appears to be inconsistent with (25). Since $m = 2 < \mathcal{N}_{\rm LS}$, (25) states that the LS receiver cannot resolve any colliding packets. However, the result shown in Fig. 6 indicates that in about one third of these 100 collision events, the LS receiver could recover at least one colliding packet with two known symbols. This “inconsistency” between the theoretical and simulation results comes from the difference in the definition of resolvability. In Proposition 4 and other theoretical results presented in this paper, we consider a packet being resolved if it is obtained exactly in the absence of noise, whereas quantization and error-control coding were considered in the simulations because of the presence of noise. The simulation result that the LS receiver had nonzero probability of recovering some of the colliding packets when $m < \mathcal{N}_{\rm LS}$ can be explained as follows. In the simulations, the colliding packets were obtained from the space spanned by the first two right singular vectors of $\boldsymbol{Y}_w(t)$ with two known symbols. There were cases when a certain delay of one colliding packets was “almost” contained in this 2-D space. After quantization and error correction, this packet can be recovered (possibly with delay). Since we can consider only a 2-D subspace of $\mathcal{R}\{\boldsymbol{Y}_w(t)\}$ when two known symbols are available, it is impossible that all three packets can be resolved by the LS receiver. This is consistent with the result in Fig. 6, where the resolvability function of the LS receiver at $k = 3$ was zero in the whole SNR range. Another observation we make from Fig. 6 is that the resolvability function of the LS receiver leveled off when SNR went to infinity. This is because when $m = 2 < \mathcal{N}_{\rm LS}$, the perfect LS filter could not be constructed to eliminate all interference. The detection

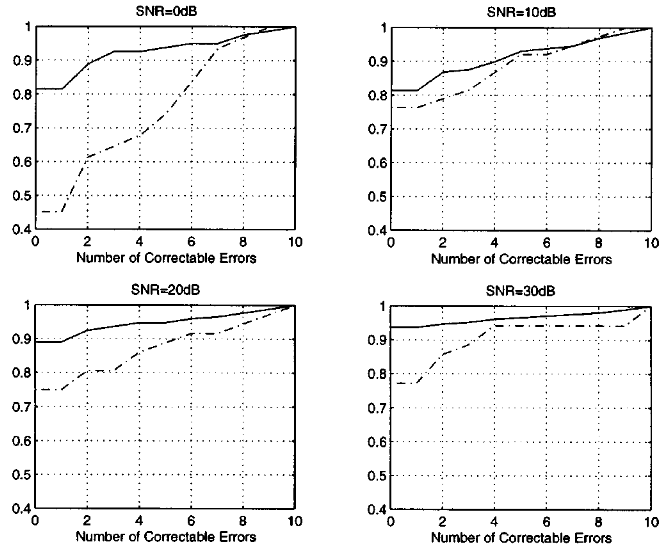

Fig. 7. Percentage of the packets detected with no more than $q(0 \leq q \leq 10)$ errors among the packets detected with no more than 10 errors ("—" : LSS, "—." : LS).

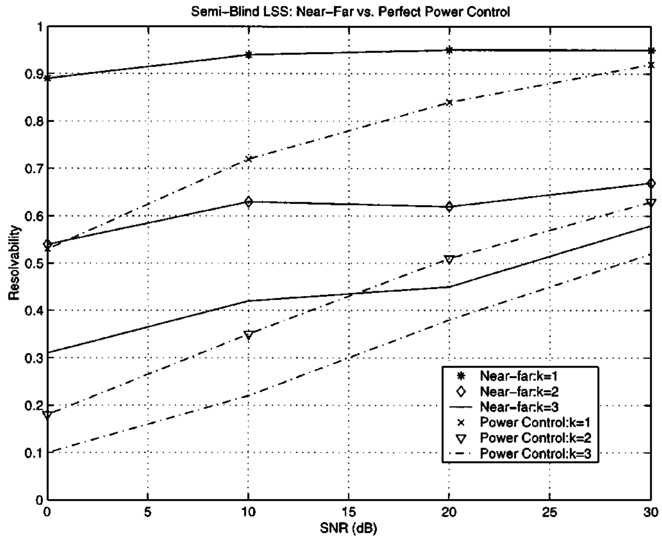

Fig. 8. Near–far versus perfect power control ($m = 2, q = 10$).

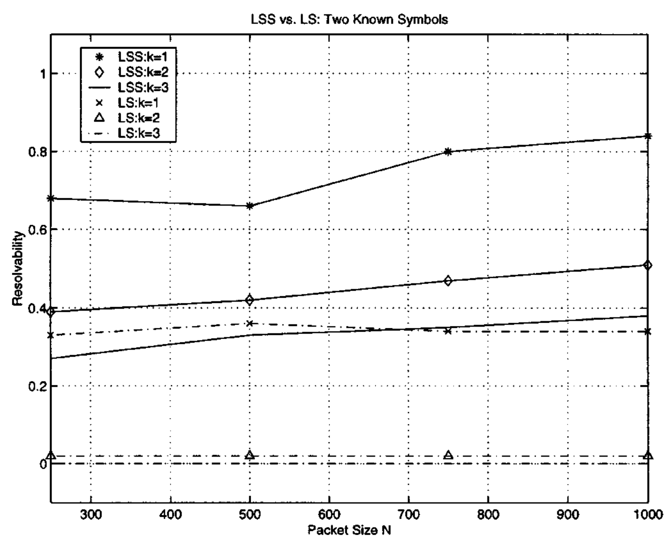

Fig. 9. Effect of packet size ($m = 2$, $q =$[3, 5, 8, 10], SNR $=$ 20 dB, perfect power control).

error was mainly determined by the residual interference rather than the additive noise.

C. Effect of Error-Control Codes

In this simulation, we study the effect of error-control codes on the resolvability of the LS receiver and the semi-blind LSS receiver. We investigated the percentage of the colliding packets detected with no more than $q(0 \leq q \leq 10)$ errors among the packets detected with no more than ten errors. The simulation result is shown in Fig. 7, where the solid and the dashed curves are results obtained with the semi-blind LSS receiver and the LS receiver, respectively. Perfect power control was assumed, and two known symbols from each user were exploited in both receivers. Fig. 7 shows that the proposed semi-blind LSS receiver had less dependence on error-control codes than the LS receiver, especially at 0 dB. From Fig. 7, we also observe that error-control codes played a less important role at higher SNR. For the semi-blind LSS receiver at 30 dB, among the colliding packets detected with no more than ten errors, about 95% packets could be detected with no error. Hence, in cases when the efficiency of bandwidth utilization is of great concern, the semi-blind LSS receiver can be implemented without error-control coding, and the decrease in the collision resolvability is only about 5%.

D. Near–Far Resistance

The near–far resistance of the proposed semi-blind LSS receiver is studied in this simulation. Recognizing that in near–far scenarios, users with smaller channel orders are usually closer to the receiver and have larger power, we set the received signal power of users with order 1 stronger by 6 dB than that of users with order 2. SNR was defined as the ratio of the strongest user's received power to the noise variance. Two known symbols from each user were exploited, and an error-control code that could correct up to 10 errors in a packet was assumed. In Fig. 8, the resolvability of the semi-blind LSS receiver with perfect power control is compared with that in the near–far scenario. Fig. 8 shows that the semi-blind LSS receiver performed better in the near–far scenario than in the perfect power control case. Even though the SNR was defined with respect to the strongest user, the probability of resolving all three colliding packets in the near–far scenario was larger than that with perfect power control. The reason for this is that the LSS receiver first extracts packets with smaller channel order that have larger power in near–far scenarios. The subtraction of stronger users from the observation facilitates the detection of weaker ones. This indicates that the semi-blind LSS receiver is near–far resistant.

E. Effect of Packet Size

Since the semi-blind LSS approach is a stochastic algorithm that relies on the convergence of the second-order statistics, it is desirable to investigate the effect of packet size on its resolvability. In Fig. 9, we plotted the resolvability of the semi-blind LSS approach as a function of packet size N at SNR $=$ 20 dB. The number of errors that could be corrected in a packet was

TABLE II
MAXIMUM THROUGHPUT OF STABILIZED SLOTTED ALOHA WITH LS AND SEMI-BLIND LSS RECEIVER

MPR Capability:	No MPR	LSS with $m=1$	LSS with $m=2$	LS with $m=12$
Maximum Throughput:	0.3679	0.8156	1.3045	0.8557

q = 3, 5, 8, and 10 for N = 250, 500, 750, and 1000, respectively. From Fig. 9, we can see that the performance of the semi-blind LSS approach degraded gracefully as packet size decreased from 1000 to 250. At $N = 250$, the semi-blind LSS approach still offered significant improvement in resolvability over the LS receiver.

F. Improvement in Network Performance

The LS receiver and the semi-blind LSS receiver can be implemented in both ad hoc networks and cellular systems for collision resolution. Not requiring feedback channels, they can be used harmoniously with existing protocol-based collision resolution algorithms. Here, we investigate the throughput improvement achieved by the LS receiver and the semi-blind LSS receiver in an infinite user cellular system using stabilized slotted Aloha.

In an uncoded ($q = 0$) system with no noise, the resolvability $\mathcal{R}_K(k, m)(K \geq 1, 0 \leq k \leq K)$ of the LS receiver and the semi-blind LSS receiver can be explicitly calculated based on P (the number of subchannels) and the distribution of the channel orders [25], [29]. The impact of the MPR capability of the LS receiver and the semi-blind LSS receiver on the maximum throughput of an infinite user cellular network using stabilized slotted Aloha can then be evaluated based on the result in [8]. Consider an example where we have $P = 8$, and the channel order of each packet is identically and independently distributed on {2, 3, 4} with equal probability. The maximum stable throughput of a slotted Aloha network with the semi-blind LSS receiver ($m = 1$, 2) and the LS receiver ($m = 12$) is listed in Table II, where we assume that collisions with multiplicity $K > 3$ cannot be resolved by either LS or semi-blind LSS receiver, i.e., $\mathcal{R}_K(k, m) = 0$ for $K > 3, 1 \leq k \leq K$, and the given m. Compared with the maximum stable throughput $e^{-1} \approx 0.3679$ of the slotted Aloha network with usual collision channel (no MPR capability), the throughput gain achieved by the MPR capability of the semi-blind LSS receiver and the LS receiver is obvious.

VII. CONCLUSION

In this paper, we present a semi-blind collision resolution technique that does not rely on the code information of colliding packets. Consequently, it can be applied to random access networks with various transmission protocols. It also provides a possible solution to packet collision in narrowband systems with fractional sampling and/or antenna array. Compared with the training-based LS receiver, significant improvement in resolvability is achieved by the proposed semi-blind LSS receiver.

The semi-blind LSS approach suggests a new way of exploiting known symbols. Different from many existing semi-blind approaches where known symbols are used to form a penalty term in certain blind criteria, the proposed algorithm separates the utilization of known symbols from the blind criterion. A two-step procedure that consists of a blind smoothing process and a nonblind linear separation is implemented in the semi-blind LSS approach. The blind smoothing process reduces the number of known symbols required for collision resolution by removing ISI and reducing MAI. The linear separator is then implemented in the second step to resolve the colliding packets by exploiting their embedded known symbols. Furthermore, unlike many existing semi-blind methods, the LSS receiver does not require known symbols to be consecutive.

The proposed semi-blind LSS approach is a stochastic algorithm that relies on the convergence of the second-order statistics. In order to improve the performance with a finite number of data samples, the LSS approach can be implemented as an interference reduction filter followed by blind or training-based methods.

APPENDIX

Proof of Lemma 1: If the (L_k+1-d)th column of $\boldsymbol{H}_w^{(k)}$ is linearly independent of other columns in $\boldsymbol{H}_w$, then there exists a vector $\boldsymbol{f} \neq \boldsymbol{0}$ such that

$$\boldsymbol{f}'\boldsymbol{H}_w = \boldsymbol{E}_D' \tag{29}$$

where $\boldsymbol{E}_D$ is the Dth unit vector with D defined as $D \triangleq L_k + 1 - d + \sum_{i=1}^{k-1}(w + L_i)$. Thus

$$\boldsymbol{f}'\boldsymbol{X}_w(t) = \boldsymbol{f}'\boldsymbol{H}_w\boldsymbol{S}_w(t) = \boldsymbol{s}_{t-d}^{(k)} \tag{30}$$

i.e.,

$$\boldsymbol{s}_{t-d}^{(k)} \in \mathcal{R}\{\boldsymbol{X}_w(t)\}. \tag{31}$$

Conversely, if $\boldsymbol{s}_{t-d}^{(k)} \in \mathcal{R}\{\boldsymbol{X}_w(t)\}$, then there exists a vector $\boldsymbol{f} \neq \boldsymbol{0}$ such that (30) holds. As a direct result, we have

$$(\boldsymbol{f}'\boldsymbol{H}_w - \boldsymbol{E}_D')\boldsymbol{S}_w(t) = \boldsymbol{0}. \tag{32}$$

Since $\boldsymbol{S}_w(t)$ is of full-row rank, we have, from (32)

$$\boldsymbol{f}'\boldsymbol{H}_w = \boldsymbol{E}_D' \tag{33}$$

which implies that

$$\begin{cases} \boldsymbol{f}^t\boldsymbol{h}_i = 0, & i = 1, \cdots, Kw + L, i \neq D \\ \boldsymbol{f}^t\boldsymbol{h}_D = 1 \end{cases} \tag{34}$$

where $\boldsymbol{h}_i$ is the ith column of $\boldsymbol{H}_w$.

Assume that

$$\boldsymbol{h}_D = \sum_{i \neq D} g_i\boldsymbol{h}_i. \tag{35}$$

Then

$$\boldsymbol{f}^t\boldsymbol{h}_D = \boldsymbol{f}^t\sum_{i \neq D} g_i\boldsymbol{h}_i = \sum_{i \neq D} g_i\boldsymbol{f}^t\boldsymbol{h}_i = 0 \tag{36}$$

which contradicts with (34). Hence, $\boldsymbol{h}_D$, which is the (L_k+1-d)th column of $\boldsymbol{H}_w^{(k)}$, is linearly independent of other columns in $\boldsymbol{H}_w$. □□□

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