Detection with Embedded Known Symbols:

Optimal Symbol Placement and Equalization

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Introduction

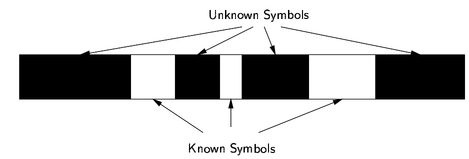

Figure 1: Typical data stream.

- For a given receiver structure, the placement of known symbols may affect the receiver performance
- This raises the following questions
  - What is the optimal placement of known symbols?
  - Does the placement of known symbols depend on the transmission channel ?
  - What is the optimal equalizer

Previous Work

- The iterative equalization problem is related to the problem of equalization in the presence of known symbols.
  - Proakis'69,Gersho and Lim'81 and Slock and De Carvalho'96, problem of designing the optimal equalizer for iterative equalization
  - Mueller and Salz'81 have addressed the same problem assuming only a subset of data symbols are known.
- Hsu'85 and more recently Kaleh'95 have addressed the equalization problem for block transmission systems
- S.Adireddy and L.Tong,CISS'00, have optimized the position of known symbol clusters for maximizing the mutual information.

Optimizing the Position of Known Symbol Clusters

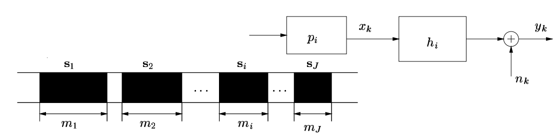

Figure 2: The Channel with a Typical Input

- $$(\mathcal{P}^*, f_{iid}^*(\mathbf{s}), p^*) = \arg \max_{\mathcal{P}, f_{iid}(\mathbf{s}), p} I(\mathbf{y}; \mathbf{x}) \quad (1)$$

  where $f_{iid}(\mathbf{s})$ is an iid probability distribution, $\mathcal{P}$ is the set of all the possible known symbol cluster placements with cluster length at least $(L+\nu)$, the order of p being ν.
- The mutual information is maximized by making the known symbol clusters as small as possible and placing them such that the unknown symbol blocks are as "equal" as possible.

System Model

- The input sequence is assumed to consist of P known and N unknown symbols.
- $\mathcal{P}$ is the index set for the known symbols.
- The unknown symbols are assumed to be independent and identically distributed with zero mean and variance σ_s^2.
- Noise is assumed to be Additive, White and Gaussian of variance σ_n^2.
- The channel is assumed to be of order L

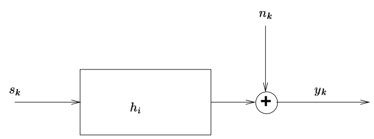

Figure 3: System Model

Generic Equalizer Structure

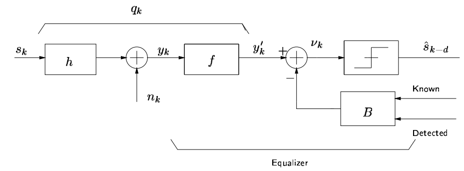

Figure 4: Equalizer Structure

- The forward filter is assumed to be time invariant and of order L_f.
- For the analysis we assume that all the past decisions are correct.

Problem Statement

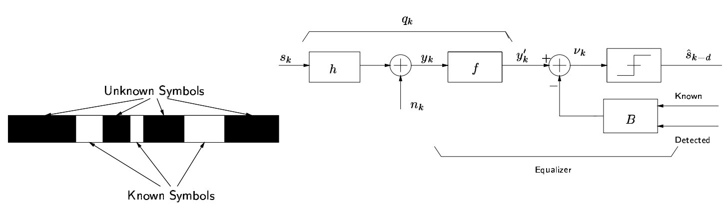

Figure 5: Equalizer Structure

- We consider the average mean square error (A-MSE) as the performance criterion

$$\mathcal{M}_a(\mathcal{P}, f, B) = \frac{1}{N} \sum_{(k-d)\notin\mathcal{P}} \mathrm{E}(\upsilon_k - s_{k-d})^2 \qquad (2)$$

- Our objective to perform the joint optimization of A-MSE

$$(\mathcal{P}^*, f^*, B^*) = \arg \min_{\mathcal{P}, f, B} \mathcal{M}_a(\mathcal{P}, f, B) \qquad (3)$$

Optimal Feedback Structure

Lemma 1 *For a given linear time invariant filter f and known symbols placement $\mathcal{P}$, the feedback structure B which subtracts the contribution of all the known and detected symbols to ISI in y'_k is optimal for any reasonable performance criterion.*

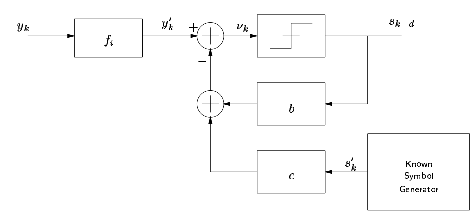

Figure 6: Precursor Cancellation DFE

Precursor Cancellation DFE

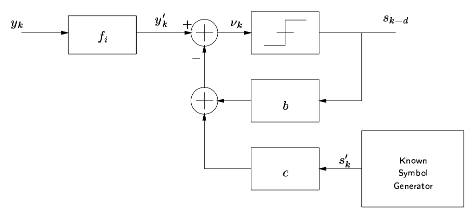

Figure 7: Precursor Cancellation DFE

- The feedback structure shown in the figure 6 is optimal if the coefficients of b and c are chosen as

$$b_i^* = \begin{cases} q_{i+d} & i = 1, \cdots, L + L_f - d \\ 0 & \text{otherwise} \end{cases}$$
$$c_i^* = \begin{cases} q_i & i = 0, \cdots, d-1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Optimal Symbol Placement

Theorem 1 *For a fixed decision delay d and a given f, if $\frac{P}{N+P} \leq \frac{1}{d+1} - \frac{d+(L_f-2d)^+}{(N+P)(d+1)}$ (where x^+ denotes the non-negative part of x) a symbol distribution $\mathcal{P}$ is optimal if and only if*

$$|x_i - x_j| > d, \forall x_i, x_j \in \mathcal{P} \text{ and } i \neq j.$$
$$x_1 \geq \max(d, L_f - d) \text{ and } x_P \leq N + P - d - 1 \quad (5)$$

If N is large enough to ignore the end effects and if the percentage of known symbols is less than $\frac{100}{(d+1)}\%$, then a distribution $\mathcal{P}$ is optimal if and only if

$$|x_i - x_j| > d, \forall x_i, x_j \in \mathcal{P} \text{ and } i \neq j. \quad (6)$$

An Example

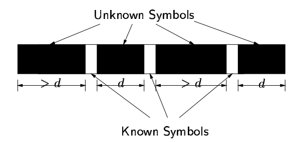

Figure 8: Example of a distribution that is Optimal

Optimal Symbol Placement(Continued)

- Conventionally, known symbols are placed in clusters
- Theorem 1 suggests that symbol detections can be improved if known symbols are distributed across the data stream.

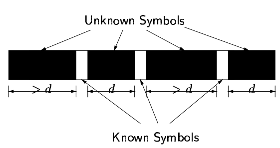

Figure 9: Example of a distribution that is Optimal

- Figure 8 shows one distribution that is optimal
- The optimal known symbol distribution is independent of the propagation channel.This property is crucial in broadcast applications

Generalized Optimal Symbol Placement

- We assume that the known symbols are placed in clusters of length at least α.
- The optimal placement of clusters for minimizing AMSE is to place the clusters at least d apart.

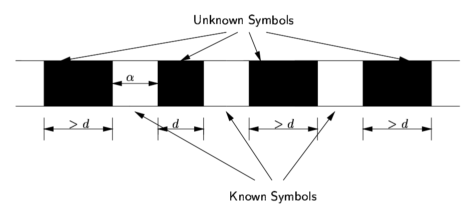

Figure 10: Example of a distribution that is Optimal

Optimal Forward Filter and Minimum AMSE

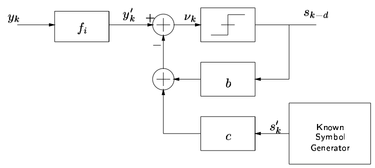

Figure 11: Precursor Cancellation DFE

$$\mathbf{f}^* = (\mathbf{H}_d^H \mathbf{\Lambda} \mathbf{H}_d + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_{L_f+1})^{-1} \mathbf{H}_d^H \mathbf{e}_d \quad (7)$$

$$\mathcal{M}_a(\mathcal{P}^*, f^*, B^*) = \mathbf{e}_d^H (\mathbf{\Lambda}^{-1} + \frac{\sigma_s^2}{\sigma_n^2} \mathbf{H}_d \mathbf{H}_d^H)^{-1} \mathbf{e}_d \quad (8)$$

where

$$\mathbf{H}_d = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ h_1 & h_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \\ h_d & h_{d-1} & \dots & h_{d-L_f} \end{bmatrix} \quad \mathbf{\Lambda} = \begin{bmatrix} 1-\frac{P}{N} & & & \\ & \ddots & & \\ & & 1-\frac{P}{N} & \\ & & & 1 \end{bmatrix} \quad \mathbf{e}_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

Simulation

- The joint optimization strategy was tested using the packet structure of the normal burst of GSM standard.
- There are 148 bits in the packet of which there are 116 information bits and 32 known symbols.
- $h = [\ 0.407\ 0.815\ 0.407]$, $L_f = 1$; $d = 1$

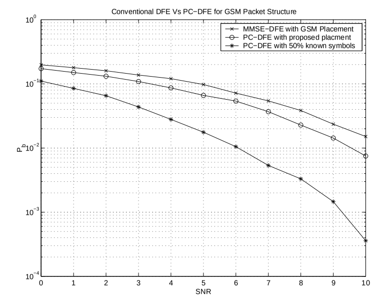

Figure 12: Comparison between Conventional strategies and proposed strategies

Conclusions

- The known symbols present in the data stream can be used to improve the performance of conventional DFE.
- We find that the performance of the receiver can be improved by jointly optimizing the equalizer and the position of known symbols.
- The algorithm of placement is quite simple.
- The gain from optimization is significant for systems with high percentage of known symbols.